Campaign Spending, Media Capture, and Political Accountability*

Vinícius Tiné†

Rafael Costa Lima‡

February, 2023

Abstract

Campaign spending and media influence are well-known instruments which are intensely utilized by special interest groups to affect political decisions, and the joint utilization of those instruments has been documented and discussed. However, both literatures on campaign spending and on media capture in political economy have developed independently, possibly missing important interconnections and therefore a broader understanding of the behavior of interest groups and their influence on democracy. In that context, we develop a theoretical analysis of the strategical relationship between campaign spending and media influence, as a step towards connecting those literatures. We develop a political agency framework which describes how special interest groups utilize campaign finance and the media to influence voters' information on the political process, in order to extract political rents. We show that when the marginal cost of political campaigns increase, campaign effort and media influence are strategical substitutes, and higher voter welfare is obtained. On the other hand, when the marginal cost of media influence increases, those instruments exhibit complementarity. In that case, higher voter welfare is not necessarily attained, and it might decrease if information levels are already too high. Finally, we analyse the effects of tighter campaign spending caps in the model, showing that higher voter welfare is obtained even when illegal campaign funds are used.

Keywords: Lobbying, Political Campaign, Rent-Seeking.

JEL: D72

---

*We thank Paulo Melo-Filho, Marcos Nakaguma, Antonin Macé, and Miguel Ballester for valuable conversations regarding this paper, as well as participants at NASMES 2022, Brazilian Political Economy Workshop 2021, LACEA-LAMES 2021, EEA-ESEM 2021, APET 2019 Strasbourg, and UFPE-PIMES 2019 for helpful comments.

†UFPE, Avenida dos Economistas, Cidade Universitária, Recife - PE, 50740-590, Phone +5581991059588, e-mail vinicius.tine@ufpe.br

‡UFPE, Avenida dos Economistas, Cidade Universitária, Recife - PE, 50740-590, Phone +5581992737992, e-mail rafael.costalima@ufpe.br

1 Introduction

In the first quarter of the twenty-first century, western civilization has experienced a growing sense of disbelief and dissatisfaction with democracy. Such a feeling emerges as a response from citizens to democracy's failure in diverse dimensions. On a global aspect, it has not supplied satisfactory solutions to the challenges imposed by the context of globalized capitalism, such as the increased inequality in income and wealth, the insecurity generated in labor markets by the reallocation of both productive activities and the work force in a global scale, and the augmented risk of social losses related to frequent financial crises, due to financial deregulation. On a local dimension, particularly in developing countries, democracy notably and chronically under-provides in essential components of a dignified societal life, such as educational quality, health care, urban sanitation, and social assistance programs. Why has democracy been failing in those respects? Instead of producing policies that are aligned with the best public interest, democracy is perceived by citizens as captured and to be promoting agendas connected to special interest groups.

Although that perception is related to the political system as a whole, it can be directed to a politician in particular. Then, the perception from voters that a given politician is committed to special interests may cause a serious damage on the public support for that politician, with particularly important consequences for electoral disputes. In fact, such belief was present in the 2016 presidential election in the United States, when Hillary Clinton was democrat party's candidate facing republicans' Donald Trump. Clinton's electoral proposal was to rewrite the rules of the american economy, in order to make it a more inclusive environment. As explained in Clinton and Kaine (2016), the current institutional framework, which denied opportunities and excluded minorities, was designed by a misguided political system that operates under the pressure of powerful interest groups. However, during the presidential campaign, Clinton herself was exposed on the wrong side of the game she has described. The website of Wikileaks released an overwhelming amount of documental evidence supporting the view that Hillary Clinton had a private position which was very different from the one expressed by her campaign platform, besides holding a degree of connection to special interests which would not be well tolerated by her electorate. For example, it was revealed that she was connected to foreign special interests in the selling of national energy assets[1] and had a very close relationship with the Wall Street financial sector[2]. Those revelations were a frequent subject during the campaign and may have been decisive for the victory of Donald Trump.

[1]The case presented by The New York Times (2015) has gained renewed interest after Wikileaks (2016a) revealed that the necessary approval for selling the strategic energy company Uranium One was closer to Hillary Clinton than previously imagined.

[2]Wikileaks (2016b) has published e-mail messages from within the Hillary Clinton circle which explicitly discuss the content of Clinton's paid speeches to financial firms, mainly from Wall Street. Those messages have revealed a clear concern from inside of Clinton's campaign about the possibility of that content becoming public. In those speeches, Clinton expressed the opinion that the financial sector itself should help the government on designing its own regulation, while reaffirming Wall Street money to be essential in her political campaign.

From the perspective of special interest groups, presidential campaign episode emphasizes the importance of investing resources to strengthen the image of their selected politicians, in order to pursue a more stable link to the political system. That process occurs in two moments in time, through different channels. First, through the electoral campaign industry, prior to the political term. Second, during the political term itself, through the myriad of forms of the media industry.

Historically, campaign spending and media influence were intensely utilized by special interest groups to affect political decisions, and the joint utilization of those instruments has been documented and discussed. A notorious example, mentioned by Petrova (2008), was the effort of extremely wealthy north-american families for abolishing the federal inheritance tax. From 1998 until 2001, when the temporary repeal of the inheritance tax was approved by the congress, eighteen families have spent an aggregate amount of (at least) 430.3 millions of dollars on different forms of activities against the inheritance tax, mainly information campaigns through the media, but also on campaign contributions. Finally, the most striking element of that story was that although the inheritance tax only applies to 1-2 per cent of americans, its abolishment was very popular among common citizens, as a strong suggestion of the potential of the information campaigns. The episode on the inheritance tax makes a clear case that, as a whole, the process of influence has an enormous potential to affect the mechanism of political accountability. Therefore, it raises evident questions regarding the regulation of both instruments.

Between the electoral campaign and media industries, it is the first one that has experienced a greater degree of regulatory change over the last decades. Among the OECD countries, a vast majority has adopted some form of regulation for electoral campaign spending (Scarrow (2007)) and the most common form has been the implementation of campaign spending caps. However, despite the existence of both theoretical and empirical work concerning the effects of campaign spending caps on important political outcomes, its effect on the utilization by interest groups of the alternative instrument, media influence, remains an open question. The lack of academic knowledge on that particular issue is due to the fact that both literatures on campaign spending and on media capture have developed independently, possibly missing important interconnections and therefore a broader understanding of the behavior of interest groups and their relation to democracy.

The objective of this paper is to develop a model which is capable of answering that question. We provide through the model a general description of how special interest groups utilize campaign finance and the mass media to influence democracy, in order to extract political rents. Therefore, the model depicts essential characteristics of the mechanics of democracy in the twenty-first century, and can be useful to answer numerous important questions related to political accountability. As a first step, we use the model to study the role of interest groups in elections and how it is related to political accountability. We show that the structure of the electoral campaign industry affects the degree of interference of interest groups on democracy, once both the costs of campaigns and the level of campaign spending caps compose the economic incentives facing interest groups, when those decide on campaign contributions and on the pressure for policy decisions. In particular, the model predicts that a tighter campaign spending cap improves political selection and political discipline, by limiting the influence of interest groups in elections and increasing the incentives of holding office through the implementation of good policies. Furthermore, that prediction is robust to the underlying possibility of illegal money contributions to campaigns, which is a common topic of concern.

Subsequently, we extend the model to clarify how interest groups operate in the mass media industry to strengthen their positions in politics. In the model, voters have imperfect information on the type of incumbent politicians. Then, when incumbents are connected to interest groups, those decide optimally on the degree of influence over the media industry, which is able to shape voters' awareness of rent-seeking politicians. That mechanism has two direct potential consequences. On one hand, political selection is affected, once those politicians may acquire improved chances for re-election. In that case, those very improved chances reduce shortsightedness in policy-making, which is a positive indirect effect on political discipline. Additionally, as the possibility of media capture raises the payoff associated to political power, it generates stronger incentives for interest groups to interfere in elections through campaign spending. Therefore, we use the model to underline a subtle connection between campaign finance, the mass media, and political accountability, in a setting with rent-seeking interest groups.

2 Related Literature

The model is connected with two different literatures. The first one discusses the role of campaign spending in democratic policy choice and originates from political science. Under a wide spectrum of settings, that literature has demonstrated how campaign spending by special interest groups may lead political decisions away from the median voter benchmark (Austen-Smith (1987), Baron (1994), Grossman and Helpman (1996), Prat (2002), Coate (2004)). The second literature related to our model studies the role of the mass media as a channel for the transmission of political information. Since different political actors (politicians, interest groups) have economic incentives to interfere on that mechanism, different authors have studied the conditions under which the capture of the mass media channel occurs and which characteristics of the society affect the probability of such event (Besley and Prat (2006), Stromberg (2004), Prat and Stromberg (2013), Petrova (2008), Beattie et al. (2021), Szeidl and Szucs (2021), Gehlbach and Sonin (2014), DellaVigna et al. (2016), Dyck, Volchkova, and Zingales (2008), Dyck and Zingales (2002)).

Those two literatures have undoubtedly enriched academic knowledge regarding the use from lobbies of the campaign spending and the mass media industries as channels of influence to politics. However, since those two investigations have been conducted separately, they may ignore important interconnections that recent work only started to unveil. For example, Bruce and Lima (2019) have presented empirical evidence about the effect of compulsory voting on the demand for political information. They show that compulsory voting has a positive impact on the probability of information consumption on the massa media, therefore potentially changing the composition of the population between informed and uninformed voters. In turn, one of the insights from the campaign spending literature is that such composition is an important determinant of campaign spending and policy decisions.

We present our model as a connection between those two literatures. We treat campaign spending and media influence as alternative instruments available for lobbies to influence the democratic process and demonstrate how lobbies' decisions on those two instruments can be interconnected. Then, our framework predicts that regulatory changes on campaign spending caps would affect the degree of media influence by interest groups.

Our model originates from the political agency framework. Since the 1970's, many valuable insights about the theme of political accountability were provided by the Barro-Ferejohn political agency tradition of models. In a seminal paper, Barro (1973) utilized a canonical political agency model to underline political income as a possible source of conflict between politicians and voters, leading to policy choices that are not in the best public interest. In that context, it was shown that periodic elections can be an effective mechanism to improve the equilibrium of the model in favour of voters. More than a decade later, Ferejohn (1986) offered an important complementary study to the former, extending the analysis to an infinite horizon case. Those two papers have set the basic ideas and the methodological foundations for the further development of the literature on political accountability. Besley and Case (1995) have shown that when voters take into account both the performances of their incumbent politician and of the incumbent politician in a neighboring jurisdiction, an interesting form of strategic interaction emerges between those, and it may affect political accountability. Coate and Morris (1995) examine the role of asymmetric information about policies and politicians in the choice of inefficient methods of resource redistribution to special interests. Persson, Roland, and Tabellini (1997) have analysed the role of separation of powers in political accountability. It was demonstrated that, under appropriated checks and balances conditions, separation of powers improves the accountability of elected officials.

3 Political Agency with Campaign Spending

Our basic political agency framework is very close to the forms presented in Berganza (2000) and Besley (2006). There are two time periods, viewed as political terms and indexed by $t \in \{1, 2\}$. The first political term starts with an electoral dispute between two politicians of different types: a congruent (or public-spirited) politician versus a dissonant (or egoistic) politician. The winner of the election is typically drawn randomly by nature in such a model and the probability of victory of the congruent politician is exogenously given as $\pi \in [0,1]$. Once declared the victor, the incumbent politician for the first term has to make a single policy decision, denoted by $e_1 \in \{0,1\}$. After this decision is made, voters decide whether to reelect the incumbent politician straight to the second term or to allow a second election to take place at the beginning of the second period, between two fresh politicians of once again different types. In principle, voters observe neither the incumbent politician's type nor his policy decision e_1 at that moment. However, at the moment of voters' decision on reelection, the incumbent politician's type is revealed to them with a probability of $\tau \in [0,1]$, while his policy choice e_1 is revealed to them with a probability of $\chi \in [0,1]$. Once the incumbent politician for the second term is determined, a second and final policy decision $e_2 \in \{0,1\}$ is made.

For each t, there is a state of nature $s_t \in \{0,1\}$, which is observed only by the incumbent politician, with equal probabilities for each possible value. Each agent's payoff depends on the observed policy decision, which is made with the state of nature in mind. Voters receive Δ if $e_t = s_t$ is observed or zero if $e_t = 1 - s_t$ is observed, and congruent politicians share voters' preferences. Hence, if the incumbent politician is congruent, he always chooses $e_t = s_t$. Dissonant politicians, on the other hand, have different preferences. If $e_t = s_t$, they receive zero. However, if $e_t = 1 - s_t$, they receive a dissonant rent, denoted by r_t, which is the realization of a random variable. r_t is continuously distributed on the interval $[0, R]$, has an expected value of μ, a pdf $g(\cdot)$, and a cdf $G(\cdot)$. The realization of r_t is revealed to the incumbent just before he has to decide on e_t. Moreover, r_1 and r_2 are independent. Finally, all agents discount the future with $\beta \in (0,1)$.

This is a standart model from the political agency tradition, which considers policy decisions to be determined by the strategic interaction between voters and politicians. In that view, the motivations of politicians play the main role in understanding public decisions.

This paper adopts a different perspective, although a paralell one, and sets interest groups, through their relationship with politicians, at the center of the discussion. In this first part of the paper, the connection from interest groups to politicians, and thus to policy choice, is campaign financing, which is assumed to affect the outcome of a very simple form of electoral competition.

Interest Groups and Campaign Financing

We now introduce the first distinctive ingredients from the basic setup. We assume that, instead of exogenously given, the probability of victory in the election t of the dissonant politician, denoted by $1 - \pi_t$, is a function of his campaign effort b_t and is given by

$$1 - \pi_t = \frac{b_t}{b_t + 1} \tag{1}$$

A lobby is now introduced as a new agent in the model. The lobby emerges as a part of a mutually advantageous deal with the dissonant politician: it provides funds for the dissonant politician's electoral campaign. If elected, the dissonant politician will follow the lobby's instructions about policy decisions. The amount of campaign effort b_t is chosen by the lobby, and each choice of b_t has an associated cost of $c \cdot b_t$ for the lobby, where $c > 0$. The amount $c \cdot b_t$ should be interpreted as the lobby's campaign spending on election t. It is assumed for convenience that $c < \mu$, in order to obtain interior solutions to the lobby's decision problems. Finally, the lobby shares the dissonant politician's preferences perfectly and also observes r_t at the moment when the policy decision on e_t must be made.

