Minimum wage and Informality in a Roy Bargaining Economy: Evidence from a Bunching Estimator

Hugo Jales * Zhengfei Yu †

Abstract

We study the determination of employment, formality, and wages using a bargaining model featuring compensating differentials and self-selection. Our framework allows us to create a novel taxonomy of formal employment that complements the taxonomy used in the literature to discuss informality (Ulyssea, 2018). This taxonomy is shown to be useful to characterize the effects of labor market policies such as the minimum wage. We use the model to estimate the effects of the minimum wage in Brazil using the "PNAD" dataset for the years 2001-2005. Our results suggest that the minimum wage decreases formality by approximately 8%.

Keywords: Minimum wage, Informality, Bunching.

JEL codes: J00, J31, J30.

1 Introduction

Labor markets in developing countries are usually characterized by a large number of informal employment relationships (Ulyssea, 2018). Rationalizing why some workers

*Corresponding author. Syracuse University, Department of economics. Email: hbjales@syr.edu.
†University of Tsukuba, Faculty of Humanities and Social Sciences.

and firms choose informal employment and understanding how informality interacts with labor market policies are the goals of this paper. This paper contributes to the literature by proposing a bargaining model that is useful in answering both of these questions. In our empirical exercise, we study the connection between minimum wages and informality in Brazil, a developing country characterized by a large informal sector.

We use a framework, closely following Roy (1951) and Rosen (1986), to explain the heterogeneity in the type of employment – formal versus informal – and wages. Workers and firms bargain over the wage *and* the sector under which production will take place. This process generates an equilibrium distribution of employment, sector choices – formal and informal –and wages. The minimum wage acts as a constraint in the bargaining process and, as a result, affects the equilibrium joint distribution of these variables.

We show that informal sector workers can be characterized according to their second-best sector alternative (formality versus non-employment) and that, although these informal workers all share the same sector choice, they differ substantially in the way that they respond to labor market policies. As a result, the economy's response to a policy that increases the costs of informality will largely depend on the fraction of workers that belong to each group. Similarly, we show that a similar taxonomy can be constructed for formal workers: They can be characterized according to their second-best alternative (informality versus non-employment). The economy's response to policies that increase the costs of operation in the formal sector (such as an increase in taxes or the minimum wage) will largely depend on the type of jobs formal workers hold.

We show that under some statistical restrictions on the variables in the model, we can obtain a tractable empirical specification of minimum wage effects. The minimum wage effects can be estimated using a two-step procedure consisting of a particular type of a Tobit regression – to recover parameters of the wage distribution – and a

Logit regression – to recover parameters of the conditional probability of formality given the wage –, where both of these regressions are run in a carefully constructed sub-sample defined by the minimum wage level and the range of spillover effects.

We derive the statistical restrictions that connect the bargaining model to our empirical exercise so that our empirical analysis is consistent with our theory. However, the validity of our empirical analysis does not require our theoretical model to be correctly specified. It only requires the functional form assumptions – on the (marginal) distribution of wages and the conditional probability of formality given the wage – to be correctly specified and that an upper bound on how far up on the wage distribution spillover effects might reach to be available. Thus, although we interpret our empirical results in the light of our model, whether or not our minimum wage effects are credible is more of a question on the validity of the functional form assumptions and limits on spillover effects we impose. Because the functional form assumptions we imposed are on (partially) observable objects – a wage distribution and a conditional probability function – these restrictions are, to a certain extent, testable.

In our empirical exercise, we estimate the effects of the Brazilian federal minimum wage. The Brazilian setting is suitable for us because the federal minimum wage is the same for every worker. State minimum wages above the federal level are rare – existent in only two out of 26 states during the period of our analysis – and seldom enforced (Corseuil et al., 2015). As a result, the country does not have rich cross-sectional minimum wage variation that can be used to infer the effects of the policy. Our empirical exercise uses a nation-wide representative dataset of the Brazilian population known as "PNAD" for the years 2001-2005. We find that the minimum wage has a "supporting effect" in the bottom of the wage distribution, pushing up the wages of low wage workers, similar to the effects documented in DiNardo et al. (1996). As a result, minimum wage increases induce increases in average wages. On the other hand, the policy decreases the size of the formal sector.

Our results are related to the vast literature concerning the economic effects of the minimum wage (Card and Krueger, 1994; Card, 1992; Neumark and Wascher, 2000; Dube et al., 2010; Clemens and Wither, 2019; Dickens et al., 1998; Lemos, 2009; Sorkin, 2015; Meer and West, 2016; Aaronson, 2001; Aaronson et al., 2012; Brochu and Green, 2013; MaCurdy, 2015; Baker et al., 1999; Flinn, 2006; Tonin, 2011; Jales, 2018; Cengiz et al., 2019; Heckman and Sedlacek, 1981; Slichter, 2015; Engbom and Moser, 2017; Monras, 2019; Saltiel and Urzúa, 2018).

Our results also relate to the literature concerning the phenomena of informality in developing countries and its relation to employment and productivity (Galiani and Weinschelbaum, 2012; Meghir et al., 2015; Rauch, 1991; Mattos and Ogura, 2009; Ulyssea, 2010, 2018). Lastly, our empirical exercise is closely related to the econometric literature that exploits bunching (Saez, 2010; Kleven and Waseem, 2013; Caetano, 2015; Caetano et al., 2020; Jales and Yu, 2017) and discontinuities (Hahn et al., 2001; Calonico et al., 2014) to identify the causal effects of policy interventions.

Our analysis has two main contributions: (i) We provide a novel taxonomy of formal workers/firms that extends the taxonomy typically used in the development literature for informal workers (Ulyssea, 2018; La Porta and Shleifer, 2014). We show that whether a formal worker will lose his job or move to an informal contract as a result of any policy that increases the costs of formality fundamentally depends on which class the worker/firm belongs to according to our proposed taxonomy, and that bounds for the proportion of workers that belong to each of these types can be obtained using policy variation such as labor taxes or the minimum wage.[1](ii) We endow our theoretical framework with statistical assumptions on the shocks of the model in a way to arrive at a tractable likelihood function. We use this to study the

[1]Our model also includes a characterization of informal contracts. This characterization nests two of the leading explanations for the nature of the formal sector proposed in the literature. This characterization is similar (but less general than) to the characterization proposed by (Ulyssea, 2018). Ulyssea's characterization nests all of the leading explanations for the nature of the formal sector in the context of a dynamic model. Since our model is static, it cannot include De Sotto's (1989) explanation, which is related to (red tape) entry costs to the formal sector.

effects of a particular policy that increases the costs of operation in the formal sector: the minimum wage. We then use the estimated effects of the minimum wage to assess the relative size of the types of formal employment according to our proposed taxonomy. We find that the combined effects of disemployment and movements to the informal sector induced by the minimum wage reduce the size of the formal sector by approximately 8%.

This paper is organized as follows: Section 2 discusses the model and its implications. Section 3 discusses the empirical strategy. Section 4 presents the results. Section 5 concludes.

1.1 Relationship with previous work

Our empirical strategy to estimate the effects of the minimum wage is closely related to the Bunching methods used in Cengiz et al. (2019), Kleven and Waseem (2013), Meyer and Wise (1983), and Jales (2018). However, there are important differences between our approach and theirs, which we discuss in this section.

Kleven and Waseem (2013) use a bunching approach to investigate the effects of notches in the tax code in Pakistan. Our approach is similar to theirs. However, we use functional forms derived from our model, whereas they use a polynomial specification instead. We also model the joint distribution of outcomes (formality and wages), whereas they model the marginal (of earnings) only.

Cengiz et al. (2019) use variation on minimum wage levels to identify the effects of the minimum wage on employment and wages by studying the distribution of jobs around the minimum wage. They look at the missing jobs below the minimum wage and the excess jobs above and at the minimum wage to impute the employment effects of the policy. Our identification is similar to theirs in the sense that missing jobs in the bottom part of the wage distribution will play a role in our estimates of the disemployment effects of the policy. Another similarity is that they also impose an upper limit on how far spillover effects of the minimum wage can reach, in the same way as we do.

In terms of differences, Cengiz et al. (2019) obtain the counterfactual by means of a control group, a state that is similar in terms of the wage distribution but did not face an increase in the minimum wage. Our strategy, in contrast, is to obtain the counterfactual by extrapolating the patterns we see in the upper part of the wage distribution towards the bottom, taking advantage of the functional form approximations we obtain from our model. Another key difference is that we model the joint distribution of formality and wages, as opposed to the marginal distribution of wages. This allows us to obtain estimates of the effects of the minimum wage on the size of the formal sector. Our strategy is then more suitable for developing country settings where informality is prevalent and settings such as the Brazilian labor market where there is not a lot of variation in minimum wages across different states, so no ideal control group is available.

Jales (2018) estimates a reduced-form model to obtain the effects of the minimum wage in the Brazilian labor market. However, the identification arguments and corresponding estimation strategy that we use here inherently different. In contrast with our approach, Jales (2018) obtain identification from the discontinuity in the wage distribution at the minimum wage level, whereas our robust estimates do not use any wage data at or below the minimum wage level to construct the counterfactual. Our counterfactual is constructed by extrapolating the functional form assumptions for the joint distribution of formality and wage derived from our bargaining model from levels strictly above the minimum wage towards zero and thus is in spirit more similar to Kleven and Waseen's bunching method. The method we use here is also suited to compute marginal effects of the minimum wage. These objects are of more relevance than the effects of completely abolishing the minimum wage, as in Jales (2018). Also, it is worth pointing Jales (2018) cannot handle spillover effects in all but one of the parameters of interest and only under an independence condition, whereas we can handle spillovers in all of them without this condition. We can also allow for a much more rich structure of effects of the minimum wage in the bottom part of the wage distribution. Lastly, since Jales (2018) uses a reduced-form, atheoretical model, he is not able to discuss the welfare effects of the minimum wage or to use the minimum wage to obtain bounds for the taxonomy of formal contracts we propose in this paper.

Our strategy is quite similar in spirit to Meyer and Wise (1983), in the sense that Meyer and Wise also impose a functional form on the latent wage distribution and obtain the disemployment effects of the policy by looking at the missing jobs in the bottom part of the wage distribution. In contrast with Meyer and Wise, however, our approach models the joint distribution of sector and wages and thus allows us to obtain the movements from one sector to another as a response to the policy. These effects are absent in Meyer and Wise's model as the model is designed with the US labor market in mind. Our approach to estimation is more robust to spillovers and can better handle misspecification of effects in the bottom part of the wage distribution than theirs since they rely only on the efficient (but not robust) maximum likelihood estimator. Also, since our model is derived from our bargaining framework, we can discuss the welfare effects of the policy and obtain bounds for the share of employment contracts of different kinds according to our proposed taxonomy.

2 A bargaining model

2.1 Environment

Let worker's utility be given by: $U(l, s, w) = l \cdot (w - \epsilon + \eta_s)$, where w denotes the wage, $s \in \{0, 1\}$ denotes the type of employment contract (one for the formal sector, zero otherwise), l denotes a binary indicator of whether the worker is employed, ϵ denotes the worker's outside option, and η_s denotes the amenity associated with employment at sector s. Let firm's profit be given by: $\Pi(l, s, w) = l \cdot (\alpha - w - \tau_s)$, where α denotes the worker's productivity, τ_1 the tax (and other costs) associated with formal sector employment, and τ_0 the costs of hiding this activity in the informal sector. The costs associated with informality, τ_0, can be thought of as a combination of fines, bribes to inspectors, and all distortions in productivity, output, and profits that are associated with hiding the worker in an informal employment contract.

Workers and firms are heterogeneous with respect to $(\alpha, \epsilon, \tau_1, \tau_0, \eta_1, \eta_0)$. These random variables can be correlated, so individuals with a higher α may also have a higher τ_0, for example. That is, the costs of informality might be larger for more productive individuals. Correlation between these random variables is an important ingredient so that the model can accommodate empirical regularities such as the fact that more productive workers are less likely to be informal (Galiani and Weinschelbaum, 2012). Assume that the economy consists of a large number of i.i.d. draws of the vector $(\alpha, \epsilon, \tau_1, \tau_0, \eta_1, \eta_0)$, where, for each draw, workers and firms behave as described below. This is implicitly the same as assuming that all labor markets are segmented (perhaps across narrowly defined city-occupation-industry cells), as in Lavecchia (2019), Lee and Saez (2012), and others.[2]

In a given sector-occupation-city cell, a (single) worker bargains with a firm. The worker's problem is to choose $\vec{l} \in \{0, 1\}$, that is, to choose whether to work or not, for any offer of the pair (w, s). The solution to this problem is characterized by a threshold associated with the worker's participation constraint $(U(w, s, l) \geq 0)$:

$$\vec{l} = \mathbb{I}\{w - \epsilon + \eta_s > 0\}.$$

Similarly, the firm's problem is to choose $\overleftarrow{l}$, that is, to employ the worker or not, for any given pair of (w, s).[3] The solution to the firm's problem is also a threshold

[2]This assumption simplifies by ruling out mechanisms such as labor-labor substitution – as in Teulings (2000) – and changes in labor market tightness – as in Flinn (2006). We discuss later how the results of the empirical exercise can be made robust to violations of this restriction.

[3]We focus on the decision of a single worker bargaining with a single firm. Thus, we think of every firm having at most one employee. There are important relationships between firm size and informality, all of which we abstract in our discussion. An interesting framework that explicitly accounts for firm size in the context of informality can be found in Galiani and Weinschelbaum (2012). Our key results still hold in a setting in which firms can hire multiple workers under a restriction on the production function technology.

associated with the firm's participation constraint ($\Pi(w,s,l) \geq 0$):

$$\overleftarrow{l} = \mathbb{I}\{\alpha - w - \tau_s > 0\}.$$

In general, there are multiple pairs of (w,s) such that labor supply equals labor demand, $\vec{l} = \overleftarrow{l}$, that is, there are many values for (w,s) that induce both worker and firm to agree on employment. To resolve this indeterminacy, assume that the equilibrium pair (w^*, s^*) is decided according to the solution of a Nash bargaining problem:

$$(w^*, s^*) = \arg\max_{(w,s)} (\Pi(l,s,w))^{1/2}(U(l,s,w))^{1/2}, \tag{1}$$

where w^* and s^* are the equilibrium wage and sector that prevail in the unconstrained case, that is, in the absence of any labor market institution such as the minimum wage. In the absence of a minimum wage policy, the bargaining process between worker and firm operates unconstrained. Thus, any wage that solves the maximization problem above, no matter how small, is valid. The solution to problem (1) is characterized as follows and is proved in Appendix A.

$$s^* = \mathbb{I}\{\eta_1 - \eta_0 > \tau_1 - \tau_0\}, \text{ and } w^* = \frac{1}{2}\left(\alpha + \epsilon - \tau_{s^*} - \eta_{s^*}\right). \tag{2}$$

The worker will be employed in the formal sector if and only if the net (aggregate) benefit is larger in the formal sector compared to the informal sector.[4] Also, we have that l^*, the equilibrium employment, has the property that $l^* = 1 \iff \min\{U(1,s^*,w^*), \Pi((1,s^*,w^*)\} > 0$, that is, the participation constraints of both worker and firm have to be met for employment to be equal to one. The equilibrium wage will be such that the total surplus from the transaction is split between worker and firm.

It is helpful to think about some special cases in the sector choice equation. When

[4]Note that this solution has the intuitive property that an increase in tax rates (or a decrease in the cost of informality) induces the marginal workers to move to the informal sector.

$\eta_0 = \eta_1$, the worker is always employed in the sector where the wage draw is larger. That is, $s^* = \mathbb{I}\{w_1 > w_0\}$, as in the standard Roy-model. When $\tau_1 = \tau_0$, workers sort themselves based on their idiosyncratic valuation of amenities, that is, $s^* = \mathbb{I}\{\eta_1 > \eta_0\}$. Note that a mechanism similar to compensating differentials is present since $\frac{\partial w_1}{\partial \eta_1} < 0$, so the worker's equilibrium wage is decreasing in η_1, the valuation of the amenities associated with working in that sector. This mechanism is absent in the standard Roy model. In the Appendix A, Section 1.1, we further discuss the relationship between our bargaining framework and these canonical models.

Define w_1 (w_0) as the equilibrium wage a worker would get if he were employed in the formal (informal) sector. From the solution to the Nash Bargaining problem, we have:

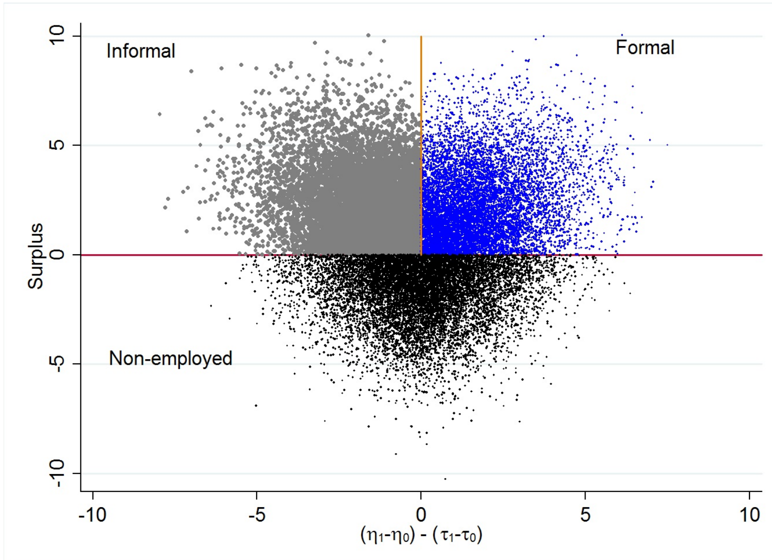
$$w_1 = \frac{1}{2}\ (\alpha + \epsilon - \tau_1 - \eta_1)\,, \text{ and } w_0 = \frac{1}{2}\ (\alpha + \epsilon - \tau_0 - \eta_0)\,. \tag{3}$$

The decision of formal versus informal employment is endogenous. It is based on the opportunity costs of each state. In equilibrium, a worker will be employed in the formal sector if $\tau_1 - \eta_1 < \tau_0 - \eta_0$. If the inequality is reversed, the worker will be employed in the informal sector. We can then re-write w^* as $w^* = s^* w_1 + (1 - s^*) w_0$ as the equilibrium wage of a employed worker.

Figure 1 displays the choice of employment and sector for different workers. The horizontal line characterizes the geometric locus of workers and firms at the margin of indifference between employment and non-employment. The vertical line characterizes the geometric locus of firms and workers that are indifferent between producing in the formal or the informal sector. Workers in the upper right quadrant are employed in the formal sector, workers in the upper left quadrant are employed in the informal sector, and workers in the bottom half are non-employed.

In general, the informal sector will have both firms that could operate in the formal sector and firms that could not, given the current level of taxes and benefits. That is, the model encompasses two of the leading explanations for informality as special

Figure 1: Employment and sector choice



Note: Surplus is defined as $r^* = r_1 s^* + r_0(1 - s^*)$, where $r_s = \alpha + \eta_s - \epsilon - \tau_s$.
Source Simulated data from the model using Gaussian draws. See text for more details.

cases.[5] To characterize these different groups, consider the thought experiment of increasing enforcement. This is the case when $\tau_0 \to \infty$. In this situation, the costs of operating in the informal sector become prohibitively large. As a result, the reaction of the firms will be to either move to the formal sector or to go out of business. Whether one or the other option will be chosen depends on the surplus associated with the choice of operating in the formal sector. Thus, the response of the firm to an increase in enforcement will depend on the ranking of the surplus of being formal, informal, or non-participating before the policy of enforcement is introduced.

$$\textit{Parasite:} \lim_{\tau_0 \to \infty} (s^*, l^*) = (1, 1) \iff \alpha - \epsilon + \eta_0 - \tau_0 \geq \alpha - \epsilon + \eta_1 - \tau_1 \geq 0.$$

$$\textit{Survival:} \lim_{\tau_0 \to \infty} l^* = 0 \iff \alpha - \epsilon + \eta_0 - \tau_0 \geq 0 \geq \alpha - \epsilon + \eta_1 - \tau_1.$$

[5] Ulyssea (2018) obtains a similar result in the context of a dynamic model. Ulyssea's result also incorporates a third view that associates informality with red tape and entry costs to the formal sector. We abstract from this issue since, in a static model, there is no distinction between the costs of formality and the costs of entering the formal sector. Another important distinction is that we only model informality of the employment relationship, not the firm's decision to register and pay profit taxes. In the terminology of Ulyssea, our model is about employment contract informality (the intensive margin), whereas Ulyssea's model is about both the extensive margin of informality (the firm's registration choice) and also the intensive margin of informality (the nature of the employment contracts). For a more detailed discussion of these issues, see Ulyssea (2018).

These equations show that some firms will react to the policy of increased enforcement by moving to the formal sector. The firms that would choose to do that are the ones in which $\alpha - \epsilon + \eta_0 - \tau_0 \geq \alpha - \epsilon + \eta_1 - \tau_1 \geq 0$. That is, the surplus in the informal sector (before enforcement is introduced) is higher than the surplus in the formal sector, which is higher than non-participating. In the terminology of La Porta and Shleifer (2008), these firms correspond to the "parasite" view of informal employment.[6]

On the other hand, there are firms that will react to the policy of increased enforcement by going out of business. The firms that would choose to do that are the ones in $\alpha - \epsilon + \eta_0 - \tau_0 \geq 0 \geq \alpha - \epsilon + \eta_1 - \tau_1$. For these firms, the surplus (before enforcement is introduced) is larger than the surplus of non-participation, and the surplus of non-participation is larger than the surplus in the formal sector. These firms will react to increased enforcement by leaving the market. They, in the terminology of La Porta and Shleifer (2008), correspond to the "survival" view of informal employment.[7]

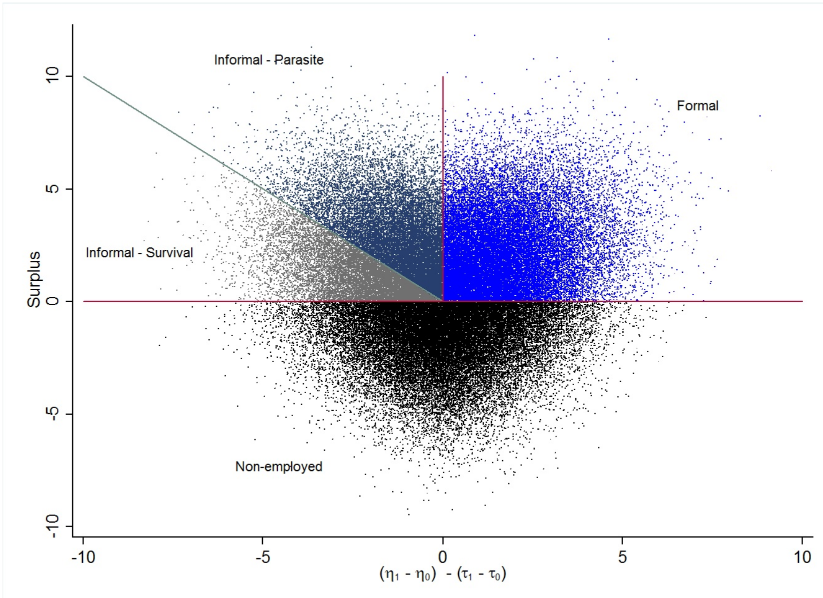
This result shows that a policy that increases enforcement will, in general, induce some workers to become formal. However, this will happen at the cost of inducing other workers to become unemployed. The geometrical locus that separates the parasite from the survival companies is given by the negative 45-degree line, the line that bisects the second and fourth quadrant. Figure 2 displays this relationship.

The model also allows for a similar taxonomy of *formal* firm-worker pairs. These firms can be divided according to their response to a change in the environment that

[6]It is worthwhile to note that La Porta and Shleifer's taxonomy is originally designed to characterize informality of the firm – that is, the decision to register and pay taxes–, whereas we are borrowing their definition and applying it to characterize informality of employment contracts. There are important distinctions between firm informality and employment informality. The interaction between these two phenomena can be economically relevant as well (Ulyssea, 2018). For the purposes of our model and empirical exercise, we will abstract completely from the process of firm informality and only focus on the informality of employment contracts. So, whenever we refer to an informal firm in our discussion, we mean a firm-worker pair for which the employment contract is informal.

[7]For an example of an actual policy that closely mimics our thought exercise of complete enforcement, see Guzman (2017).

Figure 2: The parasite and the survival view of Informality



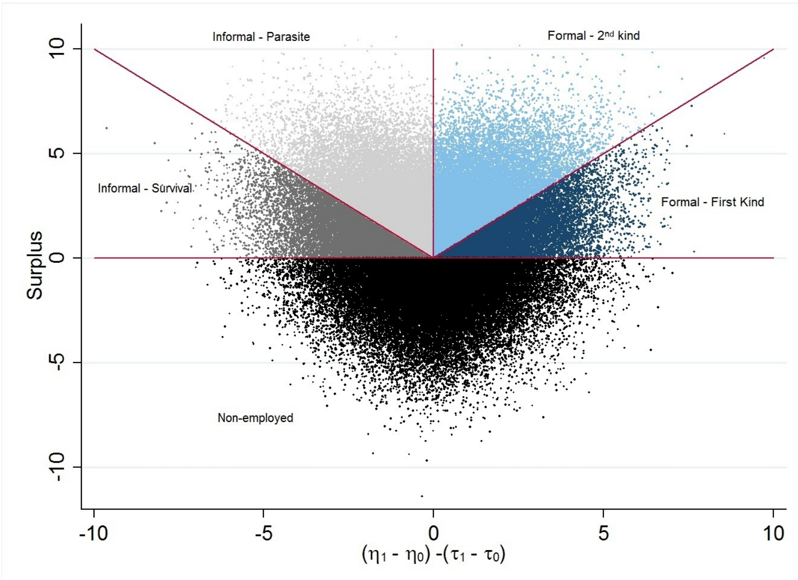
Note: Surplus is defined as $r^* = r_1 s^* + r_0(1 - s^*)$, where $r_s = \alpha + \eta_s - \epsilon - \tau_s$.
Source Simulated data from the model using Gaussian draws. See text for more details.

increases the costs of operating in the formal sector. We will denote the firm (and worker pairs) that respond to an increase in τ_1 ($\tau_1 \rightarrow \infty$) by going out of business as "formal contracts of the first kind". For these firms, there is no surplus from operating in the informal sector. As a result, their next best option is not to operate at all. Alternatively, we will denote by "formal contracts of the second kind" the formal worker and firm pairs that respond by moving to the informal sector (when $\tau_1 \rightarrow \infty$). For these firms, the surplus of operating in the informal sector is positive. They choose to operate in the formal sector because at the current value of τ_1 the surplus is larger when they are formal.[8]

This characterization is helpful when thinking about the effects of the minimum wage, as the minimum wage policy introduces a constraint on firm-worker pairs operating in the formal sector. Employment relationships of the second kind will never

[8]The model predicts that attempts to increase the number of formal sector jobs by increasing the costs of operating in the informal sector (such as strengthening enforcement) will invariably generate some employment losses, whereas reducing the costs of operating in the formal sector (by decreasing taxes) will move marginal workers from the informal to the formal sector *and*, in addition, create new jobs (Ulyssea, 2010).

Figure 3: The taxonomy of formal and informal employment



Note: Surplus is defined as $r^* = r_1 s^* + r_0(1 - s^*)$, where $r_s = \alpha + \eta_s - \epsilon - \tau_s$.
Source Simulated data from the model using Gaussian draws. See text for more details.

respond to the policy by terminating the contract. The worker and firm only need to decide between staying in the formal sector and paying the minimum wage or operating in the informal sector. On the other hand, workers and firms that belong to the other class – that is, those of employment of the first kind – will never operate in the informal sector. Thus, their choice is limited to remaining in the formal sector and paying the minimum wage or terminating the contract. As one can see, in an economy characterized by the possibility of informality, the effects of the minimum wage will largely depend on how many contracts in the formal sector are characterized by parasite firms and how many are characterized by survival firms (and, naturally, how many workers are affected by the minimum wage). Figure 3 displays the geometric locus that separates these types of firms.

Formal employment of the first kind:

$\lim_{\tau_1 \to \infty} l^* = 0 \iff \alpha - \epsilon + \eta_1 - \tau_1 \geq 0 \geq \alpha - \epsilon + \eta_0 - \tau_0$

Formal employment of the second kind:

$\lim_{\tau_1 \to \infty} (l^*, s^*) = (1, 0) \iff \alpha - \epsilon + \eta_1 - \tau_1 \geq \alpha - \epsilon + \eta_0 - \tau_0 \geq 0$

2.2 The Minimum Wage

Suppose a minimum wage policy is introduced in this economy. Assume that the policy is only binding in the formal sector. Thus, if a worker is employed in the formal sector, his wage must be above the minimum wage. However, if the worker is employed in the informal sector, then his wage can be greater or smaller than the minimum wage. Denote by $(\tilde{w}, \tilde{s})$ the pair of wage and sector that would prevail in this economy in the presence of the minimum wage.

The minimum wage policy introduces a new constraint in the Nash bargaining problem: $\mathbb{1}\{w < m\}sl = 0$, where m is the minimum wage level. This constraint simply states that it cannot be the case that $s = 1$, $l = 1$, and $\mathbb{1}\{w < m\} = 1$. The worker must be either informal or unemployed, or, if he is employed and formal, it has to be the case that his wage is greater than or equal to the minimum wage. The introduction of the minimum wage changes the equilibrium joint distribution of sector and wages. This equilibrium can be characterized by solving the constrained Nash bargaining problem.

The new equilibrium has the property that if in the absence of the minimum wage, the worker's wage was larger than the minimum wage ($w^* > m$), then the worker earns exactly the same wage once the policy is introduced. This is straightforward to see since if the constraint imposed by the minimum wage is not binding for (l^*, s^*, w^*) – the triplet employment, sector, and wage that solves the unconstrained Nash Bargaining problem – then it must be the case that (l^*, s^*, w^*) also solves the constrained optimization problem.

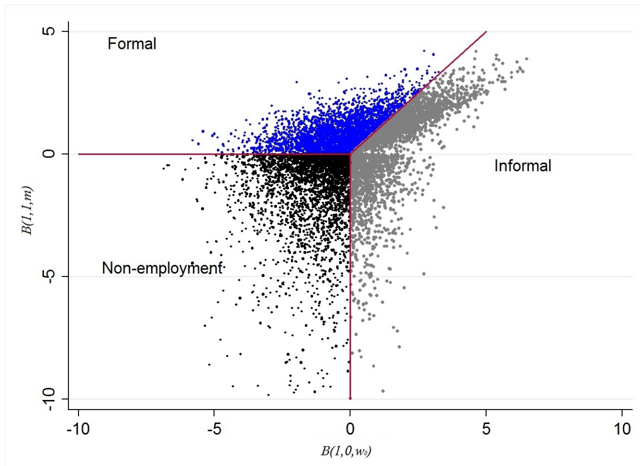
For the workers whose constraint $\mathbb{1}\{w < m\}sl = 0$ is binding when evaluated at the equilibrium wage, sector, and employment, the solution will be different than it would be in the absence of the minimum wage. Define the Nash Product by $B(l, s, w) = (\Pi(l, s, w))^{1/2}(U(l, s, w))^{1/2}$. The constrained solution to the Bargaining problem is given by: $w = m$, $s = 1$, and $l = 1$ if $\max\{B(1, 1, m), B(1, 0, w_0), B(0, \cdot, \cdot)\} = B(1, 1, m)$. This means that the worker remains employed in the formal sector and earns the minimum wage if the Nash product of this action is still positive (so it dominates being fired) and also it is greater than the Nash product obtained when employing him in the informal sector assigning him to the optimal informal sector wage. For later use in our empirical exercise, we denote the proportion of low wage formal workers that "bunch" at the minimum wage by $\pi_m^{(1)}$. This is an object we hope to estimate from the data.

Conversely, the worker will move to the informal sector if $\max\{B(1,1,m), B(1,0,w_0), B(0,\cdot,\cdot)\} = B(1,0,w_0)$, that is, if the costs of moving the worker to the informal sector are smaller than keeping him in the formal sector and changing his wage. In this case, we have that $\tilde{w} = w_0$; that is, the worker will earn in the presence of the minimum wage the wager associated with his assignment to the *informal sector*, w_0. Lastly, if both $B(1,1,m)$ and $B(1,0,w_0)$ are negative, then the constrained solution to the bargaining problem will be to let $\tilde{l} = 0$, that is, the worker becomes unemployed. This will be the case if the worker's cost of informality is high and $\Pi(1,1,m)$ is negative, that is, the (net of taxes and opportunity costs) productivity of the worker is below the minimum wage. We denote the fraction of low-wage formal workers that move to the informal sector as a result of the policy by $\pi_d^{(1)}$. Lastly, we let the remaining fraction of low-wage formal workers that end up losing their jobs as a result of the minimum wage policy by $\pi_u^{(1)}$.

Figure 4 displays the responses of different formal sector workers for which the minimum wage constraint is binding. The southeastern corner consists of workers whose Nash product in the informal sector is still positive and larger than the Nash product evaluated at formal employment at the minimum wage. These workers move to the informal sector as a response to the minimum wage policy. As shown in Figure 3, all these workers are in formal employment contracts of the second kind. The northwestern corner consists of workers for which the Nash product when assigned to the formal sector at the minimum wage $B(1,1,m)$ is positive and larger than the Nash product when assigned to the informal sector $B(1,0,w_0)$. These workers remain

Figure 4: Formal workers' response to the minimum wage

Employment and sector choice



Note: $B(l,s,w) = \Pi(l,s,w)^{0.5}U(l,s,w)^{0.5}$.
Simulated data from the model using Gaussian draws. See text for more details.

formal and earn the minimum wage. These workers may belong to either formal-parasite or formal-survival firms. The southwest corner consists of the workers for which both the Nash product when assigned to the formal sector at the minimum wage $B(1,1,m)$ *and* the Nash product when assigned to the informal sector $B(1,0,w_0)$, are both negative. These workers lose their jobs once the minimum wage is introduced. All these workers are, as shown in Figure 3, in formal employment contracts of the first kind. The red lines characterize the geometric locus of the boundary of indifference between these choices.

The difference in the surplus of workers and firms when operating in the formal and informal sector is given by $\Delta = \frac{1}{2}(\eta_1 - \eta_0 - \tau_1 + \tau_0)$. The presence of Roy's comparative advantage (or unobserved heterogeneity) and Rosen's compensating wage differentials lead workers and firms to get distinct rents when they operate in different sectors. That implies that workers that end up moving to the informal sector due to the presence of the minimum wage could have utility levels that are, in principle, quite different than the wages that they would have in the formal sector. Our next result, however, shows that for small values of m, formal and informal sector surpluses must be approximately the same for the workers that move. Consider the effect of introducing a "small" minimum wage that approaches the lowest wage equilibrium in the formal sector; that is, m is slightly above $\inf w_1$. For simplicity, assume $\inf w_1 = 0$.

Proposition 1. *Let c denote some positive constant, we have:*

(a) For an arbitrary small $c > 0$, if the minimum wage $m \leq c^2$, then $\Pr\left[\Delta \leq c | \max\{B(1,1,m), B(1,0,w_0), B(0,\cdot,\cdot)\} = B(1,0,w_0)\right] = 1$.

(b) For all m, $\lim_{c\downarrow 0} \Pr\left[\Delta \leq c | \max\{B(1,1,m), B(1,0,w_0), B(0,\cdot,\cdot)\} = B(1,1,m)\right] = 0$.

(c) For all m, $\lim_{c\downarrow 0} \Pr\left[\Delta \leq c | \max\{B(1,1,m), B(1,0,w_0), B(0,\cdot,\cdot)\} = B(0,\cdot,\cdot)\right] = 0$.

(d) For an arbitrary small $c > 0$, if the minimum wage $m \leq c$, then $\Pr\left[U(1,1,w_1) + \Pi(1,1,w_1) \leq c | \max\{B(1,1,m), B(1,0,w_0), B(0,\cdot,\cdot)\} = B(0,\cdot,\cdot)\right] = 1$.

Proposition 1 is proved in Appendix B, Section 2.2. Part (a) implies that the probability that the difference in workers' utility between the formal and informal sectors (i.e., $\Delta = U(1,1,w_1) - U(1,0,w_0)$) is close to zero, conditional on switching from the formal to the informal sector as a result of a small minimum wage is 100%. In other words, for small values of m and for the subset of workers that move to the informal sector due to the minimum wage, we have that $U(1,1,w_1)$ is approximately the same as $U(1,0,w_0)$. As a result, the welfare cost of such a movement for these worker-firm pairs is negligible. Parts (b) and (c) show that the same probability is zero for the workers that bunch at m and also for those that lose their jobs. Part (d) shows that, when m is small, if the result of the minimum wage policy is to induce a worker-firm pair to dissolve the employment relationship, then the surplus of this employment relationship must be small. The next proposition summarizes the expected welfare change of formal workers due to the minimum wage.

Proposition 2. *When the minimum wage is small, the change in the formal worker's expected welfare is approximately given by $m \times \pi_m^{(1)}$ times the fraction of formal workers* *for which the policy is binding. This object can be bounded above by the minimum wage level times the fraction of formal workers that are observed with wages equal to the minimum wage. The upper bound coincides with the actual welfare approximation if the minimum wage generates no unemployment.*

Proposition 2 immediately follows from parts (a) and (d) of Proposition 1. Formal workers who are affected by the minimum wage react in one of three ways: (i). shift to the informal sector; (ii) bunch at the minimum wage; and (iii) become unemployed. Part (a) and (d) of Proposition 1 imply that at a small minimum wage, the expected welfare change associate with reactions (i) and (iii) are close to zero. Therefore, the expected welfare change comes solely from workers with reaction (ii), that is, those who bunch at the minimum wage.

3 Empirical Strategy

One key challenge in bringing this model to the data is that we do not typically observe certain objects, such as the value of time at home (ϵ), or the worker's valuation of formal job amenities (η_1). In this section, we discuss the simplifying restrictions we must impose on the model's structure so we obtain a tractable likelihood function that can be successfully computed when we only observe data on the pair of sector and wage, but not on the fundamental shocks that drive the model ($\alpha, \epsilon, \tau_1, \eta_1, \tau_0, \eta_0$). We show that, under certain restrictions, the model collapses to a two-sector structural version of the model used by Meyer and Wise (1983) to evaluate the effects of the minimum wage in the USA. We warn beforehand that, although we believe that most of these restrictions are justified, one fruitful avenue for future research would be to attempt to estimate the model in a setting in which the econometrician observes a richer set of drivers of the variation in wage and sector choice, which would render disposable some of the restrictions we impose below.

Assumption 1 (Productivity distribution)**.** *The worker's productivity α is drawn* *from a log-normal distribution.*

The assumption of log-normality is commonly invoked to model wage data (Meyer and Wise (1983); Laroque and Salanié (2002), and many others). Roy (1950) developed theoretical arguments that suggest that productivity should approximately follow a log-normal distribution.

Assumption 2. *Let u_s be the orthogonal projection residuals of $\epsilon - \eta_s - \tau_s$ on α and β_s be the corresponding projection coefficient. The variance of u_s is small in the following sense: There exists two small positive numbers t and k such that,* $\Pr[|\frac{u_s}{\alpha(1+\beta_s)}| > t] \leq k$.

This assumption states that the distribution of wages is, up to some location and scale shifts, *approximates* the distribution of worker's productivity. A more detailed description of this projection argument can be found in Appendix C, Section 3.1. Importantly, this assumption does not impose that the remaining forces in the model, such as the value of amenities, taxes, and time at home, are necessarily small. It allows workers to be out of the labor force ($\epsilon \to \infty$), it allows for some workers to arbitrarily large costs of formality or informality. It also allows amenities (η) and costs (τ) to be a non-trivial fraction of the worker's productivity or wages. What the assumption restricts is the probability that the component of these costs that varies independently of the worker's productivity becomes too large. Intuitively, what it states is that when we look at workers at quantiles far apart on the wage distribution, we are inclined to believe that there is a high likelihood that such large differences in earnings are due to differences in the marginal product of labor. The most extreme opposite of this assumption would be to believe that workers with large differences in earnings must have similar productivities but large differences in the tax treatments of their occupations (differences in τ) or large differences in the non-pecuniary aspects of the job (η). We stress that we do not bound how large η, τ, or ϵ can be, so workers with prohibitively large costs of formality or informality are not ruled out by this assumption, neither are workers that are out of the labor force.[9] In that sense, when

[9] For example, if the costs of formality for workers in a similar occupation are the sum of a constant

we look at the distribution of wages and observe workers with drastically different wages, we do not assume that these differences should be immediately attributed to large differences in the valuation of amenities, neither to differences in their costs of operating in the formal/informal sector.

Assumption 3 (Common-Support). *In the absence of the minimum wage, the support of* w_s *is* $\mathbb{R}^+$ *for both sectors.*

This assumption states that, without the minimum wage, the effective lower bound for both wage distributions is zero. This assumption is standard in the policy evaluation literature. The key implication of this assumption is that, in the absence of the minimum wage, the conditional probability of formality given the wage never touches zero or one. If this assumption were violated, in the absence of the minimum wage, one could predict with 100% accuracy the likelihood that a worker is formal or informal just by knowing his latent wage, which would make the conditional probability model degenerate. We believe that this is not a controversial assumption. In Appendix C, Section 3.1, we show that, in the absence of such an assumption, we only need to add one extra parameter to the model. The added parameter characterizes the lower end of the support of the formal sector latent wage distribution.[10]

Together, Assumptions 1, 2, and 3 stated above place strong restrictions in the shape of the joint distribution of sector and wages, as we demonstrate in Proposition 3 and 4 below. As a result, these assumptions are partially testable. If the model cannot fit the data, then either the assumed shape for the joint distribution of sector and wage is misspecified, or the minimum wage effects are. However, the converse is not necessarily true: It is possible for the model to fit the observed data and still

fraction of their productivity plus a firm-worker specific costs of bookkeeping, then Assumption 2 implies that the distribution of worker-firm specific costs of formality (the bookkeeping costs) are small relative to the distribution of worker's productivity.

[10] We stress that in the presence of the minimum wage, the support of the wage distributions will likely not be the same since the formal sector wage distribution has a lower bound at m, whereas the informal sector wage distribution has a lower bound at zero. Our assumption is about the support of these wage distributions in the absence of a price floor.

be misspecified. In that sense, observing a good fit is a necessary but not sufficient condition to trust the counterfactuals obtained from the model. However, if our functional form assumptions are severely misspecified, the model fit will likely be poor. Thus, when we look at the model fit, we are testing a condition that is implied by these assumptions but not equivalent to them. We cannot form a consistent test in this case because we do not observe the latent wage distribution directly. We show in our empirical application that this is indeed the case for the Brazilian labor market, although we cannot say that this must be the case in other settings.

3.1 Implications

Under the assumptions 1, 2, and 3 presented in the previous section, we can characterize the joint distribution of sector and wage both in the absence and in the presence of the minimum wage. In this section, we show this characterization and how it implies a particular structure for the effects of the minimum wage, which can be thought of as a two-sector extension of the empirical framework proposed by Meyer and Wise (1983).

Given that our interest lies in estimating the causal effect of the minimum wage, in the following discussion that follows, it is useful to employ the Rubin potential outcomes notation. In the presence of the minimum wage, a worker i is characterized by a wage $W_i(1)$ and a sector $S_i(1)$, which is equal to one if the worker is employed in the formal sector and zero otherwise.

Also, let the pair $(W_i(0), S_i(0))$ denote the counterfactual – or latent – wage and sector that prevail in the absence of the minimum wage. Finally, define $F_0(w)$ ($f_0(w)$) as the c.d.f (p.d.f) of latent wages $W(0)$ and $F(w)$ ($f(w)$) as the c.d.f (p.d.f) of observed wages ($W_i(1)$ or, using shorter notation, W_i). We assume that the econometrician observes a random sample of i.i.d. draws of the pair $(W(1), S(1))$. That is, the econometrician only observes the data in the presence of the policy.

Proposition 3 (Latent wage distribution)**.** *Suppose Assumptions 1 to 3 hold. In the absence of the minimum wage, the density of potential wages in the formal sector* $f_{w_1}(x)$ *and in the informal sector* $f_{w_0}(x)$ *follow a log-normal distribution. In addition, if* $(\log w_s, \Delta)'$ *approximately have a joint normal distribution, then the density of log-wages for those that choose to work in each sector – that is, the formal sector density* $f_{\log w_1}(x|S(0) = 1)$ *and the informal sector density* $f_{\log w_0}(x|S(0) = 0)$ *– follow skew-normal distributions. The unconditional log-wage distribution* $f_{\log w}(x)$ *follows a mixture of skew-normals. In addition, if* $S(0)$ *is independent of* α*, then this mixture of skew-normals is well-approximated by a mixture of normals. Instead, if* $w_1 \approx w_0$*, then this mixture of skew-normals collapse to a (single component) normal.*

The exact form of wage distributions mentioned in Proposition 3 and its proof can be found in Appendix C, Section 3.1. Proposition 3 specifies the shape of the distribution of wages, both across sectors and also unconditionally, under different restrictions. Our parametric family nest well-known parametric families used to model the distribution of earnings. For example, when $\log w_1 \approx \log w_0$ and also $S(0)$ is independent of α, then the distribution of wages collapses to the log-normal wage distribution that has been used by Roy (1950). If we relax these two constraints, then the wage distribution we obtain is identical to the canonical form of the Roy (1951) model of self-selection. Finally, if we drop the common-support assumptions, the wage distribution becomes a mixture of skewed *and shifted* log-normals. Defining θ_0 to be the parameters of this distribution and f_0 to be the latent density of wages, we have that, $f_0(w) = f_0(w; \theta_0)$.

The advantage of using a parameterized functional form for the latent distribution of wages is that it allows for maximum likelihood estimation of the model parameters. On the other hand, the robustness of the empirical results to deviations from log-normality is lost. Thus, there is a trade-off. In our empirical application, we show that the Brazilian log-wage distribution seems to be well approximated by a normal distribution, so even our most restrictive assumption provides an accurate approximation of the target distribution. The parametric assumption does not seem to impose a significant loss of credibility in this particular exercise.

Proposition 4 (Conditional probability of the (latent) sector given the wage)**.** *The conditional distribution of the latent sector given the latent wage can be approximated by the Logit parametric family* $\{\Lambda(w, \delta) : \delta \in B \subset \mathbb{R}^k\}$*. That is,* $\Pr[S(0) = 1|W(0) = w] = \Lambda(w, \delta_0)$ *for some* $\delta_0 \in B$*.*

This result states that the conditional probability of formality given the wage, in the absence of the minimum wage policy, can be approximated using a logit model. See Appendix C, