Accountability and Political Competition*

Braz Camargo† Arianna Degan‡

February 1, 2023

Abstract

Is increasing political competition good for voters? We study this question in the political career concerns framework. We first show that the relationship between political competition, viewed as the cost of challenging incumbent politicians, and *effective accountability*, i.e., the politicians' incentive to behave in the voters' interest, is undetermined in the sense that an increase in political competition can be associated both with an increase and with a decrease in effective accountability. We also show that effective accountability need not be maximized when challenging incumbents is costless. We then discuss conditions under which an increase in political competition has an unequivocal impact on effective accountability and show that as the number of potential challengers to an incumbent politician increases, the conditions under which this impact is unequivocally negative become more likely. Overall, our results show that, unlike in the marketplace, where an increase in competition typically benefits consumers, increasing competition in the political sector can adversely affect voters.

*We are grateful to Scott Ashworth, Alexander Hirsch, Davin Rahia, and Alice Uribe-McGuire for excellent discussions of the paper. We would also like to thank audience members at the 2017 CPEG Meeting, the 76th Annual MPSA Conference, the 2018 Quebec Political Economy Conference, the 2018 Empirical Microeconomics Workshop at Banff, the 2019 SAET Meetings, the 3rd Great Lakes Political Economy Conference, the 2020 World Congress of the Econometric Society, the 2020 SITE conference on political economy, the 12th conference on Economic Design, and the 29th Wallis Institute Annual Conference, as well as seminar participants at FGV EESP, FGV EPGE, MSU, and UQÀM for comments and suggestions. Braz Camargo gratefully acknowledges financial support from CNPq.

†Sao Paulo School of Economics – FGV. Address: Rua Itapeva 474, São Paulo, SP 01332-000, Brazil. E-mail: braz.camargo@fgv.br

‡ESG-UQÀM, Départment des sciences économiques. Address: 320, rue Sainte-Catherine Est Montréal, Québec, H2X 3X2, Canada. E-mail: degan.arianna@uqam.ca

1 Introduction

"An ideal political democracy is defined as: an institutional arrangement for arriving at political decisions in which individuals endeavor to acquire political office through perfectly free competition for the votes of a broadly based electorate." (Becker 1958, p.106).

By and large, economists view competition in a positive light. In the marketplace, the argument goes that competition among firms for consumers and the resulting risk of loss of market share ensures that firms behave in the consumers' best interest and only the best firms survive. Becker [1958] argues that in an ideal political democracy, free competition for votes ensures that politicians in power act in the electorate's best interest and only the best politicians survive. There are, however, two important imperfections in the political sector, namely, the entry costs associated with the large scale of operation in this sector and information and agency problems. Wittman [1989, 1995] argues that political markets nevertheless respond to information and agency problems, with competition for political office and the implied risk of political takeover playing a crucial role.[1]

In this paper, we study whether political competition indeed benefits voters. We do so in the political career concerns framework.[2] In this framework, an office-motivated incumbent politician privately makes an effort choice. The incumbent's performance depends on their effort and unknown ability. A (representative) voter observes the incumbent's performance, which provides information about the incumbent's ability, and then decides whether to retain the incumbent. The incumbent politician thus have an incentive to exert effort and benefit the voter so as to positively influence the voter's assessment of their ability and increase their likelihood of retention. The political career concerns framework captures

[1]More generally, Wittman [1989, 1995] argues that competition in the political sector, in much the same way as market competition, produces (constrained) efficient outcomes; see also Stigler [1972]. The analogy with economic markets is useful. In the same way that agents trade goods in economic markets, in an election voters trade votes for public-policy outcomes. The rules for determining electoral outcomes constitute the market-clearing mechanism: they determine which "goods" are traded in equilibrium.

[2]Political career concerns models adapt the career concerns model of Holmström [1999] to a political economy setting. They are used to study how politicians' concerns for their future career affect their behavior.

in a parsimonious way the presence of principal-agent and informational frictions in the political sector and is by now the standard electoral accountability framework.

We introduce competition in the political career concerns framework in the form of a citizen who decides whether to run for office against the incumbent. The citizen's ability is also unknown and the voter observes a signal about this ability before deciding whether to retain or replace the incumbent (should the citizen decide to run for office). Running for office is costly, though. Similarly to entry barriers in the marketplace, this cost affects the amount of political competition faced by the incumbent.[3] Other than the citizen's entry decision, our model is a standard political career concerns model.

We establish two sets of results. First, we show that the relationship between effective accountability, i.e., the incumbent's incentive to exert effort, and political competition is undetermined in the sense that an increase in political competition can be associated both with an increase and with a decrease in effective accountability. We also show that effective accountability need not be maximized when entry is costless. Then, we discuss economically meaningful conditions under which the relationship between political competition and effective accountability is determined in the sense that an increase in political competition has the same effect on effective accountability in every equilibrium. In particular, we discuss conditions under which an increase in political competition unequivocally *reduces* effective accountability.

In order to understand our indeterminacy results, notice that the incumbent's incentive to exert effort is tied to their likelihood of retention given their performance in office. By making it easier for the citizen to run for office against the incumbent, an increase in political competition has an ambiguous impact on effective accountability. On the one hand, it reduces the likelihood that an incumbent with a good performance is retained, which is bad for incentives. On the other hand, it reduces the likelihood that an incumbent with a poor performance is retained, which is good for incentives. We show that both the situation

[3]Our measure of political competition captures the ex-ante electoral advantage of incumbents. Another measure of such advantage is the margin of victory. There are other notions of political competition used in political economy and political science. See Bardhan and Yang [2004] for a discussion of these measures.

in which the positive effect dominates the negative effect and the situation in which the negative effect dominates the positive effect are compatible with equilibrium behavior. Our determinacy results consist in deriving conditions under which the positive effect always dominates or is always dominated by the negative effect. Loosely speaking, these conditions imply that entry after a good performance is always more or always less responsive to changes in political competition than entry after a poor performance.

Our baseline setting is such that the incumbent politician either faces one challenger, when the citizen enters, or faces no challenger, when the citizen does not enter. We also consider the case in which the incumbent politician always faces at least one challenger and there exists more than one potential entrant. We show that our indeterminacy results remain the same in this other case. More interestingly, the presence of an established challenger and of more than one potential entrant strengthens the negative impact of an increase in political competition on effective accountability by making entry after a good performance more responsive to an increase in political competition than entry after a poor performance. In particular, under mild technical assumptions, an increase in political competition *always* reduces effective accountability if the number of potential entrants is sufficiently large.

The rest of the paper is organized as follows. We discuss the related literature in the remainder of this section. In Section 2, we introduce our model and define equilibria. In Section 3, we characterize equilibria. In Section 4, we establish a number of results that are useful for our analysis and discuss our approach to comparative statics. In Section 5, we establish our results about the relationship between political competition and effective accountability. In Section 6, we consider the many-challengers extension. Section 7 concludes and the Appendix contains omitted proofs and details. Camargo and Degan [2023] contains additional details and extensions.

Related Literature. Several papers study theoretically or empirically (or both) the relationship between political competition and a number of economic and political outcomes, often obtaining conflicting results. De Paola and Scoppa [2011] and Gallaso and Nannicini [2011] establish a positive relationship between political competition and politician quality, while Dal Bó and Finan [2018] shows that this relationship can go both ways. Polo [1998] shows that political competition can increase political rents, while Svaleryd and Vlachos [2009] finds the opposite. Ashworth et al. [2014] finds that political competition increases the efficiency of municipal administration, while Afridi et al. [2021] finds that political competition may increase corruption. In the context of redistributive politics, Myerson [1993] and Lizzeri and Persico [2005] show, respectively, that a higher number of candidates is associated with more unequal redistribution and greater distortion in the provision of public goods. Arvate [2013] finds, instead, that more candidates increases the supply of local public goods. Theoretical and empirical studies of the relation between political competition and economic development and growth also reach mixed conclusions. Acemoglu and Robinson [2006] propose a model with a U-shaped relationship between political competition and economic development. Padovano and Ricciuti [2009] and Besley et al. [2010] find a positive relationship between political competition and economic growth, while Alfano and Baraldi [2015, 2016] find an inverted U-shaped relationship.

Starting with Barro [1973] and Ferejohn [1986], a large literature has studied how reelection concerns motivate politicians to behave in the voters' interest.[4] To our knowledge, our paper is the first to study the effect of political competition on effective accountability in a political career concerns setting. Our results show, contrary to Wittman [1989, 1995], that increasing political competition can lower the incentive of politicians in office to behave in the voters' interest.[5] More generally, our results show that without additional assumptions on how entry of politicians responds to changes in political competition there exists no relationship between political competition and effective accountability.

The analysis in our paper also speaks to the literature on incumbency advantage.[6] A common view in this literature is that higher retention probabilities due to office holding

[4]See Ashworth [2012] and Duggan and Martinelli [2017] for reviews of the literature, and Besley and Case [1995], Alt et al. [2011], and Ferraz and Finan [2011] for evidence on this.

[5]Kartik et al. [2015] show, also contrary to Wittman [1989, 1995], that political competition can fail to promote efficient aggregation of politicians' policy-relevant private information.

[6]See, e.g., Cox and Katz [1996], Levitt and Wolfram [1997], Stone et al. [2004], Gordon et al. [2007], Ashworth and Bueno de Mesquita [2008], and Hall and Snyder [2015]

hurt voters by lowering effective accountability and reducing the voters' ability to replace low-quality incumbents.[7] A corollary of this view is that increasing political competition is beneficial to voters. Our analysis shows that this conclusion is not warranted.[8]

2 Model

In this section, we present our model, define equilibria, and make some remarks about our modelling choices.

Agents. There are three agents, namely, an incumbent, a citizen who can run for office against the incumbent, and a representative voter. We refer to the incumbent and the citizen as the politicians. Politicians can be of one of two types: a low-ability type and a high-ability type. We denote a politician's type by τ, where $\tau = L$ if the politician is of low ability and $\tau = H$ otherwise. A politician's type is unknown to all agents, including the politician, and is independent of the other politician's type. The (ex-ante) probability a politician is of high ability is $\pi_0 \in (0, 1)$. We refer to the probability that the other agents assign to a politician being of high ability as the politician's reputation.

Output. A politician's output in office depends on their private choice of effort $a \in A = [0, \overline{a}]$ and on their type τ, and is either $y = h$, a success, or $y = \ell$, a failure. The probability a politician of type τ who exerts effort a succeeds is $f(a, \tau) > 0$, where f is a twice continuously differentiable, strictly increasing, and strictly concave function of a with $\partial f(0, \tau)/\partial a$ finite for all τ. Moreover, $f(a, H) > f(a, L)$ for all $a \in A$, so high-ability politicians are more likely to succeed than low-ability ones no matter their effort. In an abuse of notation, we let $f(a, \pi) = \pi f(a, H) + (1 - \pi)f(a, L)$ be the ex-ante probability a politician of reputation π succeeds when their effort is a. The assumption of binary output simplifies the exposition without changing our message; see Camargo and Degan [2023] for details.

[7] In a diverging view, Ashworth et al. [2019] shows that incumbency advantage can arise solely from the fact that office holding provides information about incumbents to voters.

[8] While not an object of interest in our analysis, we can extend our indeterminacy results to show that there exists no relationship between political competition and incumbency advantage in our setting.

Learning. The incumbent's performance in office is observable, and so can be used by all agents to update their beliefs about the incumbent's type. Before deciding whether to run for office, the citizen observes a public signal about their ability.[9] Since observing this signal and the citizen's reputation are equivalent, we take the citizen's signal to be their reputation. Let Ω be the c.d.f. describing the unconditional distribution of the citizen's reputation. We assume Ω has support $[0,1]$ and a continuous density ω in $(0,1)$.[10]

Preferences. Politicians care only about holding office and effort is costly for them. The benefit of holding office is $B > 0$ and the cost of exerting effort a is $c(a)$. The function c is twice continuously differentiable, increasing and convex, and satisfies the 'Inada' conditions $c(0) = c'(0) = 0$ and $c'(\overline{a}) > B\partial f(0, \pi_0)/\partial a$. The voter's payoff from having a politician in office is $F = 0$ if the politician fails and $S > 0$ otherwise. In addition, the voter's payoff from keeping the incumbent in office depends on an additive shock $z \in \mathbb{R}$. This shock represents a dimension of horizontal differentiation that co-exists with the dimension of vertical differentiation at the core of our analysis and is distributed according to a continuously differentiable c.d.f. Γ with support $[-\eta, \eta]$ where $\eta > 0$.

Entry. The citizen pays a cost $\kappa \in [0, \overline{\kappa}]$ with $\overline{\kappa} < B$ if they run for office. This cost measures the extent of political competition faced by the incumbent: lowering κ increases political competition by making a challenge to the incumbent less difficult.[11] In what follows, we use a change in the entry cost and a change in political competition interchangeably.

Timing. Action takes place in two periods. In the first period, the incumbent privately chooses their effort, output is realized, and agents update their beliefs about the incumbent's type. In the second period, the citizen observes their reputation and decides whether to run for office. Following that, the voter chooses which politician to put in office after privately

[9]So, there exists no asymmetry of information between the citizen and the voter at the election stage. This implies that the citizen's decision to run for office has no signaling content.

[10]So, from an ex-ante perspective, any reputation is possible for the citizen. Our results extend to the case in which the support of Ω is contained in $[0,1]$, and either sufficiently low or sufficiently high reputations (or both) are not possible *a priori*. We gain no insights by doing so.

[11]The cost of entry represents not only the direct monetary costs the citizen has to pay to mount a challenge to the incumbent but also any opportunity costs incurred by the citizen when they run for office.

observing the incumbent-specific preference shock and, if the citizen runs for office, the citizen's reputation.[12] The incumbent is automatically retained if the citizen does not run for office. Finally, the politician in office in the second period makes an effort choice.

Strategies and Equilibria. A strategy profile is a list $(a, \sigma, \rho, a^I, a^C)$ where: (*i*) $a \in A$ is the incumbent's first-period effort; (*ii*) $\sigma : [0,1] \times \{\ell, h\} \mapsto [0,1]$ is a function such that $\sigma(\pi, y)$ is the probability the citizen enters if their reputation is π and the incumbent's first-period output is y; (*iii*) $\rho : [0,1] \times \{\ell, h\} \times [-\eta, \eta] \mapsto [0,1]$ is a function such that $\rho(\pi, y, z)$ is the probability the voter retains the incumbent if the citizen runs for office, the citizen's reputation is π, the incumbent's first-period output is y, and the shock to the voter's payoff is z; (*iv*) $a^I : A \times \{\ell, h\} \times ([0,1] \cup \{\emptyset\}) \mapsto A$ is a function such that $a^I(a, y, \pi)$ is the incumbent's second-period effort if retained by the voter given the incumbent's first-period effort a and output y and the citizen's reputation π, where $\pi = \emptyset$ if the citizen does not enter; and (*v*) $a^C : [0,1] \times \{\ell, h\} \mapsto A$ is a function such that $a^C(\pi, y)$ is the citizen's second-period effort if they enter and are put in office by the voter given their reputation π and the incumbent's first-period output y. A belief system is a map $\pi : \{\ell, h\} \mapsto [0,1]$ such that $\pi(y)$ is the incumbent's reputation as a function of their first-period output.

We consider pure-strategy sequential equilibria. Consistency of beliefs is guaranteed by Bayes' rule since both output realizations are possible regardless of the incumbent's first-period effort choice. Clearly, politicians have no incentive to exert effort in the second period. So, in what follows, we take the politicians' choice of effort in the second period to be always zero and omit it from our description of equilibria. Given this, we refer to the incumbent's choice of effort in the first period simply as the incumbent's effort.

Remarks. We depart from the canonical political career concerns model by endogenizing the set of agents who run for office against the incumbent. Two features of our model are important in this regard, namely, the presence of horizontal differentiation and the citizen's random reputation. Horizontal differentiation introduces uncertainty in the voter's

[12]Our results do not change if the shock z is public. The key assumption is that the incumbent does not know the realization of this shock when making an effort choice in the first period.

retention decision. Without this uncertainty, the citizen's entry decision does not respond to changes in the entry cost: either the citizen always enters or the citizen never enters. Without a random reputation for the citizen, small changes in the entry cost do not change the incumbent's probability of retention conditional on their output. Indeed, if the citizen's reputation is not random, then, except in knife-edge cases, the citizen who is indifferent between entering or not entering is not marginal with respect to the voter's retention decision.

We assume that politicians care only about holding office. This amounts to assuming that a politician's ability has no value outside of the political sector. We can extend our analysis to the case in which a politician's ability has value outside of the political sector. When this is the case, the citizen has a reputation-dependent outside option.[13] This implies that, unlike in our setting, the citizen's entry decision need not be a cutoff rule in that the citizen enters if, and only if, their reputation is above a certain threshold.[14] The analysis of this more general case is less transparent and yields no new insights. For this reason, we relegate it to Camargo and Degan [2023].

3 Equilibrium Characterization

In this section, we characterize equilibria. We proceed by backward induction. First, we consider the voter's appointment decision. Then, we consider the citizen's entry decision. Finally, we consider the incumbent's effort choice. The main result is a characterization of the incumbent's equilibrium choice of effort, which is the basis of our subsequent analysis.

3.1 Appointment Decision

First note that the voter reappoints the incumbent if the citizen does not run for office. Consider then the case in which the citizen runs for office and let π_I and π_C be, respectively, the incumbent's and the citizen's reputation in the second period. Since politicians exert no

[13]Moreover, as in the presence of incentive pay, the incumbent benefits from a success even if not retained in office, increasing their incentive for exerting effort.

[14]In particular, a citizen with a high enough reputation may not find it optimal to enter, so that a reduction in the entry cost can, unlike our setting, improve the expected quality of the politician who runs for office.

effort in office in the second period, the voter replaces the incumbent if, and only if

$$Sf(0, \pi_I) + z \leq Sf(0, \pi_C),$$

where z is the shock to the voter's preferences.[15] Now let $\theta = \eta/S[f(0,H) - f(0,L)] > 0$. The next result follows from straightforward algebra.

Lemma 1. *Suppose the citizen runs for office. The voter replaces the incumbent if, and only if, $z \leq (\pi_C - \pi_I)\eta/\theta$.*

Let $G : \mathbb{R} \mapsto [0,1]$ be such that $G(x) = \Gamma((\eta/\theta)x)$. The function G is nondecreasing and continuously differentiable, with $G(-\theta) = 0$ and $G(\theta) = 1$. Moreover, G is strictly increasing in $[-\theta, \theta]$. It follows from Lemma 1 that the probability the incumbent is replaced is $G(\pi_C - \pi_I)$. This probability increases with the reputation difference $\pi_C - \pi_I$, is zero if this difference is sufficiently negative, and is one if this difference is sufficiently positive.

The ratio θ measures the importance of the shock to the voter's preferences relative to vertical differentiation between politicians. When θ is small, and vertical differentiation is important, the probability $G(\pi_C - \pi_I)$ is responsive even to small changes in $\pi_C - \pi_I$. On the other hand, when θ is large, $G(-1) > 0$ and $G(1) < 1$, and the voter can find it optimal to replace a high-ability incumbent even if the citizen is of low ability and keep a low-ability incumbent even if the citizen is of high ability. We focus our analysis in the case in which θ is small. In Camargo and Degan [2023], we discuss the large-θ case and show that the substance of our results remains the same.

Assumption 1. $\theta < \min\{\pi_0, 1 - \pi_0\}$.

3.2 Entry Decision

The citizen's expected payoff from running for office when their reputation is π_C, the incumbent's reputation is π_I, and the entry cost is κ is

$$BG(\pi_C - \pi_I) - \kappa.$$

[15] Since the shock z is continuously distributed, it is without loss to assume that the voter replaces the incumbent when indifferent between the incumbent and the citizen.

We assume the citizen does not run for office when indifferent between running and not running. This assumption is without loss when the entry cost is positive, since in this case the probability the citizen is indifferent between running and not running for office is zero. This assumption constrains the citizen's behavior when entry is costless, though. Indeed, when $\kappa = 0$, it is (weakly) optimal for the citizen to run for office even if their probability of being selected is zero. The assumption that the citizen enters only if it is strictly optimal to do so is a reasonable equilibrium refinement. It rules out situations in which the citizen's equilibrium behavior when $\kappa = 0$ cannot be approximated by the citizen's equilibrium behavior when κ is positive but small.

Let $H : [0, 1] \to [-\theta, \theta]$ be the inverse of the restriction of G to $[-\theta, \theta]$. The function H is continuous, strictly increasing, continuously differentiable, and such that $H(0) = -\theta$ and $H(1) = \theta$. Since $\kappa/B \in [0, 1]$ regardless of the entry cost κ, the indifference condition $G(\pi_C - \pi_I) = \kappa/B$ is equivalent to $\pi_C = \pi_I + H(\kappa/B)$. Let

$$\Pi_C(\pi_I, \kappa) = \pi_I + H\left(\frac{\kappa}{B}\right). \tag{1}$$

The next result follows immediately.

Lemma 2. *The citizen enters if, and only if,* $\pi_C > \Pi_C(\pi_I, \kappa)$.

Lemma 2 implies that the citizen runs for office with an interior probability if, and only if, $\Pi_C(\pi_I, \kappa) \in (0, 1)$—the citizen enters with probability one if $\Pi_C(\pi_I, \kappa) \leq 0$ and enters with probability zero if $\Pi_C(\pi_I, \kappa) \geq 1$. Since $\Pi_C(\pi_I, \kappa)$ is strictly increasing in κ, the citizen's probability of entry is responsive to changes in political competition when this probability is interior.[16] Note from Assumption 1 that $\Pi_C(\pi_0, \kappa) \in (0, 1)$ for all $\kappa \in [0, \overline{\kappa}]$.

3.3 Incumbent's Effort

To conclude our equilibrium characterization, we consider the incumbent's choice of effort in the first period. We begin by determining the probability the incumbent is retained as

[16] When there exists no horizontal differentiation among politicians, the citizen's entry decision does not respond to changes in κ. Indeed, when $\eta = 0$, the citizen enters if, and only if, $\pi_C \geq \pi_I$.

a function of their reputation in the second period. We then discuss how the incumbent's second-period reputation depends on their performance in the first period given the conjecture about their effort. Together, these two pieces of information allow us to the determine the incumbent's payoff given their effort and the conjecture about their effort. This, in turn, allows us to characterize the incumbent's equilibrium effort since in equilibrium the incumbent's optimal choice of effort must coincide with the conjecture about their effort.

Retention. The incumbent is retained either when $\pi_C \leq \Pi_C(\pi_I, \kappa)$, and the citizen does not run for office, or when $\pi_C > \Pi_C(\pi_I, \kappa)$ but $z > (\pi_C - \pi_I)\eta/\theta$, and the citizen runs for office but is not chosen by the voter. The probability $Q(\pi_I, \kappa)$ that the incumbent is retained when their reputation is π_I and the entry cost is κ is then equal to

$$Q(\pi_I, \kappa) = \Omega(\Pi_C(\pi_I, \kappa)) + \int_{\max\{0, \Pi_C(\pi_I, \kappa)\}}^{1} [1 - G(\pi - \pi_I)]\omega(\pi) d\pi,$$

where we adopt the convention that the above integral is zero if $\Pi_C(\pi_I, \kappa) > 1$. The next result establishes some properties of $Q(\pi_I, \kappa)$; see the Appendix A.1 for a proof.

Lemma 3. *The probability $Q(\pi_I, \kappa)$ is continuous, nondecreasing with π_I and κ, and strictly increasing with π_I and κ if $\Pi_C(\pi_I, \kappa) \in (0, 1)$. Moreover, $Q(\pi_I, \kappa)$ is continuously differentiable in the set $\{(\pi_I, \kappa) \in [0, 1] \times [0, \overline{\kappa}] : \Pi_C(\pi_I, \kappa) \neq 0, 1\}$ with*

$$\frac{\partial Q}{\partial \kappa}(\pi_I, \kappa) = \omega(\Pi_C(\pi_I, \kappa)) \frac{\kappa}{B} \frac{\partial \Pi_C}{\partial \kappa}(\pi_I, \kappa) \tag{2}$$

for all $(\pi_I, \kappa) \in [0, 1] \times [0, \overline{\kappa}]$ such that $\Pi_C(\pi_I, \kappa) \in (0, 1)$.

Given that $\Pi_C(\pi_I, \kappa)$ is strictly increasing with π_I and κ, it immediately follows that $Q(\pi_I, \kappa)$ is nondecreasing with π_I and κ and strictly increasing with π_I and κ as long as $\Pi_C(\pi_I, \kappa)$ is interior. The intuition for (2) is simple. When $\Pi_C(\pi_I, \kappa) \in (0, 1)$ and the probability of entry for the citizen is interior, the marginal increase in the probability the incumbent is retained following a marginal increase in the cost of entry is proportional to: (*i*) the probability the citizen is on the margin between entering and not entering; and (*ii*) the increase in the cutoff of entry. Since $\omega(\pi) > 0$ for all $\pi \in (0, 1)$, we then have that the incumbent's probability of retention is responsive to changes in the cost of entry if the citizen's probability of entry is interior.[17]

Belief Updating. Let a^e be the voter's and the citizen's conjecture about the incumbent's effort and $\pi^+(y, a^e)$ be the incumbent's reputation in the second period when their output in the first period is y given the conjecture a^e. Bayes' rule implies that

$$\pi^+(y, a^e) = \frac{\mu(y, a^e, H)\pi_0}{\mu(y, a^e, H)\pi_0 + \mu(y, a^e, L)(1 - \pi_0)},$$

where $\mu(h, a, \tau) = f(a, \tau)$ and $\mu(\ell, a, \tau) = 1 - \mu(h, a, \tau)$. Given that $f(a, H) > f(a, L)$ for all $a \in A$, it follows that $\pi^+(\ell, a) < \pi_0 < \pi^+(h, a)$ for all $a \in A$. So, the incumbent's performance in office is always informative about their ability.

Payoffs and Equilibrium Effort. The incumbent's payoff depends not only on their effort a but also on the conjecture about their effort a^e. Indeed, as discussed above, the conjecture a^e determines how the incumbent's reputation responds to their performance in the first period. When the entry cost is κ, the incumbent's payoff given a and a^e is

$$U(a, a^e, \kappa) = B\left[f(a, \pi_0)Q(\pi^+(h, a^e), \kappa) + (1 - f(a, \pi_0))Q(\pi^+(\ell, a^e), \kappa)\right] - c(a).$$

Since, by Lemma 3, $Q(\pi^+(h, a^e), \kappa) \geq Q(\pi^+(\ell, a^e), \kappa)$ regardless of a^e and κ, it follows that $U(a, a^e, \kappa)$ is concave in a for all $a^e \in A$ and $\kappa \in [0, \overline{\kappa}]$.

The effort a^* is an equilibrium choice of effort for the incumbent when the entry cost is κ if a^* maximizes $U(a, a^*, \kappa)$; i.e., $U(a^*, a^*, \kappa) \geq U(a, a^*, \kappa)$ for all $a \in A$. The next result provides a characterization of the incumbent's equilibrium choices of effort.

Lemma 4. *The effort a^* is an equilibrium choice of effort for the incumbent when the entry cost is κ if, and only if,*

$$B\frac{\partial f}{\partial a}(a^*, \pi_0)\left[Q(\pi^+(h, a^*), \kappa) - Q(\pi^+(\ell, a^*), \kappa)\right] = c'(a^*). \tag{3}$$

[17] When π_C is deterministic, $Q(\pi_I, \kappa) = 1$ if $\Pi_C(\pi_I, \kappa) > \pi_C$ and $Q(\pi_I, \kappa) = 1 - G(\pi_C - \pi_I)$ otherwise. So, in this case, unless $\Pi_C(\pi_I, \kappa) = \pi_C$, the probability $Q(\pi_I, \kappa)$ does not respond to small changes in κ.

The interpretation of (3) is straightforward. The left-hand side is the marginal benefit of effort to the incumbent when the entry cost is κ, their effort is a^*, and the citizen and the voter correctly anticipate the incumbent's behavior. The right-hand side is the marginal cost of effort a^*. The concavity of $U(a, a^e, \kappa)$ in a for all $a^e \in A$ and $\kappa \in [0, \overline{\kappa}]$ implies that (3) is sufficient for $a^* \in A$ to maximize $U(a, a^*, \kappa)$. In Appendix A.2, we show that (3) is also necessary for $a^* \in A$ to maximize $U(a, a^*, \kappa)$.

Since, by Lemma 3, the left-hand side of (3) is continuous in a^*, the intermediate value theorem and the Inada conditions on c imply that (3) has a solution for all κ.[18] The Inada conditions on c also imply that $a^* \in A$ can be an equilibrium choice of effort for the incumbent only if $a^* < \overline{a}$ and that $a^* = 0$ is an equilibrium choice of effort for the incumbent only if the incumbent's probability of retention does not depend on their output when $a^e = 0$. The latter, however, is not possible as $\Pi_C(\pi_0, \kappa) \in (0, 1)$ for all $\kappa \in [0, \overline{\kappa}]$, and so the incumbent's probability of retention responds to their performance in office regardless of a^e; see Appendix A.3 for details. Lemma 4 thus admits the following corollary.

Corollary 1. *An equilibrium choice of effort for the incumbent always exists and is interior.*

4 Preliminaries

Here, we lay down the groundwork for our analysis of the relationship between effective accountability and political competition. We first discuss the scope for equilibrium multiplicity and our approach to comparative statics. We then show that any interior choice of effort for the incumbent is consistent with equilibrium behavior and provide sufficient conditions for two different interior effort choices to be consistent with equilibrium behavior.

Before beginning with our analysis, we introduce some notation and definitions. Let

$$MB(a, \kappa) = B \frac{\partial f}{\partial a}(a, \pi_0) \left[Q(\pi^+(h, a), \kappa) - Q(\pi^+(\ell, a), \kappa) \right]$$

be the incumbent's marginal benefit of exerting effort a when the entry cost is κ and the

[18] Indeed, the left-hand side of (3) is bounded below by $0 = c'(0)$ and above by $B\partial f(0, \pi_0)/\partial a < c'(\overline{a})$.

other agents correctly anticipate the incumbent's behavior,

$$\delta(a,\kappa) = Q(\pi^{+}(h,a),\kappa) - Q(\pi^{+}(\ell,a),\kappa)$$

be the increase in the incumbent's probability of retention following a success when their conjectured choice of effort is a and the entry cost is κ, and

$$\Delta(a,\kappa) = MB(a,\kappa) - c'(a).$$

Let $\underline{\pi}^{+}(\ell) = \inf_a \pi^{+}(\ell,a)$ and $\overline{\pi}^{+}(h) = \sup_a \pi^{+}(h,a)$. We say that the informativeness of output is bounded if $\underline{\pi}^{+}(\ell) > 0$ and $\overline{\pi}^{+}(h) < 1$. Note that $\overline{\pi}^{+}(h) < 1$ since $f(a,L) > 0$ for all $a \in A$ and success is never perfectly informative of high ability. It is easy to see that $\underline{\pi}^{+}(\ell) > 0$ if, and only if, $f(\overline{a},H) < 1$ and failure is never perfectly informative of low ability. Finally, we say that effort increases the informativeness of output if $\pi^{+}(h,a)$ is increasing with a and $\pi^{+}(\ell,a)$ is decreasing with a and say that effort decreases the informativeness of output if the opposite takes place.

4.1 Equilibrium Multiplicity

Clearly, there exists a unique equilibrium choice of effort for the incumbent when the entry cost is κ if $MB(a,\kappa)$ is nondecreasing with a. On the other hand, multiple equilibrium choices of effort for the incumbent can arise when $MB(a,\kappa)$ is not decreasing with a, in which case (3) can have multiple solutions; see Figure 1. To see why $MB(a,\kappa)$ need not be decreasing with a, note that while the strict concavity of f implies that $\partial f(a,\pi_0)/\partial a$ is strictly decreasing with a, the difference $\delta(a,\kappa)$ need not be decreasing with a. A necessary condition for this is that effort does not decrease the informativeness of output.[19] At the end of this section, we show that if the average marginal benefit of effort, $MB(a,\kappa)/a$, is nondecreasing with a in $(0,\overline{a})$, then, for a suitable choice of the cost function, there exist multiple equilibrium choices of effort for the incumbent when the entry cost is κ. We

[19] We show in Camargo and Degan [2023] that effort increasing the informativeness of output is not sufficient for $MB(a,\kappa)$ to be nondecreasing with a. So, the equilibrium can be unique even when effort increases the informativeness of output.

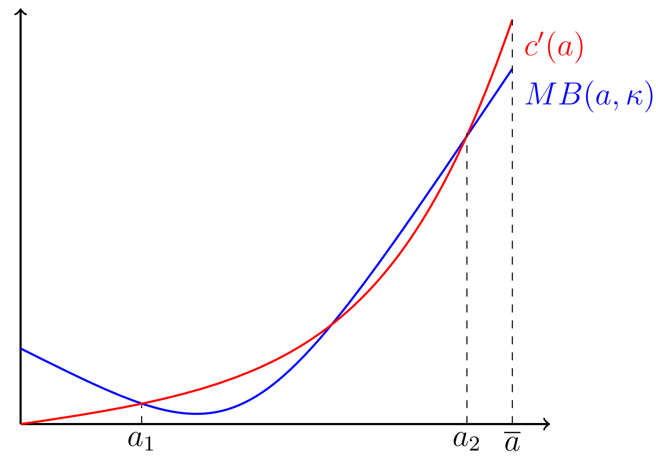

Figure 1: Equilibrium Multiplicity

provide an example of a production function for which $MB(a_1, \kappa)/a < MB(a_2, \kappa)/a_2$ for some $0 < a_1 < a_2 < \overline{a}$ in the proof of Proposition 2 in the next section.

4.2 Comparative Statics

It follows from Lemma 4 that the incumbent's incentive to exert effort when their conjectured effort is a and the entry cost is κ is proportional to the increase $\delta(a, \kappa)$ in their probability of retention following a success. By affecting the citizen's entry decision, a change in the entry cost affects the incumbent's probability of retention conditional on their reputation, thus changing $\delta(a, \kappa)$ for all $a \in A$. This, in turn, alters the incumbent's incentive to exert effort, and so their equilibrium choice of effort.

When the incumbent's equilibrium choice of effort is unique for every entry cost, one can show that the response to a change in political competition is unambiguous: if a^* is the incumbent's equilibrium choice of effort given an entry cost κ, then a change in κ that increases (respectively, decreases) $\delta(a^*, \kappa)$ leads to higher (respectively, lower) effort.[20] However, as discussed above, multiple equilibria arise naturally in our setting. Thus, in our analysis, we rely on local comparative statics analysis using the implicit function theorem.

[20]See Camargo and Degan [2023] for details.

Suppose a^* is an equilibrium choice of effort for the incumbent when the entry cost is $\kappa_0 \in (0, \overline{\kappa})$. If $\Delta(a, \kappa)$ is continuously differentiable in a neighborhood of (a^*, κ_0) and $\partial\Delta(a^*, \kappa_0)/\partial a \neq 0$, then the implicit function theorem implies that there exist open sets $E \subseteq (0, \overline{a})$ and $F \subseteq (0, \overline{\kappa})$ with $(a^*, \kappa_0) \in E \times F$ and a continuously differentiable function $a^* : F \to E$ such that $\Delta(a, \kappa) = 0$ for $(a, \kappa) \in E \times F$ if, and only, if $a = a^*(\kappa)$; i.e., $a^*(\kappa)$ is the unique equilibrium choice of effort for the incumbent in a neighborhood of a^* when the entry cost κ is in a neighborhood of κ_0. Totally differentiating the equation $\Delta(a^*(\kappa), \kappa) = 0$ at $\kappa = \kappa_0$ and solving for $da^*(\kappa_0)/da$ yields

$$\frac{da^*}{d\kappa}(\kappa_0) = -\frac{\partial\Delta(a^*, \kappa_0)/\partial\kappa}{\partial\Delta(a^*, \kappa_0)/\partial a} = -\frac{B\partial f(a^*, \kappa_0)/\partial a}{\partial\Delta(a^*, \kappa_0)/\partial a} \cdot \frac{\partial\delta}{\partial\kappa}(a^*, \kappa_0). \tag{4}$$

The derivative in (4) describes the effect on the incumbent's equilibrium choice of effort of a local change in the entry cost. It makes explicit how the incumbent's equilibrium behavior responds to changes in $\delta(a, \kappa)$.

As is well known, in the presence of multiple equilibria, local comparative statics analysis using the implicit function theorem can lead to ambiguous results in the sense that if a_1^* and a_2^* are equilibrium choices of effort for the incumbent given an entry cost κ_0, then local changes in the entry cost that change $\delta(a_1^*, \kappa_0)$ and $\delta(a_2^*, \kappa_0)$ in the same direction can lead to opposite changes in the incumbent's effort. So, as is standard, we restrict attention to *stable* equilibria. When the entry cost is κ_0, an equilibrium in which the incumbent's choice of effort is a^* is stable if $\Delta(a, \kappa)$ is continuously differentiable in a neighborhood of (a^*, κ_0) and $\partial\Delta(a^*, \kappa_0)/\partial a < 0$.

Restricting attention to stable equilibria ensures that a local change in political competition leads to an unambiguous and economically meaningful response in the incumbent's behavior: if the incumbent's equilibrium choice of effort is a^* when the entry cost is κ_0, then a local change in the entry cost that increases (respectively, decreases) $\delta(a^*, \kappa_0)$ leads to higher (respectively, lower) effort. So, our indeterminacy results are not driven by the selection of equilibria in which the incumbent's behavior responds in an implausible way to chances in the increase in the probability of retention following a success.

We conclude this part with some remarks about our equilibrium refinement. First note that if $MB(a,\kappa)$ is nondecreasing with a, then the unique equilibrium is stable if $MB(a,\kappa)$ is continuously differentiable. Also note from Lemma 3 that if $\Pi_C(\pi^+(a^*,y),\kappa_0) \neq 0, 1$ for all y, then $\Delta(a,\kappa)$ is continuously differentiable in a neighborhood of (a^*,κ_0). So, the requirement that $\Delta(a,\kappa)$ is continuously differentiable in a neighborhood of (a^*,κ_0) if a^* is an equilibrium choice of effort for the incumbent when the entry cost is κ_0 can fail to hold only in knife-edge cases. Moreover, since $H(\kappa/B) \in [-\theta,\theta]$ for all $\kappa \in [0,\overline{\kappa}]$, it follows that $\Pi_C(\pi^+(y,a),\kappa) \in (0,1)$ regardless of a, y, and κ when $\theta < \min\{\underline{\pi}^+(\ell), 1-\overline{\pi}^+(h)\}$. Thus, $\Delta(a,\kappa)$ is continuously differentiable if θ is small enough when the informativeness of output is bounded. Finally, note that if a^* is the unique equilibrium choice of effort for the incumbent when the entry cost is κ_0—or, more generally, if a^* is the highest equilibrium choice of effort for the incumbent when the entry cost is κ_0—and $\Pi_C(\pi^+(y|a),\kappa_0) \neq 0, 1$ for all y, then one can perturb the cost function c so that a^* remains the unique (or highest) equilibrium effort choice for the incumbent and the equilibrium is stable.[21]

4.3 Rationalization

We first show that regardless of the entry cost, every $a^* \in (0,\overline{a})$ can be an equilibrium choice of effort for the incumbent for a suitably chosen cost function. Moreover, we can choose this cost function in such a way that if the entry cost is κ_0, then a^* is the incumbent's unique equilibrium choice of effort and, in case $MB(a,\kappa)$ is continuously differentiable in a neighborhood of (a^*,κ_0), the equilibrium is stable. In what follows, we say that a cost function c is *admissible* if it is twice continuously differentiable, increasing, convex, and satisfies the Inada conditions described in Section 2. Recall that besides the cost function and the entry cost, the primitives of the model are the probability π_0 that politicians are of high type, the production function f, the distribution of the citizen's reputation Ω, the benefit of holding office B, the voter's payoff from a success S, and the distribution of the incumbent-specific preference shocks Γ.

[21] See Camargo and Degan [2023] for details.

Lemma 5. *Fix all the model's primitives but the cost function. For each* $a^* \in (0, \overline{a})$*, there exists an admissible cost function* c *such that* a^* *is the unique equilibrium choice of effort for the incumbent when the cost function is* c*. Moreover, for all* $\kappa_0 \in [0, \overline{\kappa}]$ *such that* $MB(a, \kappa)/\partial a$ *is continuously differentiable in a neighborhood of* (a^*, κ_0)*, we can choose* c *so that the equilibrium is stable when the entry cost is* κ_0*.*

Lemma 5 is intuitive. Indeed, let $a^* \in (0, \overline{a})$ and suppose the entry cost is κ_0. Since $MB(a^*, \kappa_0)$ is positive by the proof of Corollary 1 and $MB(a, \kappa)$ is bounded above by $B\partial f(0, \pi_0)/\partial a$, an admissible cost function c with $c'(a^*) = MB(a^*, \kappa_0)$ clearly exists. In Appendix A.4, we show that we can choose c to be such that $c'(a) \neq MB(a, \kappa_0)$ for all $a \neq a^*$, in which case a^* is the unique equilibrium choice of effort for the incumbent. Moreover, we show that if $MB(a, \kappa)$ is continuously differentiable in a neighborhood of (a^*, κ_0), then we can choose c so that $\partial MB(a^*, \kappa_0)/\partial a < c''(a^*)$ and the equilibrium is stable as well. A sketch of the argument is as follows. First, we show that there exists a nondecreasing piecewise linear function $\phi : A \to \mathbb{R}$ with $\phi(0) = 0$, $\phi(a) < MB(a, \kappa_0)$ for all $a < a^*$, $\phi(a) > MB(a, \kappa_0)$ for all $a > a^*$, and $\phi(\overline{a}) > B\partial f(0, \pi_0)/\partial a$; see Figure 2. Moreover, ϕ is such that $\partial MB(a^*, \kappa_0)/\partial a < \phi'(a^*)$ if $\partial MB(a^*, \kappa_0)/\partial a$ exists.[22] We then show that we can approximate ϕ by a continuously differentiable function λ with the same properties. The function $c : A \mapsto \mathbb{R}$ with $c(a) = \int_0^a \lambda(s)ds$ is the desired cost function.[23]

We now show that if the average marginal benefit of effort $MB(a, \kappa)/a$ is nondecreasing with effort, then multiple effort choices for the incumbent are consistent with equilibrium behavior. Also, if $MB(a, \kappa)$ is continuously differentiable, then these effort choices are equilibrium effort choices in stable equilibria under an additional technical assumption.

Lemma 6. *Fix all the model's primitives but the cost function and let* κ_0 *be the entry cost. If there exist* $0 < a_1 < a_2 < \overline{a}$ *with* $MB(a_1, \kappa_0)/a_1 < MB(a_2, \kappa_0)/a_2$*, then there exists*

[22] For such a function to exist, it is sufficient that $r(a) = (MB(a, \kappa_0) - MB(a^*, \kappa_0))/(a - a^*)$ is bounded in $A \setminus \{a^*\}$. For simplicity, we assume in the proof of Lemma 5 that $\partial MB(a^*, \kappa_0)/\partial a$ exists, so that $r(a)$ has the desired property. In Camargo and Degan [2023], we show that this assumption is not necessary.

[23] The cost function c is neither strictly increasing nor strictly convex. We can alter c so that it becomes strictly increasing and strictly convex by changing the function ϕ so that it, and thus λ, is strictly increasing. It is clear from the proof of Lemma 5 that the cost function c is not unique.

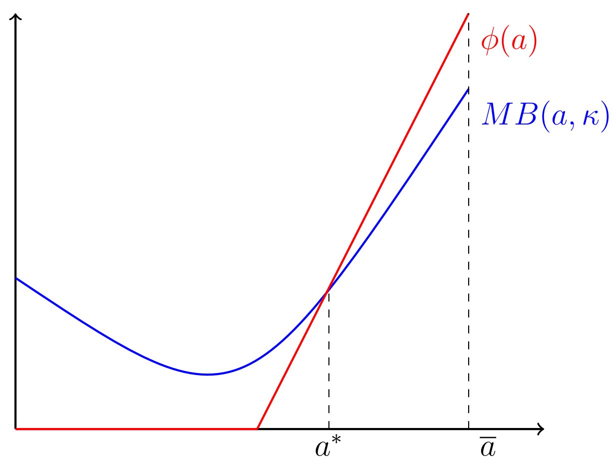

Figure 2: Rationalization of Unique Stable Equilibrium

an admissible cost function c such that a_1 and a_2 are equilibrium effort choices for the incumbent when the cost function is c. Moreover, if $MB(a, \kappa)$ is continuously differentiable and $\partial MB(a_1, \kappa_0)/\partial a < MB(a_2, \kappa_0)/a_2$, then we can choose c such that a_1 and a_2 are effort choices for the incumbent in stable equilibria.

A sketch of the proof of Lemma 6 is as follows; details are in Appendix A.5. Let the entry cost be κ_0 and suppose $0 < a_1 < a_2 < \overline{a}$ satisfy $MB(a_1, \kappa_0)/a_1 < MB(a_2, \kappa_0)/a_2$. We can construct a nondecreasing piecewise linear function $\phi : A \to \mathbb{R}$ with $\phi(0) = 0$, $\phi(a_i) = MB(a_i, \kappa_0)$ for $i = 1, 2$, and $\phi(\overline{a}) > B\partial f(0, \pi_0)/\partial a$; see Figure 3. As in the proof of Lemma 5, we can approximate ϕ by a continuously differentiable nondecreasing function λ with $\lambda(0) = 0$, $\lambda(a_i) = MB(a_i, \kappa_0)$ for each i, and $\lambda(\overline{a}) > B\partial f(0, \pi_0)/\partial a$. Hence, $c : A \to \mathbb{R}$ with $c(a) = \int_0^a \lambda(s)ds$ is an admissible cost function such that a_1 and a_2 are equilibrium choices of effort for the incumbent when the cost function is c. We show that if $MB(a, \kappa)$ is continuously differentiable, then we can construct ϕ, and thus λ, so that a_2 is an equilibrium choice of effort for the incumbent in a stable equilibrium when the cost function is c. Moreover, we show that if $\partial MB(a_1, \kappa_0)/\partial a < MB(a_2, \kappa_0)/a_2$, then we can take c to be such that a_1 is also an equilibrium effort choice for the incumbent in a stable equilibrium if the cost function is c.

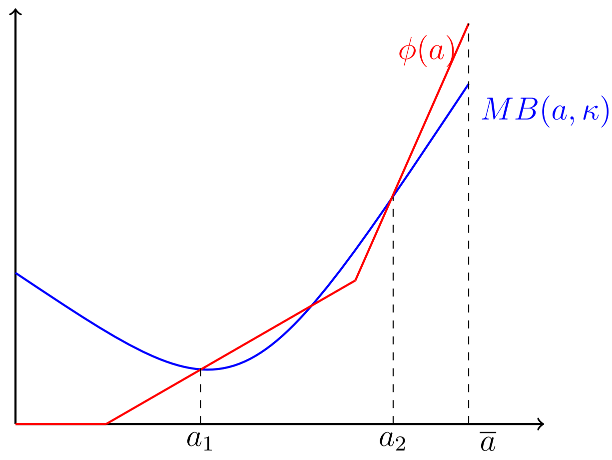

Figure 3: Rationalization of Multiple Stable Equilibria

The second part of Lemma 6 requires the stronger assumption that $MB(a, \kappa)$ is continuously differentiable. We know from above that this is the case if the informativeness of output is bounded as long as horizontal differentiation is small enough. In Camargo and Degan [2023], we strengthen Lemma 3 to show that if the distribution of the citizen's reputation Ω is such that $\omega(0) = \omega(1) = 0$, then $Q(\pi_I, \kappa)$ is continuously differentiable.[24] This, in turn, implies that $MB(a, \kappa)$ is continuously differentiable when $\omega(0) = \omega(1) = 0$.

5 Effective Accountability and Political Competition

We now study the relationship between effective accountability and political competition. Since we do local comparative statics analysis using the implicit function theorem, in what follows we understand changes in political competition as local changes.

5.1 Indeterminacy

We know from Lemma 3 that the incumbent's probability of retention after a given output realization does not increase with the entry cost and decreases strictly with the entry cost

[24]Intuitively, $\omega(0) = \omega(1) = 0$ implies that the probability the citizen is indifferent between entering or not does not jump discontinuously when the cutoff of entry $\Pi_C(\pi_I, \kappa)$ crosses the boundary of the unit interval.

if the citizen's probability of entry is interior. So, for any $a \in (0, \overline{a})$ and $\kappa \in (0, \overline{\kappa})$, the impact of a reduction in κ on $\delta(a, \kappa)$ is ambiguous: it depends on whether the reduction in the citizen's probability of retention after a success is greater or smaller than the reduction in the citizen's probability of retention after a failure. So, *a priori*, the impact of an increase in political competition on effective accountability can either be positive or negative. We leverage this observation to establish our indeterminacy results.

The first indeterminacy result we establish is that if there exist $a^* \in (0, \overline{a})$ and $\kappa_0 \in (0, \overline{\kappa}]$ such that the probability of entry for the citizen is interior regardless of the incumbent's performance when the incumbent's conjectured effort is a^*, then the incumbent exerting effort a^* is consistent with equilibrium behavior both in an stable equilibrium in which an increase in political competition increases effective accountability and in an stable equilibrium in which an increase in political competition has the opposite effect. Moreover, we can ensure that a^* is the unique equilibrium choice of effort in both cases. In what follows, we say that a cumulative distribution function Ω is an *admissible* distribution of the citizen's reputation if it has support $[0, 1]$ and a continuous density in $(0, 1)$.

Proposition 1. *Let the entry cost be $\kappa_0 \in (0, \overline{\kappa}]$ and fix all other primitives of the model but the cost function c and the distribution of the citizen's reputation Ω. If there exists $a^* \in (0, \overline{a})$ with*

$$0 < \Pi_C(\pi^+(\ell, a^*), \kappa_0) < \Pi_C(\pi^+(h, a^*), \kappa_0) < 1, \tag{5}$$

then there exist admissible cost functions c_1 and c_2 and admissible distributions of the citizen's reputation Ω_1 and Ω_2 such that: (i) a^* is the incumbent's unique equilibrium choice of effort either when $c = c_1$ and $\Omega = \Omega_1$ or when $c = c_2$ and $\Omega = \Omega_2$; (ii) the equilibrium is stable in both cases; and (iii) an increase in political competition increases effective accountability in one case but decreases it in the other.

Given that $|H(\kappa/B)| \leq \theta$ and $\pi^+(y, a)$ is interior, for any $a^* \in (0, \overline{a})$ and $\kappa_0 \in (0, \overline{\kappa}]$, condition (5) holds if θ is small enough. Moreover, if the informativeness of output is bounded, then (5) holds for all $a^* \in (0, \overline{a})$ and $\kappa_0 \in (0, \overline{\kappa}]$ if $\theta < \min\{\underline{\pi}^+(\ell), 1 - \overline{\pi}^+(h)\}$.

Proof of Proposition 1. Consider $a^* \in (0, \overline{a})$ for which (5) holds. Let Ω_1 and Ω_2 be admissible distributions of the citizen's reputation with densities ω_1 and ω_2 with the property that $\omega(\Pi_C(\pi^+(h, a^*), \kappa)) < \omega(\Pi_C(\Pi^+(\ell, a^*), \kappa))$ if $\omega = \omega_1$ and the opposite inequality holds if $\omega = \omega_2$. Lemma 5 implies that for each Ω_i, there exists an admissible cost function c_i such that a^* is the unique equilibrium choice of effort for the incumbent in stable equilibria when the cost function is c_i and the distribution of the citizen's reputation is Ω_i. Now let $\delta_i(a, \kappa) = Q_i(\pi^+(h, a), \kappa) - Q_i(\pi^+(\ell, a), \kappa)$, where $Q_i(\pi_I, \kappa)$ is the incumbent's probability of retention as a function of their second-period reputation and the entry cost when the distribution of the citizen's reputation is Ω_i. The desired result follows since $\partial \Pi_C(\pi_I, \kappa)/\partial \kappa$ is independent of π_I and so, by Lemma 3,

$$\frac{\partial \delta_i}{\partial \kappa}(a^*, \kappa_0) \propto \omega_i(\Pi_C(\pi^+(h, a^*), \kappa)) - \omega_i(\Pi_C(\pi^+(\ell, a^*), \kappa_0)). \qquad \square$$

The idea behind Proposition 1 is simple. The response of the incumbent's effort to a change in political competition depends on how sensitive to changes in political competition is the citizen's entry decision after each output realization by the incumbent. An increase in political competition increases effective accountability if the citizen's entry decision after a failure is more responsive to this change in the entry cost than the citizen's entry decision after a success, and decreases otherwise. Lemma 5 shows that both situations are compatible with the *same* equilibrium choice of effort for the incumbent.

The second indeterminacy result we establish is that if horizontal differentiation is sufficiently small, then, regardless of the entry cost, multiple stable equilibria can exist in which effective accountability reacts in opposite directions to an increase in political competition.

Proposition 2. *There exists $\overline{\theta} > 0$ such that if $\theta < \overline{\theta}$, then for every entry cost $\kappa \in (0, \overline{\kappa}]$ there exist choices of the other primitives of the model for which multiple stable equilibria exist with the property that an increase in political competition increases effective accountability in one of these equilibria but decreases effective accountability in another.*

A sketch of the proof of Proposition 2 is as follows; see Appendix A.6 for the details. First, we show that for every entry cost κ and distribution of the citizen's reputation Ω, we can find a production function f and interior effort choices $a_1 < a_2$ with the property that $MB(a_1, \kappa)/a_1 < MB(a_2, \kappa)/a_2$ and $\partial MB(a_1, \kappa)/\partial a < MB(a_2, \kappa)/a_2$ if the production function is f.[25] So, by Lemma 6, the efforts a_1 and a_2 are equilibrium effort choices for the incumbent in stable equilibria for a suitable choice of cost function. Moreover, similarly to the proof of Proposition 1, we show that when horizontal differentiation is sufficiently small, we can choose Ω in such a way that the incumbent's effort choices in these two equilibria respond in opposite directions to a decrease in the entry cost.

Propositions 1 and 2 do not concern how the set of equilibrium effort choices for the incumbent depends on the entry cost. The final result we establish in this subsection complements these propositions by showing that effective accountability need not be maximized in stable equilibria when the entry cost is zero.

Proposition 3. *Fix all the model's primitives but the cost function, the distribution of the citizen's reputation, and the entry cost. If $\theta < \overline{\pi}^+(\ell) = \sup_a \pi^+(\ell, a)$, then there exist an admissible cost function c and an admissible distribution of the citizen's reputation Ω such that effective accountability is not maximized in stable equilibria when entry is costless if the cost function is c and the distribution of the citizen's reputation is Ω.*

A sketch of the proof of Proposition 3 is as follows; see Appendix A.7 for the details. Since $\theta < \overline{\pi}^+(\ell)$, there exists $a^* \in (0, \overline{a})$ with $\theta < \pi^+(\ell, a^*)$. Suppose the distribution of the citizen's reputation has a density ω with $\omega(\pi^+(h, a^*) - \theta) > \omega(\pi^+(\ell, a^*) - \theta)$ and choose the cost function to be such that a^* is the unique equilibrium choice of effort for the incumbent in stable equilibria when entry is costless. We show in Appendix A.7 that the stability and uniqueness of equilibria when $\kappa = 0$ implies that there exists $\underline{\kappa} \in (0, \overline{\kappa})$ such that if the entry cost is $\kappa \in (0, \underline{\kappa})$, then the incumbent's equilibrium choice of effort is unique, differentiable with respect to κ, and such that it converges to a^* when κ decreases to zero. Totally differentiating the equilibrium condition $\Delta(a^*(\kappa), \kappa) = 0$ with respect to

[25] The production function f in the proof of Proposition 2 has the property that $\pi^+(h, a)$ does not depend on a. It is possible to extend the argument in the proof to the case in which f is such that $\pi^+(h, a)$ is strictly increasing with a.

κ, we obtain that

$$\frac{\partial \Delta}{\partial a}(a^*(\kappa),\kappa)\frac{da^*}{d\kappa}(\kappa)+\frac{\partial \Delta}{\partial \kappa}(a^*(\kappa),\kappa)=0$$

for all $\kappa \in (0,\underline{\kappa})$. Since $\partial\Delta(a^*,0)/\partial a < 0$ by construction, $\partial\Delta(a^*(\kappa),\kappa)/\partial a < 0$ if κ is small enough, ensuring stability of equilibria when κ is close to zero. Moreover, since

$$\frac{\partial \Delta}{\partial \kappa}(a^*(\kappa),\kappa) \propto \omega\left(\pi^+(h,a^*(\kappa))+H\left(\frac{\kappa}{B}\right)\right)-\omega\left(\pi^+(\ell,a^*(\kappa))+H\left(\frac{\kappa}{B}\right)\right)$$

and $\lim_{\kappa\to 0} H(\kappa/B) = -\theta$, it follows that $\partial\Delta(a^*(\kappa),\kappa)/\partial\kappa > 0$ reducing κ further if necessary. So, $da^*(\kappa)/d\kappa > 0$ for κ close to zero, which establishes the desired result.

5.2 Determinacy

As it turns out, there are conditions under which the response of $\delta(a,\kappa)$ to a change in the entry cost is unambiguous. For instance, suppose there exists $\kappa_0 \in (0,\overline{\kappa}]$ such that

$$0 < \Pi_C(\pi^+(\ell,a),\kappa_0) < 1 < \Pi_C(\pi^+(h,a),\kappa_0) \text{ for all } a \in (0,\overline{a}) \tag{6}$$

and the citizen never enters after the incumbent succeeds when the entry cost is κ_0. In this case, $\delta(a,\kappa_0)$ is locally decreasing with the entry cost for all $a \in (0,\overline{a})$, and an increase in political competition always increases effective accountability. Likewise,

$$\Pi_C(\pi^+(\ell,a),\kappa_0) < 0 < \Pi_C(\pi^+(h,a),\kappa_0) < 1 \text{ for all } a \in (0,\overline{a}) \tag{7}$$

for some $\kappa_0 \in (0,\overline{\kappa}]$, then an increase in political competition always decreases effective accountability when the entry cost is κ_0. Conditions (6) and (7) are rather stringent, though. In particular, as discussed after the statement of Proposition 1, they never hold if horizontal differentiation is sufficiently small.

We now show that sharper determinacy results are possible if one imposes conditions on the distribution Ω of the citizen's reputation. The proof of Proposition 4 below follows immediately from the fact that if the cutoffs $\Pi_C(\pi^+(\ell,a^*),\kappa_0)$ and $\Pi_C(\pi^+(h,a^*),\kappa_0)$ are interior, then the difference $\omega(\Pi_C(\pi^+(h,a^*),\kappa_0)) - \omega(\Pi_C(\pi^+(\ell,a^*),\kappa_0))$ is positive if Ω is strictly convex in $[0,1]$ and negative if Ω is strictly concave $[0,1]$.

Proposition 4. *Let the entry cost be $\kappa_0 \in (0, \overline{\kappa}]$ and suppose $a^* \in (0, \overline{a})$ is such that*

$$0 < \Pi_C(\pi^+(\ell, a^*), \kappa_0) < \Pi_C(\pi^+(h, a^*), \kappa_0) < 1.$$

If the distribution of the citizen's reputation is strictly concave (respectively, strictly convex), then any stable equilibrium in which a^* is the incumbent's equilibrium choice of effort has the property that an increase in political competition increases (respectively, decreases) effective accountability.

The intuition for Proposition 4 is straightforward. When Ω is strictly concave in $[0, 1]$, the citizen's entry decision after a success is less responsive to changes in political competition than the citizen's entry decision after a failure. On the other hand, when Ω is strictly convex in $[0, 1]$, the citizen's entry decision after a success is more responsive to changes in political competition than the citizen's entry decision after a failure. The logic of the proof of Proposition 1 shows that in the first case, an increase in political competition always increases effective accountability, while in the second case an increase in political competition always decreases effective accountability. The next result follows immediately from Proposition 4 and the discussion that follows Proposition 1.[26]

Corollary 2. *Suppose output has bounded informativeness. If $\theta < \min\{\underline{\pi}^+(\ell), 1 - \overline{\pi}^+(h)\}$, then an increase in political competition increases (respectively, decreases) effective accountability in any stable equilibrium when the distribution of the citizen's reputation is strictly concave (respectively, strictly convex).*

6 Many-Challengers Extension

In this section, we extend our analysis to the case in which the number of citizens can be greater than one and the incumbent faces one challenger for sure. We show that our

[26]When $MB(a, \kappa)$ is decreasing with a for all κ, such as when effort decreases the informativeness of output, one can extend Corollary 2 to show that if θ is small enough, then effort is maximized (respectively, minimized) in stable equilibria when the distribution of the citizen's reputation is strictly concave (respectively, strictly convex).

indeterminacy results remain the same. However, all else constant, entry after a success becomes more responsive to an increase in political competition than entry after a failure. Moreover, under mild technical assumptions, for *any* distribution of the citizen's reputation, an increase in political competition decreases effective accountability in every stable equilibrium when the number of potential entrants is sufficiently large.

Consider first the case in which besides the citizen, who needs to pay an entry cost to become a challenger, there exists an established challenger, who always runs for office. The established challenger's reputation is also random and drawn from the same distribution as the citizen's reputation. These reputations are independent and publicly revealed at the beginning of the second period, before the citizen decides whether to enter. All other features of the model are the same, including the shock z to the voter's payoff from keeping the incumbent in office. So, except for the zero-probability case in which the citizen and the established challenger have the same reputation, the voter's choice is either between the incumbent and the established challenger or between the incumbent and the citizen.

Let π_{EC} be the established challenger's second-period reputation and, as in the baseline model with only the citizen, let π_C and π_I be, respectively, the citizen's and the incumbent's second-period reputations. Now let: (*i*) $Q(\pi_I, \kappa; \pi_{EC})$ be the incumbent's probability of retention when their reputation is π_I and the entry cost is κ, conditional on the established challenger's reputation; and (*ii*) $Q(\pi_I, \kappa)$ be the incumbent's probability of retention when their reputation is π_I and the entry cost is κ. Note that

$$Q(\pi_I, \kappa) = \int_0^1 Q(\pi_I, \kappa; \pi)\omega(\pi)d\pi.$$

We determine $Q(\pi_I, \kappa)$ in what follows.

First, note that the citizen enters if, and only if, $\pi_C > \max\{\pi_{EC}, \Pi_C(\pi_I, \kappa)\}$ and the voter's choice is between the citizen and the incumbent—we assume, without loss, that if $\pi_C = \pi_{EC}$ and the voter is indifferent between the citizen and the established challenger, then the voter selects the latter when they prefer to replace the incumbent.[27] Now note that

[27] Indeed, the citizen only enters if $\pi_C > \pi_{EC}$. In this case, the citizen's payoff from entering is the same as in the baseline model, $BG(\pi_C - \pi_I) - \kappa$, in which case the citizen enters if, and only if, $\pi_C > \Pi_C(\pi_I, \kappa)$.

the incumbent is retained either when the citizen does not enter and the voter prefers the incumbent to the established challenger or when the citizen enters and the voter prefers the incumbent to the citizen. So, for all $\pi_{EC} \in [0,1]$,

$$Q(\pi_I, \kappa; \pi_{EC}) = \Omega(\max\{\pi_{EC}, \Pi_C(\pi_I, \kappa)\})[1 - G(\pi_{EC} - \pi_I)] + \int_{\max\{\pi_{EC}, \Pi_C(\pi_I,\kappa)\}}^{1} [1 - G(\pi - \pi_I)]\omega(\pi)d\pi;$$

recall that if the challenger's reputation is π, then the voter retains the incumbent with probability $1 - G(\pi - \pi_I)$. From this, it follows that

$$Q(\pi_I, \kappa) = \int_0^1 [1 - G(\pi - \pi_I)]\omega(\pi')d\pi' + \int_0^1 \left(\int_{\max\{\pi', \Pi_C(\pi_I,\kappa)\}}^{1} [G(\pi' - \pi_I) - G(\pi - \pi_I)]\omega(\pi)d\pi \right) \omega(\pi')d\pi'.$$

The interpretation of $Q(\pi_I, \kappa)$ is straightforward. The first term is the (ex-ante) probability of retention for the incumbent when the citizen does not enter. The second, negative, term is the decrease in the incumbent's probability of retention due to the citizen's entry.

As in the baseline model with no established challenger, $Q(\pi_I, \kappa)$ is continuous, nondecreasing with π_I and κ, strictly increasing with π_I and κ if the cutoff $\Pi_C(\pi_I, \kappa)$ is interior, and continuously differentiable in $\{(\pi_I, \kappa) \in [0,1] \times [0, \overline{\kappa}] : \Pi_C(\pi_I, \kappa) \neq 0, 1\}$. Now let

$$\lambda(\pi_I, \kappa) = \int_0^{\max\{0, \Pi_C(\pi_I,\kappa)\}} \left[1 - \frac{B}{\kappa} G(\pi - \pi_I)\right] \omega(\pi)d\pi.$$

Note that $\lambda(\pi_I, \kappa) > 0$ if, and only if, $\Pi_C(\pi_I, \kappa) > 0$. We show in Appendix A.8 that

$$\frac{\partial Q}{\partial \kappa}(\pi_I, \kappa) = \lambda(\pi_I, \kappa)\omega(\Pi_C(\pi_I, \kappa)) \frac{\kappa}{B} \frac{\partial \Pi_C}{\partial \kappa}(\pi_I, \kappa) \tag{8}$$

for all $(\pi_I, \kappa) \in [0,1] \times [0, \overline{\kappa}]$ such that $\Pi_C(\pi_I, \kappa) \in (0, 1)$.

The partial derivative in (8) differs from the one in (2) by the factor $\lambda(\pi_I, \kappa)$. This term adjusts $\partial Q(\pi_I, \kappa)/\partial \kappa$ to account for the fact that now the incumbent always faces at least one challenger. Since $\lambda(\pi_I, \kappa) < 1$ when $\Pi_C(\pi_I, \kappa) \in (0, 1)$, the incumbent's probability of retention is less responsive to changes in κ than in the baseline model. The intuition for this is simple: the citizen's entry has a smaller impact on the incumbent's probability of retention when there exists an established challenger than when only the citizen is present. Also note that $\lambda(\pi_I, \kappa)$ is strictly increasing with π_I if $\Pi_C(\pi_I, \kappa) \in (0, 1)$. So, holding all else constant, the incumbent's probability of retention is more responsive to changes in the entry cost when the incumbent's reputation is high than when the incumbent's reputation is low. The intuition for this second fact is as follows. When π_I is low, the presence of the established challenger ensures that the incumbent's probability of retention is small regardless of the citizen's entry decision. So, changes in the entry cost have a small impact on $Q(\pi_I, \kappa)$. On the other hand, when π_I is high, whether the citizen becomes a challenger or not matters for the incumbent's probability of retention, as the citizen enters only if their reputation is high enough for the race for office to be competitive.

We now show that our indeterminacy results extend to the case considered here. We begin with Proposition 1. Let the entry cost be $\kappa_0 \in (0, \overline{\kappa}]$ and suppose $a^* \in (0, \overline{a})$ is such that the cutoffs $\Pi_C(\pi^+(\ell, a^*), \kappa_0)$ and $\Pi_C(\pi^+(h, a^*), \kappa_0)$ are both interior. Then, by (8),

$$\frac{\partial \delta}{\partial \kappa}(a^*, \kappa_0) \propto \lambda(\pi^+(h, a^*), \kappa_0)\omega(\Pi_C(\pi^+(h, a^*), \kappa_0)) \\ -\lambda(\pi^+(\ell, a^*), \kappa_0)\omega(\Pi_C(\pi^+(\ell, a^*), \kappa_0)). \tag{9}$$

Given that $\lambda(\pi^+(h, a^*), \kappa_0) > \lambda(\pi^+(\ell, a^*), \kappa_0)$, the right-hand side of (9) is positive if $\omega(\Pi_C(\pi^+(h, a^*), \kappa_0)) > \omega(\Pi_C(\pi^+(\ell, a^*), \kappa_0))$. On the other hand, since $\lambda(\pi^+(\ell, a^*), \kappa_0)$ does not depend on the values of $\omega(\pi)$ for $\pi > \Pi_C(\pi^+(\ell, a^*), \kappa_0)$ and $\lambda(\pi^+(h, a^*), \kappa_0)$ is bounded above, we can choose Ω so that $\lambda(\pi^+(\ell, a^*), \kappa_0)\omega(\Pi_C(\pi^+(\ell, a^*), \kappa_0))$ is bounded away from zero while $\lambda(\pi^+(h, a^*), \kappa_0)\omega(\Pi_C(\pi^+(h, a^*), \kappa_0))$ is arbitrarily close to zero. In this second case, the right-hand side of (9) is negative. So, we can extend the proof of Proposition 1 to the established-challenger case. It is easy to see from this discussion that we can also extend the proof of Proposition 3 to the established-challenger case.

The proof of Proposition 2 does not readily extend to the established-challenger case, though. We know that for every entry cost κ and distribution of the citizen's reputation Ω, one can choose the production function f so that interior effort choices $a_1 < a_2$ with

$MB(a_1, \kappa)/a_1 < MB(a_2, \kappa)/a_2$ and $\partial MB(a_1, \kappa)/\partial a < MB(a_2, \kappa)/a_2$ exist—the presence of the established challenger does not matter for this result. So, a_1 and a_2 are equilibrium choices of effort for the incumbent in stable equilibria for a suitable choice of cost function. The argument in the proof of Proposition 2 that we can choose Ω so that the incumbent's effort choices in these two equilibria respond in opposite directions to a decrease in the entry cost does not apply in the established-challenger case, though. Nevertheless, we show in Appendix A.9 that we can adapt it to the established-challenger case.

Since the adjustment factor $\lambda(\pi_I, \kappa)$ increases with the incumbent's reputation, the determinacy results of Section 5 extend only partially to the established-challenger case. Indeed, $\lambda(\pi_I, \kappa)\omega(\Pi_C(\pi_I, \kappa))$ is strictly increasing with π_I if $\Pi_C(\pi_I, \kappa)$ is interior when Ω is weakly convex in $[0, 1]$. So, if the distribution of the citizen's reputation is convex, then, when the entry cost is $\kappa_0 \in (0, \overline{\kappa}]$, an increase in political competition reduces effective accountability in any stable equilibrium in which $a^* \in (0, \overline{a})$ is the incumbent's equilibrium choice of effort if $0 < \Pi_C(\pi^+(\ell, a^*), \kappa_0) < \Pi_C(\pi^+(h, a^*), \kappa_0) < 1$. However, unlike in the baseline model, we no longer can say that the opposite holds when the distribution of the citizen's reputation is concave, as now the incumbent's probability of retention is more sensitive to increases in political competition when the incumbent's reputation is high.

We conclude the discussion in this section by considering the case in which besides the established challenger there are $N \geq 2$ citizens who decide whether to run for office in the second period. The timing of events in the first period is as in the baseline model. In the second period, the citizen's reputations, which are independent of each other and independent of the established challenger's reputation, become publicly known and the citizens simultaneously decide whether to run for office. Following the citizen's entry decisions, the voter observes the realization of the shock z to their payoff from keeping the incumbent and chooses which politician to put in office.

In order to compute the retention probability $Q(\pi_I, \kappa)$, note first that only the citizen with the highest reputation considers entering.[28] Also, the entry decision of this citizen

[28]We ignore the zero-probability event in which two or more citizens have the same reputation.

is the same as the citizen's entry decision in the established-challenger extension with a single citizen: enter if, and only if, their reputation is greater than $\max\{\pi_{EC}, \Pi_C(\pi_I, \kappa)\}$. Let Ψ be the cumulative distribution function of the highest-reputation citizen's reputation; by definition, $\Psi(\pi) = \Omega(\pi)^N$. The same argument as above shows that

$$Q(\pi_I, \kappa; \pi_{EC}) = \Psi(\max\{\pi_{EC}, \Pi_C(\pi_I, \kappa)\})[1 - G(\pi_{EC} - \pi_I)] + \int_{\max\{\pi_{EC}, \Pi_C(\pi_I, \kappa)\}}^{1} [1 - G(\pi - \pi_I)]\psi(\pi) d\pi,$$

where ψ is the density of Ψ. From this point on, the analysis is the same as above with Ψ and ψ in place of Ω and ω, respectively. In particular, if $\Pi_C(\pi_I, \kappa) \in (0, 1)$, then

$$\frac{\partial Q}{\partial \kappa}(\pi_I, \kappa) = \nu(\pi_I, \kappa)\Psi(\Pi_C(\pi_I, \kappa))\frac{\kappa}{B}\frac{\partial \Pi_C}{\partial \kappa}(\pi_I, \kappa),$$

where

$$\nu(\pi_I, \kappa) = \int_0^{\max\{0, \Pi_C(\pi_I, \kappa)\}} \left[1 - \frac{B}{\kappa}G(\pi - \pi_I)\right]\psi(\pi) d\pi$$

is strictly increasing in π_I if $\Pi_C(\pi_I, \kappa)$ is interior.

It is immediate to see that our indeterminacy results extend to this second case as there exists a one-to-one map between Ψ and Ω, and thus between ψ and ω. Since for all $N \geq 2$ the function Ψ is strictly convex in $[0, 1]$ if Ω is, the partial determinacy results in the established-challenger extension also hold when the number of citizens is greater than one. In fact, stronger determinacy results are possible under mild technical assumptions on the distribution of the citizen's reputation. Suppose Ω is twice continuously differentiable in $[0, 1]$ and such that its density ω is bounded away from zero.[29] A straightforward argument shows that $\Psi = \Omega^N$ is strictly convex if N is large enough.[30] So, even when the distribution of the citizen's reputation is concave, an increase in political competition can decrease effective accountability in all stable equilibria if the number of citizens is large enough. This is also true in the absence of the established challenger, as

[29] When Ω is concave in $[0, 1]$, this condition holds if $\omega'(0)$ is finite and $\omega(1) > 0$.

[30] Indeed, $\Psi''(\pi) \propto (N-1)(\omega(\pi))^2 + \Omega(\pi)\omega'(\pi)$. Since $\inf_{\pi \in [0,1]} \omega(\pi) > 0$ and $\sup_{\pi \in [0,1]} |\omega'(\pi)| < \infty$, it follows that $\Psi''(\pi) > 0$ for all $\pi \in [0, 1]$ if N is large enough. Note that Ω^N can be convex for N large even if Ω does not satisfy the conditions discussed in the text, such as when $\Omega(\pi) \propto \pi^\alpha$ with $\alpha \in (0, 1)$.

$\partial Q(\pi_I, \kappa)/\partial \kappa \propto \Psi(\Pi_C(\pi, \kappa))\partial \Pi_C/\partial \kappa$ in this case and so the analysis proceeds as in the baseline model with $\Psi = \Omega^N$ in place of Ω.

7 Concluding Remarks

In the political sector, the wedge between the private and social returns to effort creates the need of institutional arrangements to provide politicians with an incentive to behave in the voters' interest. In democracies, these incentives are provided by the possibility of reelection. For reelection incentives to work, however, voters need to credibly commit to reward a good performance and punish a bad one. Political competition helps voters punish a bad performance by providing them with alternatives to an incumbent politician. Political competition makes the promise of rewarding a good performance through retention less credible, though. We show that this tension between the good and the bad aspects of political competition implies that the relationship between political competition and effective accountability is undetermined in the sense that an increase in political competition can be associated with both an increase and a decrease in effective accountability. Moreover, effective accountability need not be maximized when challenging an incumbent politician is costless. We also provide economically meaningful conditions under which increasing political competition has a monotonic impact on effective accountability and show that increasing the number of potential challengers to incumbent politicians increases the scope for an increase in political competition to unequivocally reduce effective accountability.

Our analysis focuses on the impact of political competition on effective accountability. Political competition also matters for electoral selection, though. Indeed, a change in political competition directly affects electoral selection by changing the citizen's decision to run for office: all else constant, a reduction in the cost of entry makes it more likely that the citizen runs for office. A change in political competition also indirectly affects electoral selection by changing the incumbent's effort. This affects the informativeness of the incumbent's performance in office, which, in turn, affects both the voter's retention decision and the citizen's entry decision—the latter depends on the incumbent's reputation (which depends on the incumbent's equilibrium effort). The impact of political competition on electoral selection begs the question of what is the relationship between political competition and voter payoffs, which depend not only on effective accountability but also on electoral selection. Clearly, all else constant, an increase in political competition increases voter payoffs by increasing the set of candidates available for the voter to choose in the second period. Nevertheless, we show in Camargo and Degan [2023] that our indeterminacy results extend to voter payoffs and that voter payoffs need not be maximized when entry is costless. So, our message extends beyond accountability considerations.

References

Daron Acemoglu and James A. Robinson. Economic backwardness in political perspective. *American Political Science Review*, 100(1):115–131, 2006.

Farzana Afridi, Sourav Bhattacharya, Amrita Dhillon, and Eilon Solan. Electoral competition, accountability and corruption: Theory and evidence from india. *CAGE Working Paper*, 2021.

Maria Rosaria Alfano and Anna Laura Baraldi. Is there an optimal level of political competition in terms of economic growth? evidence from italy. *European Journal of Law and Economics*, 39(2):263–285, 2015.

Maria Rosaria Alfano and Anna Laura Baraldi. Democracy, political competition and economic growth. *Journal of International Development*, 28(8):1199–1219, 2016.

James Alt, Ethan Bueno de Mesquita, and Shanna Rose. Disentangling accountability and competence in elections: Evidence from u.s. term limits. *The Journal of Politics*, 73(1): 171–186, 2011.

Paulo Roberto Arvate. Electoral competition and local government responsiveness in brazil. *World Development*, 43:67–83, 2013.

John Ashworth, Benny Geys, Bruno Heyndels, and Fanny Wille. Competition in the political arena and local government performance. *Applied Economics*, 46(19):2264–2276, 2014.

Scott Ashworth. Electoral accountability: Recent theoretical and empirical work. *Annual Review of Political Science*, 15(1):183–201, 2012.

Scott Ashworth and Ethan Bueno de Mesquita. Electoral selection, strategic challenger entry, and the incumbency advantage. *The Journal of Politics*, 70(4):1006–1025, 2008.

Scott Ashworth, Ethan Bueno de Mesquita, and Amanda Friedenber. Incumbency and information. *Unpublished Manuscript*, 2019.

Pranab Bardhan and Tsung-Tao Yang. Political competition in economic perspective. *UC Berkeley Economics Department Working Paper Series*, 2004.

Robert J. Barro. The control of politicians: An economic model. *Public Choice*, 14(1): 19–42, 1973.

Gary S. Becker. Competition and democracy. *The Journal of Law & Economics*, 1:105–109, 1958.

Timothy Besley and Anne Case. Does electoral accountability affect economic policy choices? evidence from gubernatorial term limits. *The Quarterly Journal of Economics*, 110(3):769–798, 1995.

Timothy Besley, Torsten Persson, and Daniel M. Sturm. Political competition, policy and growth: Theory and evidence from the us. *The Review of Economic Studies*, 77(4): 1329–1352, 2010.

Braz Camargo and Ariana Degan. Accountability and political competion. *UQÀM Working Paper*, 2023.

Gary W. Cox and Jonathan N. Katz. Why did the incumbency advantage in u.s. house elections grow? *American Journal of Political Science*, 40(2):478–497, 1996.

Ernesto Dal Bó and Frederico Finan. Progress and perspectives in the study of political selection. *Annual Review of Economics*, 10(1):541–575, 2018.

Maria De Paola and Vincenzo Scoppa. Political competition and politician quality: evidence from italian municipalities. *Public Choice*, 148(3/4):547–559, 2011.

John Duggan and Cesar Martinelli. The political economy of dynamic elections: Accountability, commitment, and responsiveness. *Journal of Economic Literature*, 55(3): 916–984, 2017.

John Ferejohn. Incumbent performance and electoral control. *Public Choice*, 50(1/3):5–25, 1986.

Claudio Ferraz and Frederico Finan. Electoral accountability and corruption: Evidence from the audits of local governments. *American Economic Review*, 101(4):1274–1311, 2011.

Vincenzo Gallaso and Tomaso Nannicini. Competing on good politicians. *The American Political Science Review*, 105(1):79–99, 2011.

Sanford C. Gordon, Gregory A. Huber, and Dimitri Landa. Challenger entry and voter learning. *American Political Science Review*, 101(2):303–320, 2007.

Andrew B. Hall and James M. Snyder. How much of the incumbency advantage is due to scare-off? *Political Science Research and Methods*, 3(3):493–514, 2015.

Bengt Holmström. Managerial incentive problems: A dynamic perspective. *The Review of Economic Studies*, 66(1):169–182, 1999.

Navin Kartik, Francesco Squintani, and Katrin Tinn. Information revelation and pandering in elections. *Unpublished Manuscript*, 2015.

Steven D. Levitt and Catherine D. Wolfram. Decomposing the sources of incumbency advantage in the u. s. house. *Legislative Studies Quarterly*, 22(1):45–60, 1997.

Alessandro Lizzeri and Nicola Persico. A drawback of electoral competition. *Journal of the European Economic Association*, 3(6):1318–1348, 2005.

Roger B. Myerson. Incentives to cultivate favored minorities under alternative electoral systems. *The American Political Science Review*, 87(4):856–869, 1993.

Fabio Padovano and Roberto Ricciuti. Political competition and economic performance: Evidence from the italian regions. *Public Choice*, 138(3/4):263–277, 2009.

Marco Polo. Electoral competition and political rents. *IGIER Working Paper*, 1998.

George J. Stigler. Economic competition and political competition. *Public Choice*, 13: 91–106, 1972.

Walter J. Stone, L. Sandy Maisel, and Cherie D. Maestas. Quality counts: Extending the strategic politician model of incumbent deterrence. *American Journal of Political Science*, 48(3):479–495, 2004.

Helena Svaleryd and Jonas Vlachos. Political rents in a non-corrupt democracy. *Journal of Public Economics*, 93(3):355–372, 2009.

Donald Wittman. Why democracies produce efficient results. *Journal of Political Economy*, 97(6):1395–1424, 1989.

Donald Wittman. *The Myth of Democratic Failure: Why Political Institutions are Efficient*. The University of Chicago Press, Chicago & London, 1995.

A Appendix: Omitted Proofs and Details

Here, we provide all proofs and details that were omitted from the main text.

A.1 Proof of Lemma 3

Notice that $Q(\pi_I, \kappa) = 1$ if $\Pi_C(\pi_I, \kappa) \geq 1$ and that

$$Q(\pi_I, \kappa) = 1 - \int_{\max\{0, \Pi_C(\pi_I, \kappa)\}}^{1} G(\pi - \pi_I)\omega(\pi)d\pi \tag{10}$$

otherwise. Since $G(\pi - \pi_I)$ is nonincreasing with π_I for all $\pi \in [0, 1]$ and $\Pi_C(\pi_I, \kappa)$ is strictly increasing with π_I and κ, it follows that $Q(\pi_I, \kappa)$ is nondecreasing with π_I and κ and strictly increasing with π_I and κ if $\Pi_C(\pi_I, \kappa) \in (0, 1)$. The continuity of $Q(\pi_I, \kappa)$ in follows from the fact that the map $(x, \pi_I) \mapsto \int_x^1 G(\pi - \pi_I)\omega(\pi)d\pi$ is continuous by dominated convergence theorem.[31]

We now compute the derivatives $\partial Q(\pi_I, \kappa)/\partial \pi_I$ and $\partial Q(\pi_I, \kappa)/\partial \kappa$ and show that they are continuous in (π_I, κ) when $\Pi_C(\pi_I, \kappa) \neq 0, 1$. First notice that $\partial Q/\partial \pi_I(\pi_I, \kappa) = 0$ and $\partial Q/\partial \kappa(\pi_I, \kappa) = 0$ if $\Pi_C(\pi_I, \kappa) > 1$ and that

$$\frac{\partial Q}{\partial \pi_I}(\pi_I, \kappa) = \int_0^1 G'(\pi - \pi_I)\omega(\pi)d\pi$$

and $\partial Q(\pi_I, \kappa)/\partial \kappa = 0$ if $\Pi_C(\pi_I, \kappa) < 0$. Consider then the case in which $\Pi_C(\pi_I, \kappa) \in (0, 1)$. Since $\Pi_C(\pi_I, \kappa)$ is differentiable in π_I and κ for all $(\pi_I, \kappa) \in [0, 1] \times [0, \overline{\kappa}]$ and $G(\Pi_C(\pi_I, \kappa) - \pi_I) = \kappa/B$, the fundamental theorem of Calculus implies that

$$\frac{\partial Q}{\partial \pi_I}(\pi_I, \kappa) = \omega(\Pi_C(\pi_I, \kappa))\frac{\kappa}{B} + \int_{\Pi_C(\pi_I, \kappa)}^1 G'(\pi - \pi_I)\omega(\pi)d\pi.$$

and

$$\frac{\partial Q}{\partial \kappa}(\pi_I, \kappa) = \omega(\Pi_C(\pi_I, \kappa))\frac{\kappa}{B}\frac{\partial \Pi_C}{\partial \kappa}(\pi_I, \kappa)$$

Clearly, $\partial Q(\pi_I, \kappa)/\partial \kappa$ is continuous if $\Pi_C(\pi_I, \kappa) \neq 0, 1$. The continuity of $\partial Q(\pi_I, \kappa)/\partial \pi_I$ in the same set follows from the dominated convergence theorem.

[31] Just note that $\int_x^1 G(\pi - \pi_I)\omega(\pi)d\pi = \int_0^1 F(x, \pi_I, \pi)d\pi$, where $F(x, \pi_I, \pi) = \mathbb{I}_{[x,1]}(\pi)G(\pi - \pi_I)\omega(\pi)$ and $\mathbb{I}_{[x,1]}$ is the indicator function of the interval $[x, 1]$.

A.2 Proof of Lemma 4

We established the sufficiency of (3) in the main text. In order to establish necessity, suppose $a^* \in A$ maximizes $U(a, a^*, \kappa)$. Note that $a^* < \overline{a}$, for $c'(\overline{a}) > B\partial f(0, \pi_0)/\partial a$ implies that $\partial U(\overline{a}, a^e, \kappa)/\partial a < 0$ for all $a^e \in A$. Now note that a necessary condition for $a^* \in [0, \overline{a})$ to maximize $U(a, a^*, \kappa)$ is that $\partial U(a^*, a^*, \kappa)/\partial a \leq 0$ with $\partial U(a^*, a^*, \kappa)/\partial a = 0$ if $a^* > 0$. Since $c'(0) = 0$ implies that $\partial U(0, a^e, \kappa)/\partial a \geq 0$ for all $a^e \in A$, it also follows that $\partial U(a^*, a^*, \kappa)/\partial a = 0$ if $a^* = 0$. Thus, condition (3) is necessary as well.

A.3 Proof of Corollary 1

Since $\pi^+(\ell, a^e) < \pi_0 < \pi^+(h, a^e)$ for all $a^e \in A$ and $\Pi_C(\pi_0, \kappa) \in (0, 1)$ for all $\kappa \in [0, \overline{\kappa}]$, it follows that $\Pi_C(\pi^+(\ell, a^e), \kappa) < 1$ and $\Pi_C(\pi^+(h, a^e), \kappa) > 0$ for all $a^e \in A$ and $\kappa \in [0, \overline{\kappa}]$. So, $Q(\pi^+(h, a^e), \kappa) > Q(\pi^+(\ell, a^e), \kappa)$ regardless of a^e and κ. This, in turn, implies that the left-hand side of (3) is positive for all $a^* \in [0, \overline{a})$, so $a^* = 0$ does not solve (3); note that $\partial f(a, \tau)/\partial a > 0$ for all $a \in [0, \overline{a})$ given the assumptions on f.

A.4 Proof of Lemma 5

Fix all the model's primitives but the cost function and define $\phi : A \to \mathbb{R}$ to be such that $\phi(a) = \max\{0, MB(a^*, \kappa_0) + \xi(a - a^*)\}$, where κ_0 is the entry cost and

$$\xi = \frac{1}{\overline{a} - a^*}\left[B\frac{\partial f}{\partial a}(0, \pi_0) - MB(a^*, \kappa_0)\right] + \max\left\{\frac{MB(a^*, \kappa_0)}{a^*}, \sup_{a \in A\setminus\{a^*\}} \frac{MB(a, \kappa_0) - MB(a^*, \kappa_0)}{a - a^*}\right\}.$$

For simplicity, assume that $\partial MB(a^*, \kappa_0)/\partial a$ exists, so that ξ is well-defined. One can show that $MB(a, \kappa)$ is locally Lipschitz in a for all $\kappa \in [0, \overline{\kappa}]$, so that ξ is well-defined even if $\partial MB(a^*, \kappa_0)/\partial a$ does not exist.[32] Notice that $\phi(a)$ is increasing with a and that $\phi(\overline{a}) > B\partial f(0, \pi_0)/\partial a$. As $\xi > MB(a^*, \kappa_0)/a^*$, there exists $a_0 \in (0, a^*)$ with $\phi(a) = 0$ if $a \leq a_0$ and $\phi(a) > 0$ otherwise. Moreover, since $\xi > (MB(a, \kappa_0) - MB(a^*, \kappa_0))/(a - a^*)$

[32] See Camargo and Degan [2023] for a proof of this.

for all $a \in A \setminus \{a^*\}$ and $\phi(a^*) = MB(a^*, \kappa_0)$, it also follows that $\phi(a) < MB(a, \kappa_0)$ for all $a < a^*$ and $\phi(a) > MB(a, \kappa_0)$ for all $a > a^*$.

Now fix $0 < \varepsilon < \min\{a_0, a^* - a_0\}$ and let $\lambda : A \to \mathbb{R}$ be such that $\lambda(a) = \phi(a)$ if $|a - a_0| \geq \varepsilon$ and $\lambda(a) = \alpha(a - a_0 + \varepsilon)^n$ if $|a - a_0| < \varepsilon$, where $\alpha(2\varepsilon)^n = MB(a^*, \kappa_0) + \xi(a_0 + \varepsilon - a^*)$ and $\alpha n(2\varepsilon)^{n-1} = \xi$.[33] Notice that $\lambda(a_0 - \varepsilon) = \phi(a_0 - \varepsilon) = 0$ and that $\lambda(a_0 + \varepsilon) = \phi(a_0 + \varepsilon) = \alpha(2\varepsilon)^n$. Moreover, $\lambda'(a_0 - \varepsilon) = 0$ and $\lambda'(a_0 + \varepsilon) = \xi$. So, $\lambda(a)$ is increasing and continuously differentiable in a. On the other hand, by letting n be sufficiently large and ε be sufficiently small, we can ensure that $\lambda(a) < MB(a, \kappa_0)$ for all $a \in A$ such that $|a - a_0| < \varepsilon$.

To conclude the proof, let $c(a) = \int_0^a \lambda(s)ds$. By construction, c is twice continuously differentiable, increasing, and convex, with $c(0) = c'(0) = 0$ and $c'(\overline{a}) > B\partial f(0, \pi_0)/\partial a$. So, c is an admissible cost function. Also, $c'(a^*) = MB(a^*, \kappa_0)$ and $c'(a) \neq MB(a, \kappa_0)$ for all $a \neq a^*$. So, a^* is the unique equilibrium choice of effort for the incumbent when the cost function is c. Finally, $c''(a^*) = \xi > \partial MB(a^*, \kappa_0)/\partial a$, so the equilibrium is stable if $MB(a, \kappa)$ is continuously differentiable in a neighborhood of (a^*, κ_0).

A.5 Proof of Lemma 6

Suppose the entry cost is κ_0 and $MB(a_1, \kappa_0)/a_1 < MB(a_2, \kappa_0)/a_2$ for $0 < a_1 < a_2 < \overline{a}$. Let $\phi_2 : A \to \mathbb{R}$ be such that $\phi_2(a) = \max\{0, MB(a_2, \kappa_0) + \xi_2(a - a_2)\}$, where

$$\xi_2 = \frac{1}{\overline{a} - a_2}\left[B\frac{\partial f}{\partial a}(0, \pi_0) - MB(a_2, \kappa_0)\right] + \frac{MB(a_2, \kappa_0) - MB(a_1, \kappa_0)}{a_2 - a_1}.$$

Moreover, let $\phi_1 : A \to \mathbb{R}$ be such that $\phi_1(a) = \max\{0, MB(a_1, \kappa_0) + \xi_1(a - a_1)\}$, where

$$\xi_1 = \frac{\phi_2(a_2 - \eta) - MB(a_1, \kappa_0)}{a_2 - a_1 - \eta}$$

and $0 < \eta < a_2 - a_1$. Since $\lim_{\eta \to 0} \phi_2(a_2 - \eta) = MB(a_2, \kappa_0)$ and

$$MB(a_1, \kappa_0) - \left(\frac{MB(a_2, \kappa_0) - MB(a_1, \kappa_0)}{a_2 - a_1}\right)a_1 \propto MB(a_1, \kappa_0)a_2 - MB(a_2, \kappa_0)a_1 < 0$$

[33] We solve for α and n as follows. First, substitute ξ by $\alpha n(2\varepsilon)^{n-1}$ in the first equation to obtain α; note that $\alpha > 0$ since $a^* > a_0 + \varepsilon$. Then obtain n residually by the second equation.

by assumption, there exists $\overline{\eta} > 0$ for which $\xi_1 > MB(a_1, \kappa_0)/a_1$ for all $\eta \in (0, \overline{\eta})$. So, $\phi_1(a) = 0$ for a sufficiently close to zero if $\eta \in (0, \overline{\eta})$.

Fix $\eta < \overline{\eta}$ and let $\phi : A \to \mathbb{R}$ be such that $\phi(a) = \max\{\phi_1(a), \phi_2(a)\}$. By definition, $\phi_1(a_2 - \eta) = \phi_2(a_2 - \eta)$. Moreover,

$$\xi_2 > \frac{MB(a_2, \kappa_0) - MB(a_1, \kappa_0)}{a_2 - a_1} > \frac{\phi_2(a_2 - \eta) - MB(a_1, \kappa_0)}{a_2 - a_1 - \eta} = \xi_1,$$

where the second inequality follows from the fact that the second ratio is strictly decreasing with η. Thus, $\phi(a) = \phi_1(a)$ if, and only if, $a \leq a_2 - \eta$. A straightforward modification of the argument in the proof of Lemma 5 now shows that there exists an admissible cost function c with $c'(a_1) = MB(a_1, \kappa_0)$ and $c'(a_2) = MB(a_2, \kappa_0)$, so that a_1 and a_2 are equilibrium choices of effort for the incumbent. This establishes the first part of Lemma 6.

To establish the second part of Lemma 6, we prove the following auxiliary result.

Lemma 7. *Fix all the model's primitives but the cost function and let κ_0 be the entry cost. Suppose there exist $0 < a_1 < a_2 < \overline{a}$ with $MB(a_1, \kappa_0)/a_1 < MB(a_2, \kappa_0)/a_2$. Moreover, suppose $MB(a, \kappa)$ is continuously differentiable. There exist an admissible cost function c and $a_0 \in (0, a_1]$ such that if the cost function is c, then a_0 and a_2 are equilibrium effort choices for the incumbent in stable equilibria.*

Proof. Let $\phi_2 : A \to \mathbb{R}$ be such that $\phi_2(a) = \max\{0, MB(a_2, \kappa_0) + \xi_2(a - a_2)\}$, where

$$\xi_2 = \frac{1}{\overline{a} - a_2}\left[B\frac{\partial f}{\partial a}(0, \pi_0) - MB(a_2, \kappa_0)\right] + \sup_{a \in A \setminus \{a_2\}} \frac{MB(a_2, \kappa_0) - MB(a, \kappa_0)}{a_2 - a};$$

note that ξ_2 is well-defined since $\partial MB(a_2, \kappa_0)/\partial a$ exists. Moreover, let $\phi_1 : A \to \mathbb{R}$ be such that $\phi_1(a) = \max\{0, MB(a_1, \kappa_0) + \xi_1(a - a_1)\}$, where

$$\xi_1 = \frac{\phi_2(a_2 - \eta) - MB(a_1, \kappa_0)}{a_2 - a_1 - \eta}$$

and $0 < \eta < a_2 - a_1$. By construction, there exists $\overline{\eta} > 0$ such that $\xi_1 > MB(a_1, \kappa_0)/a_1$ if $\eta \in (0, \overline{\eta})$, and so $\phi_1(a) = 0$ for a sufficiently close to zero if $\eta \in (0, \overline{\eta})$.

Fix $\eta < \overline{\eta}$ and define $\phi : A \to \mathbb{R}$ to be such that $\phi(a) = \max\{\phi_1(a), \phi_2(a)\}$. Since $\phi_1(a_2 - \eta) = \phi_2(a_2 - \eta)$ and $\xi_1 > \xi_2$, we have that $\phi(a) = \phi_1(a)$ if, and only if, $a \leq a_2 - \eta$.

A straightforward modification of the argument in the proof of Lemma 5 now shows that there exists an admissible cost function c with $c'(a_1) = MB(a_1, \kappa_0)$, $c'(a_2) = MB(a_2, \kappa_0)$, and $c''(a_2) = \xi_2 > \partial MB(a_2, \kappa_0)/\partial a$. So, there exists a stable equilibrium in which a_2 is the incumbent's equilibrium choice of effort when the cost function is c.

Now note that if $c''(a_1) = \xi_1 > \partial MB(a_1, \kappa_0)/\partial a$, then there also exists a stable equilibrium in which a_1 is the incumbent's equilibrium choice of effort when the cost function is c. Consider then the case in which $\xi_1 \leq \partial MB(a_1, \kappa_0)/\partial a$. We can assume without loss that $\xi_1 < \partial MB(a_1, \kappa_0)/\partial a$, otherwise we can increase η in the definition of ϕ_1 slightly, which reduces ξ_1. Since $MB(a, \kappa_0) > 0$ for all $a \in A$ and $\phi_1(a) = 0$ if a is sufficiently close to zero, $\phi_1(a_1) = MB(a_1, \kappa_0)$ and $\phi'(a_1) = \xi_1 < \partial MB(a_1, \kappa_0)/\partial a$ together imply that there exists $a_0 \in (0, a_1)$ such that $MB(a_0, \kappa_0) = \phi_1(a_0)$ and $MB(a, \kappa_0) > \phi_1(a)$ for all $a \in [0, a_0)$. So, $\partial MB(a_0, \kappa_0)/\partial a \leq \xi_1$, otherwise there would exist $\tilde{a} \in (0, a_0)$ for which $MB(\tilde{a}, \kappa_0) = \phi_1(\tilde{a})$, a contradiction.

If $\partial MB(a_0, \kappa_0)/\partial a < \xi_1$, then we can take the cost function c in the first paragraph of the proof to be such that $c'(a_0) = \phi_1(a_0) = MB(a_0, \kappa_0)$. In this case, a_0 is an equilibrium choice of effort for the incumbent in a stable equilibrium when the cost function is c. Consider then the case in which $\partial MB(a_0, \kappa_0)/\partial a = \xi_1$ and define $\phi_0 : A \to \mathbb{R}$ to be such that $\phi_0(a) = \max\{0, MB(a_0, \kappa_0) + \xi_0(a - a_0)\}$, where

$$\xi_0 = \frac{\phi_2(a_2 - \eta) - MB(a_0, \kappa_0)}{a_2 - a_0 - \eta}.$$

Since $\phi_0(a_2 - \eta) = \phi_2(a_2 - \eta) = \phi_1(a_2 - \eta)$ and $\phi_0(a_0) = MB(a_0, \kappa_0) = \phi_1(a_0)$, it follows that $\xi_1 = \xi_0$, and so $\phi_0(a) = \max\{0, \phi_1(a_0) + \xi_1(a - a_0)\} = \max\{0, \phi_1(a)\} = \phi_1(a)$. Reducing η, and so increasing ξ_0, we then have that $\partial MB(a_0, \kappa_0)/\partial a < \xi_0$ and $\phi_0(a) = 0$ if a is close enough to zero. Let then $\lambda : A \to \mathbb{R}$ be such that $\lambda(a) = \max\{\phi_0(a), \phi_2(a)\}$. By making the reduction in η is small enough, we have that $\xi_0 < \xi_2$, and so $\lambda(a) = \phi_0(a)$ if, and only if, $a \leq a_2 - \eta$. The same argument as in the first part of the proof now shows that there exists an admissible cost function with the desired properties. $\square$

To conclude the proof of Lemma 6, note that since $MB(a_1, \kappa_0)/a_1 < MB(a_2, \kappa_0)/a_2$

is equivalent to $(MB(a_2,\kappa_0)-MB(a_1,\kappa_0))/(a_2-a_1) > MB(a_2,\kappa_0)/a_2$, we have that ξ_1 in the proof of the Lemma 7 is greater than $MB(a_2,\kappa_0)/a_2$ if η is small enough. So, when $\partial MB(a_1,\kappa_0)/\partial a < MB(a_2,\kappa_0)/a_2$, we can take the cost function in the proof of Lemma 7 to be such that $c''(a_1)=\xi_1 > \partial MB(a_1,\kappa_0)/\partial a$. This concludes the proof of Lemma 6.

A.6 Proof of Proposition 2

Suppose $f(a,H)=g(a)$ and $f(a,L)=\mu g(a)$, where $\mu \in (0,1)$ and $g : A \to (0,1]$ is a twice continuously differentiable, strictly increasing, and strictly concave function with $g(\overline{a})=1$ and $\lim_{a\to\overline{a}} g'(a) > 0$. By construction,

$$\pi^+(h,a) \equiv \pi^+(h) = \frac{\pi_0}{\pi_0+\mu(1-\pi_0)}.$$

Now let $a_1=\overline{a}/2$ and define $a_2 \in (0,\overline{a})$ implicitly by $g(a_2)=\mu$. By construction,

$$\pi^+(\ell,a_1) = \frac{\pi_0}{\pi_0+\dfrac{1-\mu g(\overline{a}/2)}{1-g(\overline{a}/2)}(1-\pi_0)} \quad \text{and} \quad \pi^+(\ell,a_2)=\frac{\pi_0}{\pi_0+(1+\mu)(1-\pi_0)}.$$

Moreover, there exists $\mu_1 \in (0,1)$ such that $a_2 > a_1$ if $\mu > \mu_1$.

Now take the distribution Ω of the citizen's reputation to be such that $\omega(0)=\omega(1)=0$, so that $MB(a,\kappa)$ is continuously differentiable, and fix the entry cost $\kappa \in (0,\overline{\kappa}]$. Since $\lim_{\mu\to 1}\pi^+(\ell,a_1)=\lim_{\mu\to 1}\pi^+(h)=\pi_0$, and thus $\lim_{\mu\to 1}\delta(a_1,\kappa)=0$, it follows that

$$\lim_{\mu\to 1}\frac{MB(a_1,\kappa)}{a_1}=0.$$

On the other hand, since $\lim_{\mu\to 1}\pi^+(h,a_2)=\pi_0 > \lim_{\mu\to 1}\pi^+(\ell,a_2)=\pi_0/[\pi_0+2(1-\pi_0)]$, Lemma 3 and the definition of a_2 imply that $\lim_{\mu\to 1}\delta(a_2,\kappa)>0$. Thus,

$$\lim_{\mu\to 1}\frac{MB(a_2,\kappa)}{a_2} = \frac{g'(\overline{a})}{\overline{a}}B[\pi_0+\mu(1-\pi_0)]\lim_{\mu\to 1}\delta(a_2,\kappa)>0.$$

Moreover, given that

$$\frac{\partial MB}{\partial a}(a_1,\kappa) \propto g''(a_1)\delta(a_1,\kappa)-g'(a_1)\frac{\partial Q}{\partial \pi_I}(\pi^+(\ell,a_1),\kappa)\frac{d\pi^+}{da}(\ell,a_1)$$

and $d\pi^+(\ell, a_1)/da \propto 1 - \mu$, there exists $\mu_2 \in (0, 1)$ with $MB(a_1, \kappa)/a_1 < MB(a_2, \kappa)/a_2$ and $\partial MB(a_1, \kappa)/\partial a < MB(a_2, \kappa)/a_2$ if $\mu > \mu_2$. By increasing μ_2 if necessary, we also have that $a_2 > a_1$ if $\mu > \mu_2$. Let $\mu \in (\mu_2, 1)$. By Lemma 6, we can take the cost function to be such that a_1 and a_2 are equilibrium effort choices for the incumbent in stable equilibria.

To conclude, suppose $\theta < \overline{\theta} = \min\{\pi_0/[\pi_0 + 2(1 - \pi_0)], 1 - \pi_0\}$, let $\mathcal{N}$ be a neighborhood of π_0 with $\pi_0^- = \pi_0/[\pi_0 + 2(1 - \pi_0)] < \pi$ for all $\pi \in \mathcal{N}$, and assume the density ω of Ω is strictly decreasing in the set $\{\pi^+ : \Pi_C(\pi^+, \kappa) \in \mathcal{N}\}$ and such that $\omega(\Pi_C(\pi_0^-, \kappa)) < \omega(\Pi_C(\pi, \kappa))$ for all $\pi \in \mathcal{N}$; since $H(\kappa/B) \in [-\theta, \theta]$, $\theta < \overline{\theta}$ implies that $\Pi_C(\pi_0^-, \kappa) > 0$ and $\Pi_C(\pi, \kappa) < 1$ for π sufficiently close to π_0. Now observe that $\lim_{\mu\to1} \pi^+(h, a_1) = \lim_{\mu\to1} \pi^+(h, a_2) = \pi_0 = \lim_{\mu\to1} \pi^+(\ell, a_1)$ and $\lim_{\mu\to1} \pi^+(\ell, a_2) = \pi_0^-$ imply that there exists $\mu_3 \in (\mu_2, 1)$ such that if $\mu > \mu_3$, then $\omega(\Pi_C(\pi^+(\ell, a_1), \kappa)) > \omega(\Pi_C(\pi^+(h, a_1), \kappa))$ and the opposite inequality holds for a_2. Given that

$$\frac{\partial \delta}{\partial \kappa}(a_i, \kappa) \propto \omega(\Pi_C(\pi^+(h, a_i), \kappa)) - \omega(\Pi_C(\pi^+(\ell, a_i), \kappa)),$$

it then follows that $\partial\delta(a_1, \kappa)/\partial\kappa$ and $\partial\delta(a_2, \kappa)/\partial\kappa$ have opposite signs if we increase μ even further so that $\mu \in (\mu_3, 1)$. This establishes the desired result.

A.7 Proof of Proposition 3

By assumption, there exists $a^* \in (0, \overline{a})$ with $\theta < \pi^+(\ell, a^*)$. Suppose the distribution of the citizen's reputation satifies $\omega(\pi^+(h, a^*) - \theta) > \omega(\pi^+(\ell, a^*) - \theta)$ and take the cost function to be such that a^* is the unique equilibrium effort choice for the incumbent in stable equilibria when entry is costless; $MB(a, \kappa)$ is continuously differentiable in a neighborhood of $(a^*, 0)$ by Lemma 3. Below, we show there exists $\underline{\kappa} \in (0, \overline{\kappa})$ such that the incumbent's equilibrium choice of effort is unique and differentiable in κ for all $\kappa \in (0, \underline{\kappa})$ and converges to a^* as κ converges to zero. Moreover, if $a^*(\kappa)$ is the incumbent's unique equilibrium effort choice when the entry cost is $\kappa \in (0, \underline{\kappa})$, then $\partial\Delta(a^*(\kappa), \kappa)/\partial a < 0$, ensuring stability of equilibria for κ close to zero. The argument in the main text concludes the proof.

We claim that the incumbent's equilibrium choice of effort is unique if $\kappa \in (0, \underline{\kappa})$ for some $\underline{\kappa} \in (0, \overline{\kappa})$. Suppose not. Then there exist a sequence $\{\kappa_n\}$ in $[0, \overline{\kappa}]$ with $\lim_n \kappa_n = 0$ and sequences $\{a_n\}$ and $\{a'_n\}$ in $(0, \overline{a})$ with $a_n \neq a'_n$ and $\Delta(a_n, \kappa_n) = \Delta(a'_n, \kappa_n) = 0$ for all $n \in \mathbb{N}$. Moving to subsequences if necessary, we can take $\{a_n\}$ and $\{a'_n\}$ to be convergent. Let a and a' be their respective limits. Since $\Delta(a, \kappa)$ is (jointly) continuous, $\Delta(a, 0) = \Delta(a', 0) = 0$. So, $a = a' = a^*$, as a^* is the unique equilibrium effort choice for the incumbent when $\kappa = 0$. Now note that the assumptions on a^* and θ imply that there exist $0 < \varepsilon < a^*$ and $0 < \delta < \overline{\kappa}$ such that $\Delta(a, \kappa)$ is continuously differentiable in $(a^* - \varepsilon, a^* + \varepsilon) \times (0, \overline{\kappa} - \delta)$. Thus, there exists $n_0 \in \mathbb{N}$ such that if $n \geq n_0$, then we can find $a''_n \in [\min\{a_n, a'_n\}, \max\{a_n, a'_n\}]$ with $\partial\Delta(a''_n, \kappa_n)/\partial a = 0$. Moreover, since $\lim_n a_n = \lim_n a'_n = a^*$, it also follows that $\{a''_n\}$ converges to a^*. So, $\partial\Delta(a^*, 0)/\partial a = \lim_n \partial\Delta(a''_n, \kappa_n)/\partial a = 0$, a contradiction with stability when entry is costless.

For each $\kappa \in (0, \underline{\kappa})$, let $a^*(\kappa)$ be the incumbent's unique equilibrium choice of effort when the entry cost is κ. We now show that $a^*(\kappa)$ is differentiable in κ for all $\kappa \in (0, \underline{\kappa})$, reducing $\underline{\kappa}$ if necessary. Note that $\lim_{\kappa \to 0} a^*(\kappa) = a^*$ as $\Delta(a, \kappa)$ is continuous. By reducing $\underline{\kappa}$ if necessary, it follows that $a^*(\kappa) \in (a^* - \varepsilon, a^* + \varepsilon)$ and $\kappa \in (0, \overline{\kappa} - \delta)$ for all $\kappa \in (0, \underline{\kappa})$. Since $\Delta(a, \kappa)$ is continuously differentiable in the set $(a^* - \varepsilon, a + \varepsilon) \times (0, \overline{\kappa} - \delta)$ and $\partial\Delta(a^*, 0)/\partial a < 0$, we then have that $\partial\Delta(a^*(\kappa), \kappa)/\partial a < 0$ for $\kappa \in (0, \underline{\kappa})$, reducing $\underline{\kappa}$ even further if necessary. The desired result now follows from the implicit function theorem.

A.8 Proof of (8)

First note that

$$\begin{aligned}
&\int_0^1 \left(\int_{\max\{\pi', \Pi_C(\pi_I, \kappa)\}}^1 [G(\pi' - \pi_I) - G(\pi - \pi_I)]\omega(\pi)d\pi \right) \omega(\pi')d\pi' \\
&\qquad = \int_0^{\Pi_C(\pi_I, \kappa)} \left(\int_{\Pi_C(\pi_I, \kappa)}^1 [G(\pi' - \pi_I) - G(\pi - \pi_I)]\omega(\pi)d\pi \right) \omega(\pi')d\pi' \\
&\qquad + \int_{\Pi_C(\pi_I, \kappa)}^1 \left(\int_{\pi'}^1 [G(\pi' - \pi_I) - G(\pi - \pi_I)]\omega(\pi)d\pi \right) \omega(\pi')d\pi'.
\end{aligned}$$

The desired result follows since, by the fundamental theorem of Calculus and the fact that $G(\Pi_C(\pi_I,\kappa)-\pi_I)=\kappa/B$, we have that

$$\begin{aligned}\frac{\partial Q}{\partial \kappa}(\pi_I,\kappa) &= -\left(\int_0^{\Pi_C(\pi,\kappa)}\left[G(\pi'-\pi_I)-\frac{\kappa}{B}\right]\omega(\Pi_C(\pi_I,\kappa))\omega(\pi')d\pi'\right)\frac{\partial \Pi_C}{\partial \kappa}(\pi_I,\kappa)\\ &= \omega(\Pi_C(\pi_I,\kappa))\frac{\kappa}{B}\frac{\partial \Pi_C}{\partial \kappa}(\pi_I,\kappa)\int_0^{\Pi_C(\pi_I,\kappa)}\left[1-\frac{B}{\kappa}G(\pi-\pi_I)\right]\omega(\pi)d\pi.\end{aligned}$$

A.9 Extending Proposition 2 to the Established-Challenger Case

Fix the entry cost κ and suppose the distribution Ω of the citizen's reputation is such that $\omega(0)=\omega(1)=0$, so that $MB(a,\kappa)$ is continuously differentiable. We know from the proof of Proposition 2 that if $f(a,H)=g(a)$ and $f(a,L)=\mu g(a)$, where $\mu\in(0,1)$ and $g:A\to(0,1]$ is twice continuously differentiable, strictly increasing, and strictly concave with $g(\overline{a})=1$ and $\lim_{a\to\overline{a}}g'(a)>0$, then there exists a choice of μ such that if $a_1=\overline{a}/2$ and $g(a_2)=\mu$, then $a_1<a_2$, $MB(a_1,\kappa)/a_1<MB(a_2,\kappa)/a_2$, and $\partial MB(a_1,\kappa)/\partial a_1<MB(a_2,\kappa)/a_2$. By Lemma 6, we can take the cost function to be such that a_1 and a_2 are equilibrium effort choices for the incumbent in stable equilibria.

Let $\pi_0^-=\lim_{\mu\to 1}\pi^+(\ell|a_2)<\pi_0$ and suppose $\theta<\min\{\pi_0^-,1-\pi_0\}$. Moreover, let $\mathcal{N}$ be a neighborhood of π_0 such $\pi_0^-<\pi$ for all $\pi\in\mathcal{N}$ and take ω to be such that ω is strictly decreasing in $\{\pi^+:\Pi_C(\pi^+,\kappa)\in\mathcal{N}\}$ and $\omega(\Pi_C(\pi_0^-,\kappa))<\omega(\Pi_C(\pi,\kappa))$ for all $\pi\in\mathcal{N}$; the restriction on θ ensures that $\Pi_C(\pi_0^-,\kappa)>0$ and $\Pi_C(\pi,\kappa)<1$ for π close enough to π_0 regardless of the function G, and so regardless of the c.d.f. Γ. The proof of Proposition 2 shows that, increasing μ if necessary, $\omega(\Pi_C(\pi^+(\ell,a),\kappa))<\omega(\Pi_C(\pi^+(h,a),\kappa))$ if $a=a_2$ and the opposite inequality holds if $a=a_1$. So, regardless of Γ,

$$\lambda(\pi^+(\ell,a),\kappa)\omega(\Pi_C(\pi^+(\ell,a),\kappa))<\lambda(\pi^+(h,a),\kappa)\omega(\Pi_C(\pi^+(h,a),\kappa)) \tag{11}$$

when $a=a_2$; recall that $\lambda(\pi_I,\kappa)$ is strictly increasing with π_I if $\Pi_C(\pi,\kappa)$ is interior. By choosing Γ so that $G(\pi-\pi^+(\ell,a_1))\approx 0$ for all $\pi\geq\Pi_C(\pi^+(\ell,a_1))$, we can ensure that $\lambda(\pi^+(h,a_1),\kappa)\approx\lambda(\pi^+(\ell,a_1),\kappa)$, in which case (11) holds with the opposite sign when $a=a_1$. Thus, Proposition 2 extends to the established-challenger case.