On the other hand, the congruent politician does not make any deal with the lobby. His probability of winning the election t will be a mere consequence of the lobby's decision of b_t, that is,

$$\pi_t = \frac{1}{b_t + 1} \tag{2}$$

It is useful at this point to outline the timing of events:

- *The lobby decides on the campaign effort* b_1
- *Nature reveals the dissonant rent* r_1
- *The lobby decides on the policy instruction* e_1
- *Voters' decision on re-election*
- *The lobby decides on the campaign effort* b_2
- *Nature reveals the dissonant rent* r_2
- *The lobby decides on the policy instruction* e_2

Then, we proceed to the equilibrium analysis of the model.

3.1 Equilibrium with Campaign Spending

We use backward induction to solve the model for a unique (bayesian) equilibrium. If the dissonant politician is the incumbent on the second term, the lobby always instructs $e_2 = 1 - s_2$. Therefore, the non-trivial part of the equilibrium analysis starts at the following decision.

The lobby decides on the campaign effort b_2

Suppose that the lobby must decide on the campaign effort for an electoral campaign on $t = 2$. His problem is

$$\max_{b_2 \geq 0} \frac{b_2}{b_2 + 1} \cdot \mu - c \cdot b_2 \tag{3}$$

The solution for that problem is

$$b_2^* = \sqrt{\frac{\mu}{c}} - 1 \tag{4}$$

Then, we define π_2^* as the equilibrium level of the probability π_2:

$$\pi_2^* = \frac{1}{b_2^* + 1} \tag{5}$$

We also define α as the lobby's expected payoff in the second period when b_2^* is chosen:

$$\alpha \equiv \frac{b_2^*}{b_2^* + 1} \cdot \mu - c \cdot b_2^* \tag{6}$$

Voters' decision on reelection

At the moment of voters' decision, there are two different scenarios which deserve special attention if the incumbent is dissonant. We analyse voters' decision in each scenario separately.

(i) Voters observe e_1 but do not observe the incumbent's type

Let λ be the probability that the dissonant incumbent chooses $e_1 = s_1$. λ is commonly referred to as the dissonant incumbent's discipline in political agency models. If the incumbent on the first term chooses $e_1 = 1 - s_1$, voters know that it can only be a dissonant politician, as congruent politicians always choose $e_1 = s_1$. Then, they do not reelect the incumbent and a second election takes place at $t = 2$. On the other hand, if $e_1 = s_1$ is chosen, it is rational for voters to re-elect the incumbent if the probability that the incumbent is congruent given that $e_1 = s_1$ was chosen is no less than ${\pi_2}^*$, that is, if

$$\frac{\pi_1 \chi}{\pi_1 \chi + (1 - \pi_1) \lambda (1 - \tau) \chi} \geq \pi_2^*, \tag{7}$$

We rearrange the inequality above and state it as follows:

Condition 1.

$$\lambda \cdot b_1 \cdot (1 - \tau) \leq {b_2}^* \tag{8}$$

In fact, as is shown in section 8.1, Condition 1 always holds. Hence, voters rationally re-elect the incumbent if $e_1 = s_1$ is chosen in scenario i.

(ii) Voters observe neither e_1 nor the incumbent's type

In this non-informative scenario, voters choose to re-elect the incumbent if the probability that the incumbent is congruent given that e_1 is not observed is no less than ${\pi_2}^*$, that is, if

$$\frac{\pi_1 (1-\chi)}{\pi_1 (1-\chi) + (1-\pi_1)(1-\chi)(1-\tau)[\lambda + (1-\lambda)]} \geq \pi_2^* \tag{9}$$

This can also be conveniently rearranged in the following way:

Condition 2.

$$b_1 (1-\tau) \leq b_2^* \tag{10}$$

There are three possibilities regarding Condition 2 (which may or may not hold), and each one of them leads to a different strategy being chosen for voters and thus to different equilibria in the model. First, if we have $b_1 (1-\tau) > b_2^*$, then it is rational for voters not to re-elect the incumbent in scenario ii. Second, if $b_1 (1-\tau) < b_2^*$, then voters do re-elect the incumbent in that scenario. Third and last, if $b_1 (1-\tau) = b_2^*$, then voters randomize between those previous two strategies, that is, they re-elect the incumbent with a probability $q \in [0,1]$. As will later be demonstrated, each one of these possibilities is consistent with specific values of the parameter τ.

Incumbent discipline: the determination of λ

Interestingly enough, incumbent discipline is insensitive to the assumption held on Condition 2. As an illustration, suppose that the dissonant politician is in power in $t = 1$ and that Condition 2 is not valid (and thus, that voters do not re-elect the incumbent in scenario ii; for the other two possibilities on Condition 2, the procedure for the determination of λ is the same as what follows). If the lobby chooses $e_1 = 1 - s_1$, his expected payoff is given by

$$r_1 + \beta\alpha \tag{11}$$

Recall that at this point the realization of r_1 is already known by the lobby. On the other hand, if the lobby chooses $e_1 = s_1$, his payoff is

$$(1-\chi)\beta\alpha + \chi[(1-\tau)\beta\mu + \tau\beta\alpha] \tag{12}$$

The lobby will then choose the congruent policy $e_1 = s_1$ if the first expression is lower than the second one, that is, if

$$r_1 < \chi (1 - \tau) \beta (\mu - \alpha) \tag{13}$$

Therefore, the lobby's discipline λ will be determined from the distribution of the dissonant rent r_1. If we define $\rho \equiv \chi (1 - \tau) \beta (\mu - \alpha)$, we have

$$\lambda = G(\rho), \tag{14}$$

The lobby decides on the campaign effort b_1

Differently from the incumbent's discipline, the choice of b_1 by the lobby is affected by the assumption on Condition 2 (and thus on voters' behavior). Therefore, we analyse the lobby's decision on b_1 for each one of the three possibilities on Condition 2. In what follows, there are three types of equilibria in the model, each one being consistent with different values of the informational parameter τ.

First, suppose that Condition 2 is not valid, that is, $b_1 (1 - \tau) > b_2^*$, and therefore voters do not re-elect the incumbent in scenario ii. For notational simplicity, let k^* be the lobby's expected payoff from winning the election at $t = 1$:

$$k^* \equiv \lambda [(1-\chi)\beta\alpha + \chi[\tau\beta\alpha + (1-\tau)\beta\mu]] + (1-\lambda) [(1-\chi) [E(r_1 \mid r_1 > \rho) + \beta\alpha] + \chi [E(r_1 \mid r_1 > \rho) + \beta\alpha]] \tag{15}$$

That expression can be simplified into

$$k^* = \lambda (\rho + \beta\alpha) + (1 - \lambda) [E(r_1 \mid r_1 > \rho) + \beta\alpha] \tag{16}$$

Then, the lobby decides on b_1 by solving:

$$\underset{b_1 \geq 0}{Max} \frac{b_1}{b_1 + 1} \cdot k^* - c \cdot b_1 \tag{17}$$

The solution to this problem has an already familiar form:

$$b_1^* = \sqrt{\frac{k^*}{c}} - 1 \tag{18}$$

As $b_1(1-\tau) > b_2^*$ was assumed and $b_1^* > b_2^*$ (see section 8.2), the present equilibrium with b_1^* is consistent only with relatively low values of τ, i.e., if $\tau < \tau^*$, for a given τ^* such that $b_1^*(1-\tau^*) = b_2^*$.

Second, suppose that Condition 2 is now strictly valid, that is, $b_1(1-\tau) < b_2^*$, and therefore voters do re-elect the incumbent in scenario ii. Once again, we define k^{**} as the lobby's expected payoff from winning the first election:

$$\begin{aligned} k^{**} &\equiv \lambda\{\rho + \chi\beta\alpha + (1-\chi)[(1-\tau)\beta\mu + \tau\beta\alpha]\} \\ &+ (1-\lambda)\{E(r_1 \mid r_1 > \rho) + \chi\beta\alpha + (1-\chi)[(1-\tau)\beta\mu + \tau\beta\alpha]\} \end{aligned} \tag{19}$$

Similarly, the lobby solves

$$\underset{b_1 \geq 0}{Max} \frac{b_1}{b_1+1} \cdot k^{**} - c \cdot b_1 \tag{20}$$

The solution is

$$b_1^{**} = \sqrt{\frac{k^{**}}{c}} - 1 \tag{21}$$

As $k^{**} > k^*$, we have $b_1^{**} > b_1^*$. Then, it is clear that given τ^{**} such that $b_1^{**}(1-\tau^{**}) = b_2^*$, we have $\tau^* < \tau^{**}$. Therefore, as $b_1(1-\tau) < b_2^*$ was assumed, this second equilibrium with b_1^{**} is consistent only with $\tau > \tau^{**}$.

Finally, the third possibility involves assuming that Condition 2 holds with equality, that is, $b_1(1-\tau) = b_2^*$. Then, as already mentioned, voters choose to re-elect the incumbent with a probability $q \in [0,1]$, a mixture of the two previous cases. Hence, the lobby solves:

$$\underset{b_1 \geq 0}{Max} \frac{b_1}{b_1+1} \cdot k^{***} - c \cdot b_1, \tag{22}$$

where $k^{***} \equiv (1-q)k^* + qk^{**}$. The solution is

$$b_1^{***} = \sqrt{\frac{k^{***}}{c}} - 1 \tag{23}$$

Letting τ^{***} be such that $b_1^{***}(1-\tau^{***}) = b_2^*$, it is clear that this third type of equilibrium is consistent only with $\tau \in [\tau^*, \tau^{**}]$.

The following result summarizes the equilibrium analysis above:

Proposition 1 (Equilibrium Analysis)**.** *The resulting equilibria of the political agency model with endogenous campaign spending involves a choice of campaign effort level* b_2^* *by the lobby at* $t = 2$*, and incumbent discipline* $\lambda = G(\rho)$*. Moreover, if* $\tau < \tau^*$*, then the lobby chooses the campaign effort level* b_1^* *in* $t = 1$*. If* $\tau > \tau^{**}$*, then* b_1^{**} *is choosen instead. Finally, if* $\tau \in [\tau^*, \tau^{**}]$*, then* b_1^{***} *is the resulting choice.*

Some points touched by the preceding equilibrium analysis relate to existing literature in political economy. Coate and Morris (1995) show that a combination of imperfect information on policy choice and on politicians' types may lead to inefficient incumbent behavior. That observation is close to the result presented above, as both parameters τ and χ affect the incumbent discipline λ. However, we offer a broader picture showing that those parameters also affect political selection through b_t. Therefore, a more precise claim on the relationship between information and political accountability requires a thorough welfare analysis, which is offered ahead.

Another key element is campaign spending, and the model connects it to both incumbent discipline and political selection. Also in the following welfare analysis, we show how changes in the electoral campaign industry environment affect political accountability via discipline and selection. This brings the paper close to the regulatory debate on campaign spending. Avis et al. (2019) find that a tighter campaign spending cap increases political competition and reduces the incumbency advantage. We contribute to that discussion by studying the gains of campaign spending caps not only in terms of political selection but also in terms of incumbent discipline. In that sense, we go ahead of the ongoing discussion and provide a richer perspective in terms of voter welfare explicitly. Moreover, Prat (2002) points out that regulating campaign spending may be ineffective, as regulations may be bypassed. For example, that might be done by using illegal money on campaigns. Nevertheless, we show that the effect of stricter campaign spending caps on political accountability is robust to such a possibility.

4 Electoral Campaigns, Information, and Political Accountability

We analyse how changes on the marginal cost of campaign effort and on voters' information affect political accountability. We do that by means of a voter welfare function, as in Besley (2006). The first step is to define voters' ex-ante welfare in the first term:

$$V_1(\lambda, \tau, \chi) = \chi [\pi_1 + (1 - \pi_1)\lambda] \Delta \tag{24}$$

In the same spirit, we define voters' ex-ante welfare in the second term:

$$V_2(\lambda,\tau,\chi) = \chi\left[\pi_1 + (1-\pi_1)(1-\lambda)\pi_2^* + (1-\pi_1)\lambda\tau\pi_2^*\right]\Delta \tag{25}$$

Then, we aggregate the previous two into voters' welfare function,

$$W(\lambda,\tau,\chi) = V_1(\lambda,\tau,\chi) + \beta V_2(\lambda,\tau,\chi) \tag{26}$$

Explicitly, we have

$$W(\lambda,\tau,\chi) = \left[\pi_1 + (1-\pi_1)\lambda\right]\chi\Delta + \beta\left[\pi_1 + (1-\pi_1)\pi_2^*\left[(1-\lambda)+\lambda\tau\right]\right]\chi\Delta \tag{27}$$

Using voters' welfare function, we present the following result.

Proposition 2 (Welfare Analysis)**.** *An increase either on c or on χ causes an improvement in voter welfare. On the other hand, the voter welfare effect of an increase on τ is indeterminate and may be negative, if information levels χ and τ are too high.*

Proof: See section 8.3.

First, an increase on c diminishes the lobby's incentives to enter electoral races, thus reducing equilibrium campaign effort and improving selection. Besides, it also improves incumbent discipline as an indirect effect: once running an election at $t = 2$ has become less attractive, it raises the lobby's incentives to behave at $t = 1$. The net effect on welfare is thus trivially positive.

The discipline effect of an increase on χ is also positive, as it reduces the lobby's opportunities of choosing $e_1 = 1 - s_1$ without being punished. In contrast, the selection effect of an increase on χ is not necessarily positive. In fact, it is negative if $\tau < \tau^*$. In spite of that possibility, the net welfare effect is also positive.

At last, an increase on τ makes the choice of $e_1 = s_1$ by the lobby a less profitable bet, thus reducing the incumbent's discipline. On the other hand, as it reduces the lobby's expected payoff from winning the first election, an increase on τ reduces b_1 and therefore improves selection. Then, we have effects on different directions and the net effect cannot be determined by means of the voter welfare function. However, if information levels χ and τ are too high, the selection gain of elevating τ disappears, and the negative discipline effect dominates the net result, which in that case is negative.

5 Campaign Spending Caps

This paper links interest groups to policy decisions, and that connection is through campaign spending. With effect, most of the OECD nations have implemented some form of regulation concerning campaign spending (see Scarrow, 2007). In that sense, a campaign spending cap is a very frequent choice of limiting the influence of interest groups into politics.

The model can be conveniently adapted to study the effect of campaign spending caps on political accountability. This is done in a very simple way by solving the model with constraints on the campaign spending $c \cdot b_t$. Specifically, we introduce a uniform binding campaign spending cap $\phi \cdot c \cdot b_2^*$ for $c \cdot b_t$, $t = 1, 2$, where $\phi \in (0, 1)$. As an illustration, the lobby's optimal choice of campaign effort on the second term now solves the following constrained problem:

$$\begin{aligned} &\underset{b_2 \geq 0}{Max} \frac{b_2}{b_2 + 1} \cdot \mu - c \cdot b_2 \\ &s.t. \;\; c \cdot b_2 \leq \phi \cdot c \cdot b_2^* \end{aligned} \tag{28}$$

The solution is

$$\overline{b_2} = \phi \cdot b_2^* \tag{29}$$

Since $\phi \in (0, 1)$, it is already clear that the introduction of a binding spending cap improves political selection in the second term. The remaining steps to determine the equilibrium of the model with campaign spending caps are identical to those described before and are presented in section 8.4.

The effects of introducing campaign spending caps are summarized in the next result, which also contains the main facts about the constrained model's equilibrium.

Proposition 3 (Campaign Spending Caps)**.** *The introduction of a uniform binding cap $\phi \cdot c \cdot b_2^*$ for $c \cdot b_t$ leads to a unique equilibrium in the constrained model, with improved political selection, incumbent discipline, and therefore voter welfare. Furthermore, a tighter campaign spending cap similarly leads to the same improvements.*

Proof: See Appendix.

The previous result contributes to the debate on campaign spending caps in several ways. In particular, it brings a positive discipline effect of spending caps to the discussion, which usually considers only selection effects, as for example in Avis et al. (2019). When deciding on policy e_1, the lobby anticipates the cap in an eventual future election and adopts an improved behavior in the present. The positive discipline and selection effects together form a positive net effect in terms of voter welfare, and this point adds precision to the discussion on caps.

5.1 Spending Caps and Illegal Campaign Money

The possibility of bypassing an existing spending cap is a common concern, pointed out for example in Prat (2002), and we address that issue using the constrained model. We show that the benefits of that form of regulation are robust to the use of non-declared campaign money by the lobby.

We let b_t' be an alternative available choice of campaign effort by the lobby. b_t' represents the role of non-declared or illegal campaign money and each choice of b_t' has an associated cost of $c' \cdot b_t'$. In order to take the higher risks of such practice into consideration, we assume that $c < c' < \mu$.

The lobby decides on both b_2 and b_2' by solving

$$\begin{aligned} &\underset{b_2, b_2' \geq 0}{Max} \frac{(b_2 + b_2')}{(b_2 + b_2') + 1} \cdot \mu - c \cdot b_2 - c' \cdot b_2' \\ &s.t.\ \ c \cdot b_2 \leq \phi \cdot c \cdot b_2^* \end{aligned} \tag{30}$$

Without the constraint, the solution would naturally be the benchmark b_2^* for b_2 and 0 for b_2'. However, if the constraint is on, the solution involves

$$\left(\overline{b_2} + b_2^{'*}\right) = \sqrt{\frac{\mu}{c'}} - 1, \tag{31}$$

where

$$\begin{aligned} &\overline{b_2} = \phi \cdot b_2^* \\ &b_2^{'*} = \sqrt{\frac{\mu}{c'}} - 1 - \phi \cdot b_2^* \end{aligned} \tag{32}$$

As $c < c'$, we have $\left(\overline{b_2} + b_2^{'*}\right) < b_2^*$. This simple exercise therefore predicts that the introduction of the spending cap reduces campaign spending on aggregate, despite generating illegal spending in equilibrium. Therefore, the positive selection effect of the spending cap is robust to the possibility of illegal campaign money.

6 Interest Groups and the Media

The political agency model with endogenous campaign spending can be conveniently extended to include endogenous influence of interest groups over the media in order to shape voters' information. That offers a richer perspective on political accountability, through which interest groups' decisions on campaign spending and on media influence may sometimes have a substitute relationship. For instance, the extended model predicts that a tighter campaign spending cap would increase interest groups' influence over the media, despite having a positive net effect on voters' welfare. The present exercise therefore connects the discussions on campaign spending caps and on media capture.

We let voters' information on the incumbent's type τ to become a function of the lobby's influence over the media, denoted by m:

$$\tau = \frac{1}{m+1} \tag{33}$$

In this extended version of the model, a low value of τ means that the lobby has a strong control of the media. The amount m is decided by the lobby just before voters' decision on re-election. The lobby would like τ to be the lowest possible, but the choice of m has an associated cost of $\eta \cdot m$, where $\eta > 0$. We assume for convenience that $\beta(\mu - \alpha) > \eta$, that is, the constant marginal cost η is not too high. For simplicity, we also assume that voters have perfect information about the policy choices, that is, $\chi = 1$.

The timing of events is extended by the lobby's decision on m:

- *The lobby decides on the campaign effort* b_1
- *Nature reveals the dissonant rent* r_1
- *The lobby decides on the policy instruction* e_1
- ***The lobby decides on the influence over the media*** m
- *Voters decide on re-election*
- *The lobby decides on the campaign effort* b_2
- *Nature reveals the dissonant rent* r_2
- *The lobby decides on the policy instruction* e_2

We present the equilibrium analysis for this extended model, which is very close to the basic one.

6.1 The Equilibrium with Media Influence and Additional Exercises

The lobby's decisions on e_2 and on b_2 are the same as the ones in the benchmark case, that is, $e_2 = 1 - s_2$ and b_2^*, respectively. Consequently, we proceed to voters' decision on re-election.

Voters decide on re-election

As in the baseline model, voters do not re-elect the incumbent if $e_1 = 1 - s_1$ is observed. However, if $e_1 = s_1$ is observed, it is once again rational for voters to re-elect the incumbent if the probability that the incumbent is congruent given that $e_1 = s_1$ was observed is no less than ${\pi_2}^*$, that is, if

$$\frac{\pi_1}{\pi_1 + (1 - \pi_1)\,\lambda\,(1 - \tau)} \geq \pi_2^*, \tag{34}$$

The inequality above can be rearranged into a familiar one:

$$\lambda \cdot b_1 \cdot (1 - \tau) \leq {b_2}^* \tag{35}$$

That is the same as Condition 1 and it also holds in the present context due to the same argument from section 8.1. Hence, voters do re-elect the incumbent if $e_1 = s_1$ is observed.

The lobby decides on the influence over the media m

If $e_1 = 1 - s_1$ is chosen, the (dissonant) incumbent is not re-elected and the lobby's payoff is $r_1 + \beta\alpha$. But, if $e_1 = s_1$ is chosen, the lobby's payoff depends on his choice of m and can be expressed as

$$\max_{m \geq 0} \left\{ (1 - \tau)\,\beta\mu + \tau\beta\alpha - \eta m \right\} \tag{36}$$

That expression is maximized by

$$m^* = \sqrt{\frac{\beta\,(\mu - \alpha)}{\eta}} - 1 \tag{37}$$

Then, we define τ^* as the resulting level of voters' information on the incumbent's type:

$$\tau^* \equiv \frac{1}{m^* + 1} \tag{38}$$

Incumbent discipline: the determination of λ

The lobby chooses the congruent policy $e_1 = s_1$ if

$$r_1 < \beta(1-\tau^*)(\mu-\alpha) - \eta m^* \tag{39}$$

Once again, the lobby's discipline $\widehat{\lambda}$ will be determined from the distribution of the dissonant rent r_1. If we define $\widehat{\rho} \equiv \beta(1-\tau^*)(\mu-\alpha) - \eta m^*$, we have

$$\widehat{\lambda} = G(\widehat{\rho}), \tag{40}$$

The lobby decides on the campaign effort b_1

Let $\widehat{k}$ be the lobby's expected payoff from winning the first election in this extended model:

$$\widehat{k} \equiv \widehat{\lambda}\left[(1-\tau^*)\beta\mu + \tau^*\beta\alpha - \eta m^*\right] + (1-\widehat{\lambda})\left[E(r_1 \mid r_1 > \widehat{\rho}) + \beta\alpha\right] \tag{41}$$

That can be simplified into

$$\widehat{k} = \widehat{\lambda}\widehat{\rho} + (1-\widehat{\lambda})E(r_1 \mid r_1 > \widehat{\rho}) + \beta\alpha \tag{42}$$

The lobby decides on b_1 by solving:

$$\underset{b_1 \geq 0}{Max} \frac{b_1}{b_1+1} \cdot \widehat{k} - c \cdot b_1 \tag{43}$$

The solution is:

$$\widehat{b_1} = \sqrt{\frac{\widehat{k}}{c}} - 1 \tag{44}$$

The following result summarizes the equilibrium analysis of the extended model:

Proposition 4 (Equilibrium Analysis with Media Influence)**.** *The equilibrium of the political agency model with endogenous campaign spending and media influence involves the baseline choice of campaign effort level* b_2^* *by the lobby in* $t = 2$*, a choice of media influence* m^* *by the lobby in* $t = 1$*, incumbent discipline* $\widehat{\lambda} = G(\widehat{\rho})$*, and a choice of campaign effort* $\widehat{b_1}$ *by the lobby in* $t = 1$*.*

The equilibrium analysis of the extended model depicts the interdependence between the lobby's decisions on campaign spending and on media influence. When deciding on the campaign effort b_1, the lobby takes into account the possibilities of strengthening its position by influencing the media in the future. On the other hand, when deciding on the media influence m, the lobby has under consideration a possible future decision on campaign effort b_2 and the expected payoff related to that decision. The next result clarifies further how connected are campaign spending and media influence as tools for controlling democratic politics in the model.

Proposition 5 (Comparative Statics and Welfare Analysis with Media Influence)**.** *An increase on c causes an increase on media influence m, while an increase on η causes a decrease on campaign effort b_1. Moreover, an increase on c improves voter welfare, whereas the voter-welfare effect of an increase on η is indeterminate and may be negative, if η is too high.*

Proof: See Appendix

Proposition 5 is predominantly an extension of Proposition 2 to the present context with endogenous media influence. However, the content of its first part offers new and important insights. It predicts in particular how the lobby would adapt to worse circumstances in the campaign industry by increasing its control over the media. An increase on c lowers the expected payoff from running an election in the second term. Therefore, the lobby has a stronger incentive to avoid losing power. Together with improving discipline, choosing a larger influence over the media is a way of achieving a more stable position in office.

The mechanism of substitution just described also inserts a trade-off into an eventual decision regarding campaign spending caps by a regulatory agency. We elaborate on that point with the following result.

Proposition 6 (Campaign Spending Caps with Media Influence)**.** *A tighter campaign spending cap in the extended model leads to improved voter welfare, although it generates more influence over the media by the lobby.*

Proof: Appendix

Proposition 6 states that the net welfare effect of a tighter spending cap remains positive in the extended model with media influence. Moreover, the second part of the result predicts how the lobby would adapt to the new environment by increasing the influence over the media. Therefore, the result has the two-fold importance of connecting the discussion on campaign spending caps to the economics of the media and contributing to the knowledge about the behavior of interest groups.

7 Conclusion

We have presented a framework with important features which deserve to be underlined. The model accomplishes its task as a general description of how interest groups influence twenty-first century democracy through campaign spending and media influence. By doing so, it combines elements from two relevant discussions, which in our view should be treated in conjunction.

The model is suited for different exercises. As demonstrated, it can be used to study the effects of campaign spending caps on voter welfare and on the behavior of interest groups. Since we examine decisions on both campaign spending and media influence within the same framework, it was possible to bring new insights on how those decisions are connected.

We also believe that the analysis leads us to important remaining questions regarding political accountability. In a setting where interest groups have a strong control over democracy, when to expect good policy decisions to emerge consistently over time? One possible answer is that it can be expected from lobbies to push for good policies when there is a relevant meeting of interests. We believe that studying such scenarios in specific policy dimensions might contribute to a better understanding of political accountability.

8 Appendix

8.1 Proof of Condition 1

Suppose that Condition 1 is not valid, that is, $\lambda \cdot b_1 \cdot (1-\tau) > {b_2}^*$. Then, voters do not re-elect the incumbent in scenario i. In turn, the lobby has no incentives for discipline, and that leads to $\lambda = 0$. By assumption, we have $0 \cdot b_1 \cdot (1-\tau) > {b_2}^*$, i.e., $0 > {b_2}^*$, a contradiction.

8.2 Proof of ${b_1}^* > {b_2}^*$

Since $b_1^* = \sqrt{\frac{k^*}{c}} - 1$ and $b_2^* = \sqrt{\frac{\mu}{c}} - 1$, the result is established by showing that $k^* > \mu$. First, we can rewrite k^* as $k^* = \lambda\rho + (1-\lambda) E(r_1 \mid r_1 > \rho) + \beta\alpha$. Then, we define an auxiliary expression $k_0 \equiv \lambda\rho + (1-\lambda) E(r_1 \mid r_1 > \rho)$, in order to write $k^* = k_0 + \beta\alpha$. This is done because showing that $k_0 > \mu$ turns out to be a convenient way to demonstrate that $k^* > \mu$. That requires two steps. We start by establishing a result on conditional distributions:

Lemma 1. $E(r_t \mid r_t \in [a,b]) = \frac{\int_a^b r_t \cdot g(r_t) dr_t}{G(b)-G(a)}; a, b \in [0,R]; a < b$

Proof. Let $\theta = Pr\left(r_t \in [a,b]\right) = G\left(b\right) - G\left(a\right)$ and let z be a continuous random variable, with support $[a,b] \subset [0,R]$, $a < b$, and pdf $f\left(z\right)$, such that the events $z \in [a',b']$ and $r_t \in [a',b']$ are equivalent, that is,

$$Pr\left(z \in [a',b']\right) = \widetilde{\theta} \cdot Pr\left(r_t \in [a',b']\right), \tag{45}$$

for all $a',b' \in [a,b]$ with $a' \leq b'$, and $\widetilde{\theta} \in \mathbb{R}$. This can be written as

$$Pr\left(z \in [a',b']\right) = \widetilde{\theta} \cdot \int_{a'}^{b'} g\left(r_t\right) dr_t \tag{46}$$

Note that, in particular,

$$Pr\left(z \in [a,b]\right) = \widetilde{\theta} \cdot \int_{a}^{b} g\left(r_t\right) dr_t = 1, \tag{47}$$

i.e.,

$$\widetilde{\theta} \cdot \theta = 1, \tag{48}$$

or,

$$\widetilde{\theta} = \frac{1}{\theta} \tag{49}$$

Then, we have

$$Pr\left(z \in [a',b']\right) = \frac{1}{\theta} \cdot Pr\left(r_t \in [a',b']\right), \tag{50}$$

i.e.,

$$\int_{a'}^{b'} f\left(z\right) dz = \frac{1}{\theta} \cdot \int_{a'}^{b'} g\left(r_t\right) dr_t, \tag{51}$$

or,

$$\int_{a'}^{b'} f\left(z\right) dz = \int_{a'}^{b'} \frac{1}{\theta} \cdot g\left(r_t\right) dr_t, \tag{52}$$

for all $a',b' \in [a,b]$ with $a' \leq b'$. Then,

$$f\left(z\right) = \frac{g\left(r_t\right)}{\theta} \tag{53}$$

As, for all $a',b' \in [a,b]$ with $a' \leq b'$, $z \in [a',b']$ and $r_t \in [a',b']$ are equivalent events, we have

$$E\left(z \mid z \in [a',b']\right) = E\left(r_t \mid r_t \in [a',b']\right) \tag{54}$$

In particular,

$$E\left(z \mid z \in [a,b]\right) = E\left(r_t \mid r_t \in [a,b]\right), \tag{55}$$

i.e.,

$$E\left(z\right) = E\left(r_t \mid r_t \in [a,b]\right) \tag{56}$$

However, as

$$E(z) = \int_a^b z \cdot f(z)\, dz = \int_a^b \frac{1}{\theta} \cdot r_t \cdot g(r_t)\, dr_t, \tag{57}$$

we have

$$E(r_t \mid r_t \in [a,b]) = \frac{\int_a^b r_t \cdot g(r_t)\, dr_t}{\theta} \tag{58}$$

As $\theta = G(b) - G(a)$, the result is established. □

The preceding result is very useful in the present context, as it allows us to obtain alternative expressions for k_0:

Corollary 1. *k_0 can be expressed as:*

(i) $k_0 = \lambda\rho + \int_\rho^R r_1 g(r_1)\, dr_1$

(ii) $k_0 = R - \int_\rho^R G(r_1)\, dr_1$

Proof. Part (i) follows from a direct application of Lemma 1 on the definition of k_0, and part (ii) follows from applying integration by parts on part (i). □

Now, take $k_0 = \lambda\rho + \int_\rho^R r_1 g(r_1)\, dr_1$ from Corollary 1, part (i). From Lemma 1, we have

$$\begin{aligned} &\lambda \cdot E(r_1 \mid r_1 < \rho) + \int_\rho^R r_1 g(r_1)\, dr_1 = \lambda \cdot \frac{\int_0^R r_1 g(r_1)\, dr_1}{G(\rho) - G(0)} + \int_\rho^R r_1 g(r_1)\, dr_1 = \\ &= \int_0^R r_1 g(r_1)\, dr_1 + \int_\rho^R r_1 g(r_1)\, dr_1 = \mu \end{aligned} \tag{59}$$

As $\rho > E(r_1 \mid r_1 < \rho)$, we have $k_0 > \mu$. Therefore, we have $k^* > \mu$ and hence ${b_1}^* > {b_2}^*$.

8.3 Proof of Proposition 2

This proof builds upon a set of auxiliary results on comparative statics. We begin with the comparative-statics effects of an increase on c.

Lemma 2. *The comparative-statics effects of an increase on c*

(i) $\frac{\partial b_2^*}{\partial c} < 0$

(ii) $\frac{\partial \alpha}{\partial c} < 0$

(iii) $\frac{\partial \rho}{\partial c} > 0$

(iv) $\frac{\partial b_1^*}{\partial c} < 0$, $\frac{\partial b_1^{**}}{\partial c} < 0$, *and* $\frac{\partial b_1^{***}}{\partial c} < 0$.

Proof. (Part (i))

We know that $b_2^* = \sqrt{\frac{\mu}{c}} - 1$. Then,

$$\frac{\partial b_2^*}{\partial c} = -\frac{\sqrt{\mu}}{2c^{\frac{3}{2}}} < 0 \tag{60}$$

(Part (ii))

α can be written as

$$\alpha = \mu - 2\sqrt{\mu c} + c = (\mu - c)^2 \tag{61}$$

Then, we have

$$\frac{\partial \alpha}{\partial c} = -2(\mu - c) < 0 \tag{62}$$

(Part (iii))

We know that $\rho = \chi(1-\tau)\beta(\mu - \alpha)$. Then, we have

$$\frac{\partial \rho}{\partial c} = \frac{\partial \rho}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial c} = 2\beta\chi(1-\tau)(\mu - c) > 0 \tag{63}$$

(Part (iv))

We know that $b_1^* = \sqrt{\frac{k^*}{c}} - 1$. Then, we have

$$\frac{\partial b_1^*}{\partial c} = -\frac{\sqrt{k^*}}{2c^{\frac{3}{2}}} < 0 \tag{64}$$

The proofs for both $\frac{\partial b_1^{**}}{\partial c} < 0$ and $\frac{\partial b_1^{***}}{\partial c} < 0$ are very similar.

□

Second, we present the comparative-statics effects of an increase on χ.

Lemma 3. *The comparative-statics effects of an increase on χ*

(i) $\frac{\partial \rho}{\partial \chi} > 0$

(ii) $\frac{\partial b_1^*}{\partial \chi} > 0$, $\frac{\partial b_1^{**}}{\partial \chi} < 0$, *and the sign of* $\frac{\partial b_1^{***}}{\partial \chi}$ *may be positive or negative. If q is sufficiently low (high), then* $\frac{\partial b_1^{***}}{\partial \chi}$ *is positive (negative).*

Proof. (Part (i))

Since $\rho = \chi(1-\tau)\beta(\mu-\alpha)$, we have

$$\frac{\partial \rho}{\partial \chi} = (1-\tau)\beta(\mu-\alpha) > 0 \tag{65}$$

(Part (ii))

(Step 1: $\frac{\partial b_1^*}{\partial \chi} > 0$)

First, from $b_1^* = \sqrt{\frac{k^*}{c}} - 1$, we have

$$\frac{\partial b_1^*}{\partial \chi} = \frac{\partial b_1^*}{\partial k^*} \cdot \frac{\partial k^*}{\partial \chi} = \frac{1}{2\sqrt{k^* c}} \cdot \frac{\partial k^*}{\partial \chi} \tag{66}$$

Therefore, in order to determine the sign of $\frac{\partial b_1^*}{\partial \chi}$, we need to study the sign of $\frac{\partial k^*}{\partial \chi}$. From section 8.2, we know that $k^* = k_0 + \beta\alpha$, where $k_0 \equiv \lambda\rho + (1-\lambda)\, E\,(r_1 \mid r_1 > \rho)$. On the other hand, from Corollary 1, part (ii) at section 8.2, we know that k_0 can be expressed as $k_0 = R - \int_\rho^R G\,(r_1)\, dr_1$. Then, we have $k^* = R - \int_\rho^R G\,(r_1)\, dr_1 + \beta\alpha$ and

$$\frac{\partial k^*}{\partial \chi} = -\frac{\partial \left[\int_\rho^R G\,(r_1)\, dr_1\right]}{\partial \chi} = -\left[-\int_\rho^{\rho + \frac{\partial \rho}{\partial \chi}} G\,(r_1)\, dr_1\right] > 0 \tag{67}$$

Hence, $\frac{\partial b_1^*}{\partial \chi} > 0$.

(Step 2: $\frac{\partial b_1^{**}}{\partial \chi} < 0$)

From $b_1^{**} = \sqrt{\frac{k^{**}}{c}} - 1$, we have

$$\frac{\partial b_1^{**}}{\partial \chi} = \frac{\partial b_1^{**}}{\partial k^{**}} \cdot \frac{\partial k^{**}}{\partial \chi} = \frac{1}{2\sqrt{k^{**} c}} \cdot \frac{\partial k^{**}}{\partial \chi} \tag{68}$$

In order to determine the sign of $\frac{\partial b_1^{**}}{\partial \chi}$, we need to study the sign of $\frac{\partial k^{**}}{\partial \chi}$. We rearrange the definition of k^{**} to rewrite it as $k^{**} = k_0 + \chi\beta\alpha + (1-\chi)\,[(1-\tau)\,\beta\mu + \tau\beta\alpha]$. Then, from Corollary 1, part (ii), we have

$$k^{**} = R - \int_\rho^R G\,(r_1)\, dr_1 + \chi\beta\alpha + (1-\chi)\,[(1-\tau)\,\beta\mu + \tau\beta\alpha] \tag{69}$$

Therefore,

$$\frac{\partial k^{**}}{\partial \chi} = \int_\rho^{\rho + \frac{\partial \rho}{\partial \chi}} G\,(r_1)\, dr_1 - \beta\,(1-\tau)\,(\mu-\alpha) \tag{70}$$

The sign of the above expression is not immediately clear. However, from basic properties of statistical distributions, we have

$$\int_\rho^{\rho + \frac{\partial \rho}{\partial \chi}} G\,(r_1)\, dr_1 < \left[\left(\rho + \frac{\partial \rho}{\partial \chi}\right) - \rho\right] \cdot 1 = \frac{\partial \rho}{\partial \chi} \tag{71}$$

Since $\frac{\partial \rho}{\partial \chi} = \beta(1-\tau)(\mu - \alpha)$, we obtain

$$\frac{\partial k^{**}}{\partial \chi} < 0 \tag{72}$$

Therefore,

$$\frac{\partial b_1^{**}}{\partial \chi} < 0 \tag{73}$$

(Step 3: $\frac{\partial b_1^{***}}{\partial \chi}$ has an indeterminate sign)

Since $b_1^{***} = \sqrt{\frac{k^{***}}{c}} - 1$ and $k^{***} = (1-q)k^* + qk^{**}$, we have

$$\frac{\partial b_1^{***}}{\partial \chi} = \frac{\partial b_1^{***}}{\partial k^{***}} \cdot \frac{\partial k^{***}}{\partial \chi} = \frac{1}{2\sqrt{k^{***}c}} \cdot \left[(1-q)\frac{\partial k^*}{\partial \chi} + q\frac{\partial k^{**}}{\partial \chi}\right] \tag{74}$$

As $\frac{\partial k^*}{\partial \chi} > 0$ and $\frac{\partial k^{**}}{\partial \chi} < 0$, we have $\frac{\partial b_1^{***}}{\partial \chi} > 0$ if q is sufficiently low and $\frac{\partial b_1^{***}}{\partial \chi} < 0$ if q is sufficiently high.

□

Third, we present the comparative-statics effects of an increase on τ.

Lemma 4. *The comparative-statics effects of an increase on* τ

(i) $\frac{\partial \rho}{\partial \tau} < 0$

(ii) $\frac{\partial b_1^*}{\partial \tau} < 0$, $\frac{\partial b_1^{**}}{\partial \tau} < 0$, *and* $\frac{\partial b_1^{***}}{\partial \tau} < 0$.

Proof. (Part (i))

Since $\rho = \chi(1-\tau)\beta(\mu-\alpha)$, we have

$$\frac{\partial \rho}{\partial \tau} = -\chi\beta(\mu - \alpha) < 0 \tag{75}$$

(Part (ii))

(Step 1: $\frac{\partial b_1^*}{\partial \tau} < 0$)

From $b_1^* = \sqrt{\frac{k^*}{c}} - 1$, we have

$$\frac{\partial b_1^*}{\partial \tau} = \frac{\partial b_1^*}{\partial k^*} \cdot \frac{\partial k^*}{\partial \tau} = \frac{1}{2\sqrt{k^*c}} \cdot \frac{\partial k^*}{\partial \tau} \tag{76}$$

We know that $k^* = R - \int_\rho^R G(r_1)\,dr_1 + \beta\alpha$. Therefore,

$$\frac{\partial k^*}{\partial \tau} = -\frac{\partial\left[\int_\rho^R G(r_1)\,dr_1\right]}{\partial \tau} = -\int_{\rho+\frac{\partial \rho}{\partial \tau}}^{\rho} G(r_1)\,dr_1 < 0 \tag{77}$$

Hence,

$$\frac{\partial b_1^*}{\partial \tau} < 0 \tag{78}$$

(Step 2: $\frac{\partial b_1^{**}}{\partial \tau} < 0$)

From $b_1^{**} = \sqrt{\frac{k^{**}}{c}} - 1$, we have

$$\frac{\partial b_1^{**}}{\partial \tau} = \frac{\partial b_1^{**}}{\partial k^{*}*} \cdot \frac{\partial k^{**}}{\partial \tau} = \frac{1}{2\sqrt{k^{**}c}} \cdot \frac{\partial k^{**}}{\partial \tau} \tag{79}$$

We know that $k^{**} = R - \int_\rho^R G(r_1)\, dr_1 + \chi\beta\alpha + (1-\chi)\left[(1-\tau)\beta\mu + \tau\beta\alpha\right]$. Therefore,

$$\frac{\partial k^{**}}{\partial \tau} = \frac{\partial k^{*}}{\partial \tau} - (1-\chi)\beta(\mu - \alpha) < 0 \tag{80}$$

Hence,

$$\frac{\partial b_1^{**}}{\partial \tau} < 0 \tag{81}$$

(Step 3: $\frac{\partial b_1^{***}}{\partial \tau} < 0$)

Since $b_1^{***} = \sqrt{\frac{k^{***}}{c}} - 1$ and $k^{***} = (1-q)\, k^* + qk^{**}$, we have

$$\frac{\partial b_1^{***}}{\partial \tau} = \frac{\partial b_1^{***}}{\partial k^{***}} \cdot \frac{\partial k^{***}}{\partial \tau} = \frac{1}{2\sqrt{k^{***}c}} \cdot \left[(1-q)\frac{\partial k^{*}}{\partial \tau} + q\frac{\partial k^{**}}{\partial \tau}\right] < 0 \tag{82}$$

□

With Lemma 2-4, we are able to calculate the selection and discipline effects of increases on c, χ, and τ:

Corollary 2. *Selection and Discipline Effects*

(i) An increase on c causes increases in both selection and discipline.

(ii) An increase on χ causes an increase in discipline, but its effect on selection may be either positive or negative. If $\tau < \tau^*$, then the selection effect is positive. If $\tau > \tau^{**}$, then the selection effect is negative. Finally, if $\tau^* < \tau < \tau^{**}$, the selection effect may be either positive or negative. In fact, if q is sufficiently low (high), then the selection effect is positive (negative).

(iii) An increase on τ causes an increase in selection and a decrease in discipline.

Proof. (Part (i))

(Discipline Effect)

We know that discipline is given by $\lambda = G(\rho)$ and that $\frac{\partial \rho}{\partial c} > 0$. Then,

$$\frac{\partial \lambda}{\partial c} = \frac{\partial G(\rho)}{\partial \rho} \cdot \frac{\partial \rho}{\partial c} > 0 \tag{83}$$

(Selection Effect)

We know, that selection in the first term is (partially) given by $\pi_1^* = \frac{1}{b_1^*+1}$. Since $\frac{\partial b_1^*}{\partial c} < 0$, we have

$$\frac{\partial \pi_1^*}{\partial c} = \frac{\partial \pi_1^*}{\partial b_1^*} \cdot \frac{\partial b_1^*}{\partial c} = -\frac{1}{(b_1^*+1)^2} \cdot \frac{\partial b_1^*}{\partial c} > 0 \tag{84}$$

The same procedure can be used to show that $\frac{\partial \pi_1^{**}}{\partial c} > 0$ and $\frac{\partial \pi_1^{***}}{\partial c} > 0$. Since $\frac{\partial \pi_2^*}{\partial c} > 0$, we conclude that the selection effect of an increase on c is positive.

(Part (ii))

(Discipline Effect)

From $\lambda = G(\rho)$ and $\frac{\partial \rho}{\partial \chi} > 0$, we have

$$\frac{\partial \lambda}{\partial \chi} = \frac{\partial G(\rho)}{\partial \rho} \cdot \frac{\partial \rho}{\partial \chi} > 0 \tag{85}$$

(Selection Effect)

Since there is no selection effect on the second term, the sign of the selection effect on the first term determines the sign of the selection effect. For $\tau < \tau^*$, we have

$$\frac{\partial \pi_1^*}{\partial \chi} = \frac{\partial \pi_1^*}{\partial b_1^*} \cdot \frac{\partial b_1^*}{\partial \chi} = -\frac{1}{(b_1^*+1)^2} \cdot \frac{\partial b_1^*}{\partial \chi} < 0, \tag{86}$$

that is, the selection effect on the first term is negative. On the other hand, for $\tau > \tau^{**}$, we have

$$\frac{\partial \pi_1^{**}}{\partial \chi} = \frac{\partial \pi_1^{**}}{\partial b_1^{**}} \cdot \frac{\partial b_1^{**}}{\partial \chi} = -\frac{1}{(b_1^{**}+1)^2} \cdot \frac{\partial b_1^{**}}{\partial \chi} > 0, \tag{87}$$

that is, the selection effect on the first term is positive. Finally, for $\tau^* < \tau < \tau^{**}$, the sign of the selection effect on the first term depends on q:

$$\frac{\partial \pi_1^{***}}{\partial \chi} = \frac{\partial \pi_1^{***}}{\partial b_1^{***}} \cdot \frac{\partial b_1^{***}}{\partial \chi} = -\frac{1}{(b_1^{***}+1)^2} \cdot \frac{\partial b_1^{***}}{\partial \chi} \cdot \frac{1}{2\sqrt{k^{***}c}} \cdot \left[(1-q)\frac{\partial k^*}{\partial \chi} + q\frac{\partial k^{**}}{\partial \chi}\right] \tag{88}$$

Since $\frac{\partial k^*}{\partial \chi} > 0$ and $\frac{\partial k^{**}}{\partial \chi} < 0$, it is clear that if q is sufficiently low (high), then the selection effect on the first term is positive (negative).

(Part (iii))

(Discipline Effect)

From $\lambda = G(\rho)$ and $\frac{\partial \rho}{\partial \tau} < 0$, we have

$$\frac{\partial \lambda}{\partial \tau} = \frac{\partial G(\rho)}{\partial \rho} \cdot \frac{\partial \rho}{\partial \tau} < 0 \tag{89}$$

(Selection Effect)

Since $\pi_1^* = \frac{1}{b_1^*+1}$, we have

$$\frac{\partial \pi_1^*}{\partial \tau} = \frac{\partial \pi_1^*}{\partial b_1^*} \cdot \frac{\partial b_1^*}{\partial \tau} = -\frac{1}{(b_1^*+1)^2} \cdot \frac{\partial b_1^*}{\partial \tau} > 0 \tag{90}$$

The same procedure can be used to show that $\frac{\partial \pi_1^{**}}{\partial \tau} > 0$ and $\frac{\partial \pi_1^{***}}{\partial \tau} > 0$. Since $\frac{\partial \pi_2^*}{\partial \tau} = 0$, we conclude that the selection effect of an increase on τ is positive.

□

The last result states first that both the selection effect and the discipline effect of an increase on c are positive. Therefore, its net effect on voter welfare is trivially positive. However, for both increases on χ and τ, the discipline and selection effects do not necessarily have the same signs. Thus, in order to determine the net effect of each of those increases on voters' welfare, we use the welfare function from section 4:

$$W(\lambda, \tau, \chi) = [\pi_1 + (1-\pi_1)\lambda]\chi\Delta + \beta[\pi_1 + (1-\pi_1)\pi_2^*[(1-\lambda)+\lambda\tau]]\chi\Delta \tag{91}$$

The welfare effect of an increase in χ is positive:

From Corollary 2, for $\tau > \tau^{**}$, both the discipline effect and the selection effect of an increase on χ are positive. Hence, the net welfare effect is trivially positive. On the other hand, for $\tau < \tau^*$, the selection effect becomes negative, and thus we need to use the welfare function to study the net effect. Then,

$$\begin{aligned} &\frac{\partial W}{\partial \chi} = \Delta[\pi_1^* + (1-\pi_1^*)\lambda + \beta[\pi_1^* + (1-\pi_1^*)\pi_2^*(1-\lambda+\lambda\tau)]] \\ &+ \chi\Delta\frac{\partial \pi_1^*}{\partial \chi}[1-\lambda+\beta[1-\pi_2^*(1-\lambda+\lambda\tau)]] + \chi\Delta\frac{\partial \lambda}{\partial \chi}(1-\pi_1^*)[1-\beta\pi_2^*(1-\tau)] \end{aligned} \tag{92}$$

We can rearrange the above expression to obtain:

$$\begin{aligned} &\frac{\partial W}{\partial \chi} = \Delta[\pi_1^*(1+\beta) + (1-\pi_1^*)[\lambda + \beta\pi_2^*(1-\lambda+\lambda\tau)]] \\ &+ \chi\Delta\frac{\partial \pi_1^*}{\partial \chi}[1-\lambda+\beta[1-\pi_2^*(1-\lambda+\lambda\tau)]] + \chi\Delta\frac{\partial \lambda}{\partial \chi}(1-\pi_1^*)[1-\beta\pi_2^*(1-\tau)] \end{aligned} \tag{93}$$

The above expression has three components, and the sign the expression is not immediately clear due to the negative term $\frac{\partial \pi_1^*}{\partial \chi}$. However, in order to show that $\frac{\partial W}{\partial \chi} > 0$, it is sufficient to show that $\pi_1^* > -\frac{\partial \pi_1^*}{\partial \chi}$, since then the first component will be clearly offsetting the second component, with a positive net result. Recall, from the proof of Corollary 2, that

$$\frac{\partial \pi_1^*}{\partial \chi} = \frac{\partial \pi_1^*}{\partial b_1^*} \cdot \frac{\partial b_1^*}{\partial \chi} = -\frac{1}{(b_1^*+1)^2} \cdot \frac{\partial b_1^*}{\partial \chi} < 0, \tag{94}$$

Since $\frac{\partial b_1^*}{\partial \chi} = \frac{1}{2\sqrt{k^* c}} \cdot \int_\rho^{\rho+\frac{\partial \rho}{\partial \chi}} G(r_1)\, dr_1$, we have

$$\frac{\partial \pi_1^*}{\partial \chi} = -\frac{1}{(b_1^*+1)^2} \cdot \frac{1}{2\sqrt{k^* c}} \cdot \int_\rho^{\rho+\frac{\partial \rho}{\partial \chi}} G(r_1)\, dr_1 \tag{95}$$

That expression can be simplified into

$$\frac{\partial \pi_1^*}{\partial \chi} = -\pi_1^* \cdot \frac{1}{2k^*} \cdot \int_\rho^{\rho+\frac{\partial \rho}{\partial \chi}} G(r_1)\, dr_1 \tag{96}$$

As mentioned, we want to show that $\pi_1^* > -\frac{\partial \pi_1^*}{\partial \chi}$, that is, that

$$\pi_1^* > \pi_1^* \cdot \frac{1}{2k^*} \cdot \int_\rho^{\rho+\frac{\partial \rho}{\partial \chi}} G(r_1)\, dr_1 \tag{97}$$

That is equivalent to

$$2k^* > \int_\rho^{\rho+\frac{\partial \rho}{\partial \chi}} G(r_1)\, dr_1 \tag{98}$$

First, note that $\rho = \chi(1-\tau)\beta(\mu-\alpha) < \mu$. Thus,

$$\rho + \frac{\partial \rho}{\partial \chi} = \chi(1-\tau)\beta(\mu-\alpha) + (1-\tau)\beta(\mu-\alpha) < 2\mu < 2k^* \tag{99}$$

Therefore, $\rho + \frac{\partial \rho}{\partial \chi} < 2k^*$. Moreover, from basic properties of statistical distributions, we have

$$\int_\rho^{\rho+\frac{\partial \rho}{\partial \chi}} G(r_1)\, dr_1 < \rho + \frac{\partial \rho}{\partial \chi} \tag{100}$$

Hence, we have $\int_\rho^{\rho+\frac{\partial \rho}{\partial \chi}} G(r_1)\, dr_1 < 2k^*$ and, therefore,

$$\frac{\partial W}{\partial \chi} > 0 \tag{101}$$

Finally, since $\frac{\partial W}{\partial \chi} > 0$ for both $\tau < \tau^*$ and $\tau > \tau^{**}$, it is therefore positive for $\tau^* < \tau < \tau^{**}$ as well. We conclude that the welfare effect of an increase in χ is clearly positive.

The welfare effect of an increase in τ is indeterminate and may be negative, if information levels χ and τ are too high:

That conclusion holds for any value of τ. We take $\tau > \tau^{**}$ as an illustration. Using the welfare function, we obtain

$$\begin{aligned} \frac{\partial W}{\partial \tau} &= \chi \Delta \frac{\partial \pi_1^{**}}{\partial \tau} \left[1 - \lambda + \beta \left[1 - \pi_2^* \left(1 - \lambda + \lambda \tau\right)\right]\right] + \chi \Delta \frac{\partial \lambda}{\partial \tau} \left(1 - \pi_1^{**}\right) \left[1 - \beta \pi_2^* \left(1 - \tau\right)\right] \\ &+ \chi \Delta \beta \left(1 - \pi_1^{**}\right) \pi_2^* \lambda \end{aligned} \tag{102}$$

The sign of the expression is unclear, since $\frac{\partial \pi_1}{\partial \tau} > 0$ and $\frac{\partial \lambda}{\partial \tau} < 0$ are effects on different directions. At this point, it is not possible to execute any useful comparison, as the one which was executed in the previous exercise. However, if we consider that information levels are too high, that is, $\chi \to 1$ and $\tau \to 1$, we obtain $\rho \to 0$ and thus $\lambda \to 0$. Consequently, both the first (positive) and the third (positive) terms disappear and we obtain a clear negative welfare effect of an increase in τ:

$$\frac{\partial W}{\partial \tau} \to -\Delta \left[g(0) \cdot \beta \cdot (\mu - \alpha) \cdot (1 - \pi_1^{**})\right] < 0 \tag{103}$$

8.4 Proof of Proposition 3

We start with the remaining steps for the equilibrium of the constrained model in section 5. We define $\overline{\alpha}$ as the lobby's payoff in the second period when $\overline{b_2}$ is chosen:

$$\overline{\alpha} \equiv \frac{\overline{b_2}}{\overline{b_2} + 1} \cdot \mu - c \cdot \overline{b_2} \tag{104}$$

From $\overline{b_2} < b_2^*$, we have $\overline{\alpha} < \alpha$. Now we analyse voters' decisions on re-election in different scenarios.

Voters' decision on reelection

Once again, at the moment of voters' decision, there are two different scenarios which deserve special attention if the incumbent is dissonant. Everything is identical to section 3.

(i') Voters observe e_1 but do not observe the incumbent's type

If $e_1 = s_1$ is chosen, it is rational for voters to re-elect the incumbent if the probability that the incumbent is congruent given that $e_1 = s_1$ was chosen is no less than $\overline{\pi_2} = \frac{1}{\overline{b_2}+1}$, that is, if

$$\frac{\pi_1 \chi}{\pi_1 \chi + (1 - \pi_1) \lambda (1 - \tau) \chi} \geq \overline{\pi_2} \tag{105}$$

That is equivalent to

Condition 3.

$$\lambda \cdot b_1 \cdot (1 - \tau) \leq \overline{b_2} \tag{106}$$

We claim that this condition holds and the proof will be presented ahead. Therefore, voters do re-elect the incumbent in scenario (i') if $e_1 = s_1$ is chosen.

(ii') Voters observe neither e_1 nor the incumbent's type

In this non-informative scenario, voters choose to re-elect the incumbent if the probability that the incumbent is congruent given that e_1 is not observed is no less than $\overline{\pi_2}$, that is, if

$$\frac{\pi_1 (1-\chi)}{\pi_1 (1-\chi) + (1-\pi_1)(1-\chi)(1-\tau)[\lambda + (1-\lambda)]} \geq \overline{\pi_2} \tag{107}$$

This can also be conveniently rearranged in the following way:

Condition 4.

$$b_1 (1-\tau) \leq \overline{b_2} \tag{108}$$

In fact, Condition 4 does hold. We present a proof of this remark, which can easily be adapted to show that Condition 3 holds.

Proof. Suppose that Condition 4 does not hold, i.e., that $b_1 (1-\tau) > \overline{b_2}$, and thus there is no re-election in scenario (ii'). Then, letting $\overline{\rho} \equiv \chi (1-\tau) \beta (\mu - \overline{\alpha}) > \rho$, it is easy to see that discipline will be given by

$$\overline{\lambda} \equiv G(\overline{\rho}) > \lambda \tag{109}$$

Moreover, the lobby's decision on b_1 solves

$$\begin{aligned} &\underset{b_1 \geq 0}{Max} \frac{b_1}{b_1+1} \cdot \left\{ \overline{\lambda} (\overline{\rho} + \beta\overline{\alpha}) + \left(1-\overline{\lambda}\right) [E(r_1 \mid r_1 > \overline{\rho}) + \beta\overline{\alpha}] \right\} - c \cdot b_1 \\ &s.t. \;\; c \cdot b_1 \leq \phi \cdot c \cdot b_2^* \end{aligned} \tag{110}$$

The solution is

$$\overline{b_1} = \phi \cdot {b_2}^* \tag{111}$$

By assumption, we had $b_1 (1-\tau) > \overline{b_2}$. Since $\overline{b_1} = \overline{b_2}$ in the constrained model, we have therefore $\phi \cdot {b_2}^* \cdot (1-\tau) > \phi \cdot {b_2}^*$. Absurd.

□

Therefore, there is re-election in scenario (ii'). It is easy to see that discipline will nevertheless be given by

$$\overline{\lambda} \equiv G(\overline{\rho}) > \lambda \tag{112}$$

Hence, it is clear that introducing spending caps improves incumbent discipline. Finally, it is easy to see that the lobby's decision on b_1 solves

$$\begin{aligned}
&\underset{b_1\geq 0}{Max}\frac{b_1}{b_1+1}\cdot\left\{\overline{\lambda}\left\{\overline{\rho}+\chi\beta\overline{\alpha}+(1-\chi)\left[(1-\tau)\beta\mu+\tau\beta\overline{\alpha}\right]\right\}\right.\\
&\left.+\left(1-\overline{\lambda}\right)\left\{E\left(r_1\mid r_1>\overline{\rho}\right)+\chi\beta\overline{\alpha}+(1-\chi)\left[(1-\tau)\beta\mu+\tau\beta\overline{\alpha}\right]\right\}-c\cdot b_1\right. \\
&s.t.\;\; c\cdot b_1\leq\phi\cdot c\cdot b_2^*
\end{aligned} \tag{113}$$

The solution is

$$\overline{b_1}=\phi\cdot {b_2}^* \tag{114}$$

Therefore, in the constrained model we have $\overline{b_1}=\overline{b_2}<{b_2}^*$ and $\overline{\lambda}>\lambda$. Hence, both selection and discipline are improved compared to the baseline case. Moreover, a tighter cap (that is, lower ϕ) would trivially lead to higher voter welfare.

8.5 Proof of Proposition 5

As in section 8.3, this proof builds upon a set of auxiliary results on comparative statics. We begin with the comparative-statics effects of an increase on c.

Lemma 5. *The comparative-statics effects of an increase on c*

(i) $\frac{\partial b_2^*}{\partial c}<0$

(ii) $\frac{\partial \alpha}{\partial c}<0$

(iii) $\frac{\partial \widehat{\rho}}{\partial c}>0$

(iv) $\frac{\partial m^*}{\partial c}>0$

(v) $\frac{\partial \widehat{b_1}}{\partial c}<0$

Proof. Except for part (iv), the proof for this result is identical to the proof of Lemma 2 in section 8.3. Therefore we present only the proof for part (iv). Since $m^*=\frac{\sqrt{\beta(\mu-\alpha)}}{\sqrt{\eta}}-1$, we have

$$\frac{\partial m^*}{\partial c}=\frac{\sqrt{\beta}}{\sqrt{\eta}}\cdot\frac{1}{2\sqrt{\mu-\alpha}}\cdot\left(-\frac{\partial\alpha}{\partial c}\right)=\frac{\sqrt{\beta}}{\sqrt{\eta}}\cdot\frac{b_2^*}{2\sqrt{\mu-\alpha}}>0 \tag{115}$$

□

Now, we present a second result, on the comparative-statics effects of an increase on η.

Lemma 6. *The comparative-statics effects of an increase on* η

(i) $\frac{\partial b_2^*}{\partial \eta} = 0$

(ii) $\frac{\partial \alpha}{\partial \eta} = 0$

(iii) $\frac{\partial \widehat{\rho}}{\partial \eta} < 0$

(iv)$\frac{\partial m^*}{\partial \eta} < 0$

(v) $\frac{\partial \widehat{b}_1}{\partial \eta} < 0$

Proof. (Parts (i)-(ii))

Since $b_2^* = \frac{\sqrt{\mu}}{\sqrt{c}} - 1$, it is clear that $\frac{\partial b_2^*}{\partial \eta} = 0$. Hence, $\frac{\partial \alpha}{\partial \eta} = 0$

(Part (iii))

The equilibrium threshold $\widehat{\rho}$ can be expressed as $\widehat{\rho} = \left(\sqrt{\beta(\mu - \alpha)} - \sqrt{\eta}\right)^2$. Then, we have

$$\frac{\partial \widehat{\rho}}{\partial \eta} = 2 \cdot \left(\sqrt{\beta(\mu - \alpha)} - \sqrt{\eta}\right) \cdot \left(-\frac{1}{2\sqrt{\eta}}\right) = -\left(\frac{\sqrt{\beta(\mu - \alpha)}}{\sqrt{\eta}} - 1\right) = -m^* < 0 \tag{116}$$

(Part (iv))

As $m^* = \frac{\sqrt{\beta(\mu-\alpha)}}{\sqrt{\eta}} - 1$, we have

$$\frac{\partial m^*}{\partial \eta} = -\frac{\sqrt{\beta(\mu - \alpha)}}{2\eta^{\frac{3}{2}}} < 0 \tag{117}$$

(Part (v))

(Step 1: $\frac{\partial \widehat{k}}{\partial \eta} < 0$)

It was already established that $\widehat{k}$ can be expressed as $\widehat{k} = R - \int_{\widehat{\rho}}^{R} G(r_1)\, dr_1 + \beta\alpha$. Then, as $\frac{\partial \widehat{\rho}}{\partial \eta} = -m^* < 0$, we have

$$\frac{\partial \widehat{k}}{\partial \eta} = -\frac{\partial}{\partial \eta}\left[\int_{\widehat{\rho}}^{R} G(r_1)\, dr_1\right] = -\int_{\widehat{\rho} + \frac{\partial \widehat{\rho}}{\partial \eta}}^{\widehat{\rho}} G(r_1)\, dr_1 = -\int_{\widehat{\rho} - m^*}^{\widehat{\rho}} G(r_1)\, dr_1 < 0 \tag{118}$$

(Step 2: $\frac{\partial b_1^*}{\partial \eta} < 0$)

As $b_1^* = \sqrt{\frac{\widehat{k}}{c}} - 1$, we have

$$\frac{\partial b_1^*}{\partial \eta} = \frac{\partial b_1^*}{\partial \widehat{k}} \cdot \frac{\partial \widehat{k}}{\partial \eta} = \frac{1}{2\sqrt{c\widehat{k}}} \cdot \left(-\int_{\widehat{\rho} - m^*}^{\widehat{\rho}} G(r_1)\, dr_1\right) < 0 \tag{119}$$

□

With Lemma 5-6, we are able to calculate the selection, discipline, and media-capture effects of increases on c and η:

Corollary 3. *Selection and Discipline Effects*

(i) An increase on c leads to better selection, better discipline, and stronger media capture.

(ii) An increase on eta leads to better selection, worse discipline, and weaker media capture.

Proof. (Part (i))

(Selection)

As $\pi_2^* = \frac{1}{b_2^*+1} = \frac{\sqrt{c}}{\sqrt{\mu}}$, we have

$$\frac{\partial \pi_2^*}{\partial c} = \frac{1}{2\sqrt{c}\sqrt{\mu}} > 0 \tag{120}$$

Moreover, as $\pi_1^* = \frac{1}{b_1^*+1}$, we have

$$\frac{\partial \pi_1^*}{\partial c} = \frac{\partial \pi_1^*}{\partial b_1^*} \cdot \frac{\partial b_1^*}{\partial c} = -\frac{1}{(b_1^*+1)^2} \cdot \frac{\partial b_1^*}{\partial c} > 0 \tag{121}$$

Hence, an increase on c leads to better selection.

(Discipline)

We know that $\lambda = G(\widehat{\rho})$ and $\frac{\partial \widehat{\rho}}{\partial c} > 0$. Therefore,

$$\frac{\partial \lambda}{\partial c} = \frac{\partial \lambda}{\partial \widehat{\rho}} \cdot \frac{\partial \widehat{\rho}}{\partial c} > 0 \tag{122}$$

Hence, an increase on c leads to better discipline.

(Media Capture)

We know that $\frac{\partial m^*}{\partial c} > 0$. Then, as $\tau^* = \frac{1}{1+m^*}$, we have

$$\begin{aligned} \frac{\partial \tau^*}{\partial c} &= \frac{\partial \tau^*}{\partial m^*} \cdot \frac{\partial m^*}{\partial c} = -\frac{1}{(1+m^*)^2} \cdot \frac{\partial m^*}{\partial c} = -\frac{\eta}{\beta(\mu-\alpha)} \cdot \frac{\sqrt{\beta}}{\sqrt{\eta}} \cdot \frac{b_2^*}{2\sqrt{\mu-\alpha}} = \\ &= -\sqrt{\frac{\eta}{\beta(\mu-\alpha)}} \cdot \frac{b_2^*}{2(\mu-\alpha)} = -\tau^* \cdot \frac{b_2^*}{2(\mu-\alpha)} < 0 \end{aligned} \tag{123}$$

Hence, an increase on c leads to stronger media capture.

(Part ii)

(Selection)

We know that $\frac{\partial b_2^*}{\partial \eta} = 0$. Therefore,

$$\frac{\partial \pi_2^*}{\partial \eta} = \frac{\partial \pi_2^*}{\partial b_2^*} \cdot \frac{\partial b_2^*}{\partial \eta} = 0 \tag{124}$$

Moreover, since $\frac{\partial b_1^*}{\partial \eta} < 0$, we have

$$\begin{aligned}\frac{\partial \pi_1^*}{\partial \eta} &= \frac{\partial \pi_1^*}{\partial b_1^*} \cdot \frac{\partial b_1^*}{\partial \eta} = -\frac{1}{\left(b_1^* + 1\right)^2} \cdot \left[-\frac{1}{2\sqrt{c\widehat{k}}} \int_{\widehat{\rho}-m^*}^{\widehat{\rho}} G\left(r_1\right) dr_1\right] \\ &= \frac{\sqrt{c}}{\sqrt{\widehat{k}}} \cdot \frac{1}{2\widehat{k}} \int_{\widehat{\rho}-m^*}^{\widehat{\rho}} G\left(r_1\right) dr_1 = \frac{\pi_1^*}{2\widehat{k}} \int_{\widehat{\rho}-m^*}^{\widehat{\rho}} G\left(r_1\right) dr_1 > 0\end{aligned} \tag{125}$$

Hence, an increase on η leads to better selection.

(Discipline)

As $\frac{\partial \widehat{\rho}}{\partial \eta} = -m^* < 0$ and

$$\widehat{\lambda} = G\left(\widehat{\rho}\right) = \int_0^{\widehat{\rho}} g\left(r_1\right) dr_1, \tag{126}$$

it follows that

$$\frac{\partial \widehat{\lambda}}{\partial \eta} = \frac{\partial \left[\int_0^{\widehat{\rho}} g\left(r_1\right) dr_1\right]}{\partial \eta} = -\int_{\widehat{\rho}-m^*}^{\widehat{\rho}} g\left(r_1\right) dr_1 = -\left[G\left(\widehat{\rho}\right) - G\left(\widehat{\rho} - m^*\right)\right] < 0 \tag{127}$$

Hence, an increase on η leads to worse discipline.

(Media Capture)

As $\tau^* = \frac{1}{1+m^*}$, it follows that

$$\begin{aligned}\frac{\partial \tau^*}{\partial \eta} &= \frac{\partial \tau^*}{\partial m^*} \cdot \frac{\partial m^*}{\partial \eta} = -\frac{1}{\left(1+m^*\right)^2} \cdot \left(-\frac{\sqrt{\beta\left(\mu-\alpha\right)}}{2\eta^{\frac{3}{2}}}\right) \\ &= \frac{\eta}{\beta\left(\mu-\alpha\right)} \cdot \frac{\sqrt{\beta\left(\mu-\alpha\right)}}{2\eta^{\frac{3}{2}}} = \frac{1}{2\sqrt{\beta\left(\mu-\alpha\right)} \cdot \sqrt{\eta}} > 0\end{aligned} \tag{128}$$

Hence, an increase on η leads to lower media capture. □

Finally, we study the net welfare effect of increases on c and on η using the voter welfare function for the extended model:

$$W\left(\pi_1^*, \pi_2^*, \widehat{\lambda}, \tau^*\right) = \left[\pi_1^* + \left(1-\pi_1^*\right)\widehat{\lambda}\right]\Delta + \beta\left[\pi_1^* + \left(1-\pi_1^*\right)\pi_2^*\left[\left(1-\widehat{\lambda}\right) + \widehat{\lambda}\tau^*\right]\right]\Delta \tag{129}$$

The welfare effect of an increase in c is positive

From the welfare function, we obtain

$$\begin{aligned}
&\frac{\partial\left[W\left(\pi_1^*,\pi_2^*,\widehat{\lambda},\tau^*\right)\right]}{\partial c}=\Delta\left[\frac{\partial\pi_1^*}{\partial c}-\frac{\partial\pi_1^*}{\partial c}\widehat{\lambda}+(1-\pi_1^*)\frac{\partial\widehat{\lambda}}{\partial c}\right]\\
&+\beta\Delta\left\{\frac{\partial\pi_1^*}{\partial c}+\left[-\frac{\partial\pi_1^*}{\partial c}\cdot\pi_2^*+(1-\pi_1^*)\frac{\partial\pi_2^*}{\partial c}\right]\cdot\left[\left(1-\widehat{\lambda}\right)+\widehat{\lambda}\tau^*\right]\right\}\\
&+\beta\Delta\left\{\left[(1-\pi_1^*)\,\pi_2^*\right]\cdot\left[-\frac{\partial\widehat{\lambda}}{\partial c}+\frac{\partial\widehat{\lambda}}{\partial c}\tau^*+\widehat{\lambda}\frac{\partial\tau^*}{\partial c}\right]\right\}
\end{aligned}\tag{130}$$

After some rearranging, this expression can be written as

$$\begin{aligned}
\frac{\partial\left[W\left(\pi_1^*,\pi_2^*,\widehat{\lambda},\tau^*\right)\right]}{\partial c}&=\underbrace{\Delta\cdot\underbrace{\frac{\partial\pi_1^*}{\partial c}}_{>0}\cdot\underbrace{\left[\left(1-\widehat{\lambda}\right)+\beta\left[1-\pi_2^*\left[\left(1-\widehat{\lambda}\right)+\widehat{\lambda}\tau^*\right]\right]\right]}_{>0}}_{(I)}\\
&+\underbrace{\Delta\cdot\underbrace{\frac{\partial\widehat{\lambda}}{\partial c}}_{>0}\cdot(1-\pi_1^*)\cdot\underbrace{\left[1-\beta\,(1-\tau^*)\,\pi_2^*\right]}_{>0}}_{(II)}\\
&+\underbrace{\Delta\cdot\underbrace{\frac{\partial\pi_2^*}{\partial c}}_{>0}\cdot\beta\,(1-\pi_1^*)\left[\left(1-\widehat{\lambda}\right)+\widehat{\lambda}\tau^*\right]}_{(III)}+\underbrace{\Delta\cdot\underbrace{\frac{\partial\tau^*}{\partial c}}_{<0}\cdot\beta\widehat{\lambda}\,(1-\pi_1^*)\,\pi_2^*}_{(IV)}
\end{aligned}\tag{131}$$

Thus, in order to show that $\frac{\partial\left[W\left(\pi_1^*,\pi_2^*,\widehat{\lambda},\tau^*\right)\right]}{\partial c}>0$, it is sufficient to establish that $(III)+(IV)>0$. It follows that

$$(III)+(IV)=\beta\,(1-\pi_1^*)\left\{\frac{\partial\pi_2^*}{\partial c}\left[\left(1-\widehat{\lambda}\right)+\widehat{\lambda}\tau^*\right]+\frac{\partial\tau^*}{\partial c}\widehat{\lambda}\pi_2^*\right\}\tag{132}$$

Substituting $\frac{\partial\pi_2^*}{\partial c}$ and $\frac{\partial\tau^*}{\partial c}$ by already known expressions leads to

$$(III)+(IV)=\beta\,(1-\pi_1^*)\left\{\frac{1}{2\sqrt{\mu c}}\left[\left(1-\widehat{\lambda}\right)+\widehat{\lambda}\tau^*\right]-\tau^*\frac{b_2^*}{2\,(\mu-\alpha)}\widehat{\lambda}\pi_2^*\right\}\tag{133}$$

After further rearranging, this can be expressed as

$$(III)+(IV)=\frac{\beta\left(1-\pi_1^*\right)}{2}\left\{\frac{1-\widehat{\lambda}}{\sqrt{\mu c}}+\widehat{\lambda}\tau^*\cdot\underbrace{\left[\frac{1}{\sqrt{\mu c}}-\frac{\left(1-\pi_2^*\right)}{(\mu-\alpha)}\right]}_{(*)}\right\}\tag{134}$$

Finally, we show that $(*)>0$:

(Step 1: $\sqrt{\mu c}<\mu-\alpha$)

It is straightforward that $\mu-\alpha=2\sqrt{\mu c}-c$. Hence, it is enough to prove that $2\sqrt{\mu c}-c>\sqrt{\mu c}$, that is, $\sqrt{\mu c}>c$. That is indeed true, since $\mu>c$ by assumption. Therefore, we have $\sqrt{\mu c}<\mu-\alpha$.

(Step 2: $(*)>0$)

From the previous step, it follows that

$$\frac{1}{\sqrt{\mu c}} - \frac{1}{\mu - \alpha} > 0 \tag{135}$$

Furthermore,

$$(*) = \frac{1}{\sqrt{\mu c}} - \frac{(1-\pi_2^*)}{(\mu - \alpha)} > \frac{1}{\sqrt{\mu c}} - \frac{1}{\mu - \alpha} > 0 \tag{136}$$

As $(*) > 0$, it follows that $(III) + (IV) > 0$ and $\frac{\partial \left[W\left(\pi_1^*, \pi_2^*, \widehat{\lambda}, \tau^*\right)\right]}{\partial c} > 0$.

The welfare effect of an increase on η is indeterminate and may be negative, if η is too high

From the welfare function, we obtain

$$\begin{aligned}
&\frac{\partial \left[W\left(\pi_1^*, \pi_2^*, \widehat{\lambda}, \tau^*\right)\right]}{\partial \eta} = \Delta \frac{\partial \pi_1^*}{\partial \eta} \left[1 - \widehat{\lambda} + \beta \left[1 - \pi_2^* \left(1 - \widehat{\lambda} + \widehat{\lambda}\tau^*\right)\right]\right] \\
&+ \Delta \frac{\partial \widehat{\lambda}}{\partial \eta} \left(1 - \pi_1^*\right) \left[1 - \beta \pi_2^* \left(1 - \tau^*\right)\right] \\
&+ \Delta \frac{\partial \tau^*}{\partial \eta} \beta \left(1 - \pi_1^*\right) \pi_2^* \widehat{\lambda}
\end{aligned} \tag{137}$$

The sign of this expression is unclear, due to the negative discipline effect. Unlike the previous welfare effect of an increase on c, we are not able to solve for the overall sign of the expression by means of useful comparison of terms. Therefore, we conclude that the welfare effect of an increase on η is indeterminate. However, we can say more about the expression. If we substitute the composing effects for known expressions obtained before, we obtain:

$$\begin{aligned}
&\frac{\partial \left[W\left(\pi_1^*, \pi_2^*, \widehat{\lambda}, \tau^*\right)\right]}{\partial \eta} = \Delta \frac{\pi_1^*}{2\widehat{k}} \int_{\widehat{\rho} - m^*}^{\widehat{\rho}} G\left(r_1\right) dr_1 \left[1 - \widehat{\lambda} + \beta \left[1 - \pi_2^* \left(1 - \widehat{\lambda} + \widehat{\lambda}\tau^*\right)\right]\right] \\
&- \Delta g(\widehat{\rho}) \left[\frac{\sqrt{\beta(\mu - \alpha)}}{2\sqrt{\eta}} + m^*\right] \left(1 - \pi_1^*\right) \left[1 - \beta \pi_2^* \left(1 - \tau^*\right)\right] \\
&+ \Delta \frac{1}{2\sqrt{\beta(\mu - \alpha)\eta}} \beta \left(1 - \pi_1^*\right) \pi_2^* \widehat{\lambda}
\end{aligned} \tag{138}$$

If the marginal cost of media influence η is too high, that is, if we let $\eta \to \beta(\mu - \alpha)$, then as a consequence we get $m^* \to 0$, $\tau^* \to 1$, $\widehat{\rho} \to 0$, and $\widehat{\lambda} \to 0$. In those circumstances, we obtain

$$\frac{\partial W}{\partial \eta} \to -\Delta \cdot \frac{g(0)(1 - \pi_1^*)}{2} < 0 \tag{139}$$

8.6 Proof of Proposition 6

Our starting point is the equilibrium analysis of the extended model with spending caps. We start at the first non-trivial decision, which is the choice of b_2. As in Section 5, that choice solves the following problem:

$$\begin{aligned} &\underset{b_2 \geq 0}{Max} \frac{b_2}{b_2+1} \cdot \mu - c \cdot b_2 \\ &s.t. \;\; c \cdot b_2 \leq \phi \cdot c \cdot b_2^* \end{aligned} \tag{140}$$

The solution is

$$\overline{b_2} = \phi \cdot b_2^* \tag{141}$$

Then, we define $\overline{\pi_2}$ as the equilibrium level of the probability π_2:

$$\overline{\pi_2} = \frac{1}{\overline{b_2}+1} \tag{142}$$

We also define $\overline{\alpha}$ as the lobby's expected payoff in the second period when $\overline{b_2}$ is chosen:

$$\overline{\alpha} \equiv \frac{\overline{b_2}}{\overline{b_2}+1} \cdot \mu - c \cdot \overline{b_2} \tag{143}$$

Since $\overline{b_2} < b_2^*$, we have $\overline{\pi_2} > \pi_2^*$ and $\overline{\alpha} < \alpha$.

Voters decide on re-election

Voters do not re-elect the incumbent if $e_1 = 1 - s_1$ is observed. However, if $e_1 = s_1$ is observed, it is once again rational for voters to re-elect the incumbent if the probability that the incumbent is congruent given that $e_1 = s_1$ was observed is no less than $\overline{\pi_2}$, that is, if

$$\frac{\pi_1}{\pi_1 + (1-\pi_1)\,\lambda\,(1-\tau)} \geq \overline{\pi_2}, \tag{144}$$

The inequality above can be rearranged into:

$$\lambda \cdot b_1 \cdot (1-\tau) \leq \overline{b_2} \tag{145}$$

We claim that the above inequality holds, due to the same argument as in Section 8.1. Hence, voters do re-elect the incumbent if $e_1 = s_1$ is observed.

The lobby decides on the influence over the media m

If $e_1 = 1 - s_1$ is chosen, the (dissonant) incumbent is not re-elected and the lobby's payoff is $r_1 + \beta\overline{\alpha}$. But, if $e_1 = s_1$ is chosen, the lobby's payoff depends on his choice of m and can be expressed as

$$\max_{m \geq 0} \{(1-\tau)\,\beta\mu + \tau\beta\overline{\alpha} - \eta m\} \tag{146}$$

That expression is maximized by

$$\overline{m} = \sqrt{\frac{\beta\,(\mu - \overline{\alpha})}{\eta}} - 1 \tag{147}$$

Then, we define $\overline{\tau}$ as the resulting level of voters' information on the incumbent's type:

$$\overline{\tau} \equiv \frac{1}{\overline{m} + 1} \tag{148}$$

Since $\overline{\alpha} < \alpha$, we have $\overline{m} > m^*$ and $\overline{\tau} < \tau^*$.

Incumbent discipline: the determination of λ

The lobby chooses the congruent policy $e_1 = s_1$ if

$$r_1 < \beta\,(1 - \overline{\tau})\,(\mu - \overline{\alpha}) - \eta\overline{m} \tag{149}$$

Once again, the lobby's discipline will be determined from the distribution of the dissonant rent r_1. If we define $\widetilde{\rho} \equiv \beta\,(1 - \overline{\tau})\,(\mu - \overline{\alpha}) - \eta\overline{m}$, discipline will be given by

$$\widetilde{\lambda} = G\,(\widetilde{\rho})\,, \tag{150}$$

The lobby decides on the campaign effort b_1

Let $\overline{k}$ be the lobby's expected payoff from winning the first election in this extended model:

$$\overline{k} \equiv \widetilde{\lambda}\,[(1-\overline{\tau})\beta\mu + \overline{\tau}\beta\overline{\alpha} - \eta\overline{m}] + (1-\widetilde{\lambda})\,[E\,(r_1 \mid r_1 > \widetilde{\rho}) + \beta\overline{\alpha}] \tag{151}$$

That can be simplified into

$$\overline{k} = \widetilde{\lambda}\widetilde{\rho} + (1-\widetilde{\lambda})E\left(r_1 \mid r_1 > \widetilde{\rho}\right) + \beta\overline{\alpha} \tag{152}$$

The lobby decides on b_1 by solving:

$$\begin{aligned} &\underset{b_1 \geq 0}{Max} \frac{b_1}{b_1+1} \cdot \overline{k} - c \cdot b_1 \\ &s.t. \;\; c \cdot b_1 \leq \phi \cdot c \cdot b_2^* \end{aligned} \tag{153}$$

The solution is

$$\overline{b_1} = \phi \cdot b_2^* \tag{154}$$

Note that in this model we have $\overline{b_1} = \overline{b_2}$ and therefore $\overline{\pi_1} = \overline{\pi_2}$.

We have solved for the equilibrium of the model with campaign spending caps and endogenous media influence. Now, we are ready to present the proof of Proposition 6. We begin with a result on the comparative statics effects of an increase on ϕ:

Lemma 7. *The comparative-statics effects of an increase on ϕ.*

(i) $\frac{\partial \overline{b_2}}{\partial \phi} > 0$

(ii) $\frac{\partial \overline{\alpha}}{\partial \phi} > 0$

(iii) $\frac{\partial \overline{m}}{\partial \phi} < 0$

(iv) $\frac{\partial \widetilde{\rho}}{\partial \phi} < 0$

(v) $\frac{\partial \overline{b_1}}{\partial \phi} > 0$

Proof. (Part (i))

Since $\overline{b_2} = \phi \cdot b_2^*$, we have

$$\frac{\partial \overline{b_2}}{\partial \phi} = b_2^* > 0 \tag{155}$$

(Part (ii))

Since $\overline{\alpha} = \frac{\overline{b_2}}{(\overline{b_2}+1)} \cdot \mu - c \cdot \overline{b_2}$, we have

$$\frac{\partial \overline{\alpha}}{\partial \phi} = \frac{\partial \overline{\alpha}}{\partial \overline{b_2}} \cdot \frac{\partial \overline{b_2}}{\partial \phi} = \underbrace{\left[\frac{1}{(\phi \cdot b_2^* + 1)^2} \cdot \mu - c \right]}_{>0} \cdot b_2^* > 0 \tag{156}$$

(Part (iii))

Since $\overline{m} = \sqrt{\frac{\beta(\mu - \overline{\alpha})}{\eta}} - 1$, we have

$$\frac{\partial \overline{m}}{\partial \phi} = \frac{\partial \overline{m}}{\partial \overline{\alpha}} \cdot \frac{\partial \overline{\alpha}}{\partial \phi} = -\frac{1}{2} \cdot \sqrt{\frac{\beta}{(\mu - \overline{\alpha})\eta}} \cdot \frac{\partial \overline{\alpha}}{\partial \phi} < 0 \tag{157}$$

(Part (iv))

From $\widetilde{\rho} = \beta(1 - \overline{\tau})(\mu - \overline{\alpha}) - \eta \overline{m}$, we obtain

$$\frac{\partial \widetilde{\rho}}{\partial \phi} = -\beta(1 - \overline{\tau}) \cdot \frac{\partial \overline{\alpha}}{\partial \phi} < 0 \tag{158}$$

(Part (v))

Since $\overline{b_1} = \phi \cdot b_2^*$, we have

$$\frac{\partial \overline{b_1}}{\partial \phi} = b_2^* > 0 \tag{159}$$

□

Using Lemma 7, we are able to calculate the selection, discipline, and media-capture effects of an increase on ϕ:

Corollary 4. *An increase on ϕ leads to worse selection, worse discipline, and weaker media capture.*

Proof. (Selection)

From $\overline{\pi_1} = \overline{\pi_2} = \frac{1}{\phi \cdot b_2^* + 1}$, we have

$$\frac{\partial \overline{\pi_1}}{\partial \phi} = \frac{\partial \overline{\pi_2}}{\partial \phi} = -\frac{1}{(\phi \cdot b_2^* + 1)^2} \cdot b_2^* = -\overline{\pi_2}^2 \cdot b_2^* < 0 \tag{160}$$

(Discipline)

Since $\widetilde{\lambda} = G(\widetilde{\rho}$, we have

$$\frac{\partial \widetilde{\lambda}}{\partial \phi} = \frac{\partial \widetilde{\lambda}}{\partial \widetilde{\rho}} \cdot \frac{\partial \widetilde{\rho}}{\partial \phi} = g(\widetilde{\rho}) \cdot \frac{\partial \widetilde{\rho}}{\partial \phi} < 0 \tag{161}$$

(Media Capture)

From $\overline{\tau} \equiv \frac{1}{\overline{m}+1}$, we have

$$\frac{\partial \overline{\tau}}{\partial \phi} = \frac{\partial \overline{\tau}}{\partial \overline{m}} \cdot \frac{\partial \overline{m}}{\partial \phi} = \frac{1}{(\overline{m}+1)^2} \cdot \frac{1}{2} \cdot \sqrt{\frac{\beta}{(\mu - \overline{\alpha})\eta}} \cdot \frac{\partial \overline{\alpha}}{\partial \phi} = \frac{1}{2(\mu - \overline{\alpha})} \cdot \overline{\tau} \cdot \frac{\partial \overline{\alpha}}{\partial \phi} > 0 \tag{162}$$

□

Since the effects above do not have the same sign, in order to calculate the sign of the overall welfare effect of an increase on ϕ we need once again to use the welfare function:

$$W\left(\overline{\pi_1}, \overline{\pi_2}, \widetilde{\lambda}, \overline{\tau}\right) = \left[\overline{\pi_1} + (1-\overline{\pi_1})\widetilde{\lambda}\right]\Delta + \beta\left[\overline{\pi_1} + (1-\overline{\pi_1})\overline{\pi_2}\left[\left(1-\widetilde{\lambda}\right) + \widetilde{\lambda}\overline{\tau}\right]\right]\Delta \tag{163}$$

At this point, we repeat the claim from Proposition 6, which we are going to prove next using Lemma 7, Corollary 4, and the welfare function:

A tighter campaign spending cap in the extended model leads to improved voter welfare

Proof. From the welfare function, we obtain

$$\frac{\partial W\left(\overline{\pi_1}, \overline{\pi_2}, \widetilde{\lambda}, \overline{\tau}\right)}{\partial \phi} = \Delta \cdot \underbrace{\frac{\partial \overline{\pi_1}}{\partial \phi}}_{<0} \cdot \underbrace{\left[1-\widetilde{\lambda}+\beta+\beta\left(1-\widetilde{\lambda}+\widetilde{\lambda}\overline{\tau}\right)(1-2\overline{\pi_1})\right]}_{>0}$$

$$+\Delta \cdot \underbrace{\frac{\partial \widetilde{\lambda}}{\partial \phi}}_{<0} \cdot \underbrace{(1-\overline{\pi_1})\cdot\left[1-\beta\overline{\pi_1}(1-\overline{\tau})\right]}_{>0} \tag{164}$$

$$+\Delta \cdot \underbrace{\frac{\partial \overline{\tau}}{\partial \phi}}_{>0} \cdot \underbrace{\beta(1-\overline{\pi_1})\overline{\pi_1}\widetilde{\lambda}}_{>0}$$

From the expression above, we see that the welfare effect has three components. In order to show that $\frac{\partial W}{\partial \phi} < 0$, it is sufficient to show that the first component (selection effect) dominates the third component (media-capture effect). That is, we show that

$$\left|\frac{\partial \overline{\pi_1}}{\partial \phi}\right| > \frac{\partial \overline{\tau}}{\partial \phi} \cdot \overline{\pi_1} \cdot \widetilde{\lambda} \tag{165}$$

and

$$\left[1-\widetilde{\lambda}+\beta+\beta\left(1-\widetilde{\lambda}+\widetilde{\lambda}\overline{\tau}\right)(1-2\overline{\pi_1})\right] > \beta \cdot (1-\overline{\pi_1}) \tag{166}$$

We begin with the first inequality. First, since $\frac{\partial \overline{\pi_1}}{\partial \phi} < 0$, we have

$$\left|\frac{\partial \overline{\pi_1}}{\partial \phi}\right| = -\frac{\partial \overline{\pi_1}}{\partial \phi} = \overline{\pi_1}^2 \cdot b_2^* \tag{167}$$

On the other hand, the expression for $\frac{\partial \overline{\tau}}{\partial \phi}$ from Corollary 4 can be developed to be written as

$$\frac{\partial \overline{\tau}}{\partial \phi} = \frac{1}{2} \cdot \overline{\tau} \cdot \overline{\pi_1} \cdot b_2^* \cdot \gamma \tag{168}$$

where

$$\gamma \equiv \frac{\mu - c \cdot (\phi b_2^* + 1)^2}{\mu + c \cdot \phi \cdot b_2^* (\phi b_2^* + 1)} \in (0,1) \tag{169}$$

Therefore, the first inequality is equivalent to

$$\overline{\pi_1}^2 \cdot b_2^* > \frac{1}{2} \cdot \overline{\tau} \cdot \overline{\pi_1} \cdot b_2^* \cdot \gamma \cdot \overline{\pi_1} \cdot \widetilde{\lambda} \tag{170}$$

that is,

$$1 > \frac{1}{2} \cdot \overline{\tau} \cdot \gamma \cdot \widetilde{\lambda} \tag{171}$$

which is true since all the terms on the right-hand side are smaller than one.

Now we turn to the second inequality. Note that $\beta + \beta \left(1 - \widetilde{\lambda} + \widetilde{\lambda}\overline{\tau}\right)(1 - 2\overline{\pi_1})$ can be expressed as

$$\beta \left[1 - \overline{\pi_1}\left(1 - \widetilde{\lambda} + \widetilde{\lambda}\overline{\tau}\right) + \left(1 - \widetilde{\lambda} + \widetilde{\lambda}\overline{\tau}\right)(1 - \overline{\pi_1})\right] \tag{172}$$

Since $\left(1 - \widetilde{\lambda} + \widetilde{\lambda}\overline{\tau}\right) < 1$, we have

$$\beta \left[1 - \overline{\pi_1}\left(1 - \widetilde{\lambda} + \widetilde{\lambda}\overline{\tau}\right) + \left(1 - \widetilde{\lambda} + \widetilde{\lambda}\overline{\tau}\right)(1 - \overline{\pi_1})\right] > \beta (1 - \overline{\pi_1}) \tag{173}$$

Therefore, since $\left(1 - \widetilde{\lambda}\right) > 0$, we have

$$\left[1 - \widetilde{\lambda} + \beta + \beta\left(1 - \widetilde{\lambda} + \widetilde{\lambda}\overline{\tau}\right)(1 - 2\overline{\pi_1})\right] > \beta \cdot (1 - \overline{\pi_1}) \tag{174}$$

and our second inequality is established. Thus, we have $\frac{\partial W}{\partial \phi} < 0$, which means that a tighter spending cap increases voter welfare. □

References

[1] Austen-Smith, D. (1987). *Interest Groups, Campaign Contributions, and Probabilistic Voting*. Public Choice, 54: 123-139.

[2] Baron, D. (1994). *Electoral Competition with Informed and Uninformed Voters*. American Political Science Review, 84 (1): 33-47.

[3] Avis,E., Ferraz, C., Finan, F., and Varjão, C. (2020). *Money and Politics: The Effects of Campaign Spending Limits on Political Entry and Competition*. Working Paper.

[4] Barro, R. (1973). *The Control of Politicians: An Economic Model*. Public Choice, 14: 19-42.

[5] Beattie, G., Durante, R., Knight, B., and Sen, An. (2000). *Advertising Spending and Media Bias: Evidence from News Coverage of Car Safety Recalls*. Management Science, 67 (2): 698-719.

[6] Berganza, J. (2000). *Two Roles for Elections: Disciplining the Incumbent and Selecting a Competent Candidate*. Public Choice, 105: 165-193.

[7] Besley, T. (2006). *Principled Agents? The Political Economy of Good Government*. Oxford University Press.

[8] Besley, T. and Case, A. (1995). *Incumbent Behavior: Vote-Seeking, Tax-Setting, and Yardstick Competition*. American Economic Review, 85: 25-45.

[9] Besley, T. and Prat, A. (2006). *Handcuffs for the Grabbing Hand? Media Capture and Government Accountability*. American Economic Review, 96: 720-736.

[10] Bruce, R. and Lima, R.C. (2019). *Compulsory Voting and T.V. News Consumption: Evidence from Brazil*. Journal of Development Economics, 138: 165-179.

[11] Clinton, H. and Caine, T. (2016). *Stronger Together. A Blueprint for America's Future*. Simon Schuster.

[12] Coate, S. (2004). *Political Competition with Campaign Contributions and Informative Advertising*. Journal of the European Economic Association, 2: 772–804.

[13] Coate, S. and Morris, S. (1995). *On the Form of Transfers to Special Interests*. Journal of Political Economy, 103: 1210-1235.

[14] DellaVigna, S., Durante, R., Knight, B., and La Ferrara, E. (2016). *Market-Based Lobbying: Evidence from Advertising Spending in Italy*. American Economic Journal: Applied Economics, 8(1): 224-256

[15] Dyck, A., Volchkova, N., and Zingales, L. (2008). *The Corporate Governance Role of the Media: Evidence from Russia*. The Journal of Finance, 63(3): 1093-1135.

[16] Dyck, A. and Zingales, L. (2002). *The Bubble and the Media*. Corporate Governance and Capital Flows in a Global Economy, Oxford University Press.

[17] Ferejohn, J. (1986). *Incumbent Performance and Electoral Control*. Public Choice, 50: 5-25.

[18] Gehlbach, S. and Sonin, K. (2014). *Government Control of the Media*. Journal of Public Economics, 118: 163-171.

[19] Grossman, G. and Helpman, E. (1996). *Electoral Competition and Special Interest Policies*. Review of Economic Studies, 63: 265–286.

[20] Persson, T., Roland, G., and Tabellini, G. (1997). *Separation of Powers and Political Accountability.* Quarterly Journal of Economics, 112: 1163-1202.

[21] Petrova, M. (2008). *Inequality and Media Capture*. Journal of Public Economics, 92: 183-212

[22] Prat, A. (2002). *Campaign Spending With Office-Seeking Politicians, Rational Voters, and Multiple Lobbies*. Journal of Economic Theory, 103: 162–189.

[23] Prat, A. (2018). *Media Power*. Journal of Political Economy, 126(4): 1747-83

[24] Prat, A. and Stromberg, D. (2013). *The Political Economy of the Mass Media, in advances in economics and econometrics: theory and applications.* In: Proceedings of the Tenth World Congress of the Econometric Society. Cambrigdge University Press.

[25] Scarrow, S. (2007). *Political Finance in Comparative Perspective*. Annual Review of Political Science, 10: 193–210.

[26] Strömberg, D. (2004). *Media Competition, Political Competition, and Public Policy.* Review of Economic Studies, 71 (1): 265-84.

[27] Szeidl, A. and Szucs, F. (2021). *Media Capture Through Favor Exchange*. Econometrica, 89 (1): 281-310.

[28] The New York Times (2015). *Cash Flowed to Clinton Foundation Amid Russian Uranium Deal*. <https://www.nytimes.com/2015/04/24/us/cash-flowed-to-clinton-foundation-as-russians-pressed-for-control-of-uranium-company.html>

[29] Wikileaks (2016a). *The Podesta Emails; Part One*. <https://wikileaks.org/podesta-emails/press-release>

[30] Wikileaks (2016b). *HRC Paid Speeches*. <https://wikileaks.org/podesta-emails/emailid/927>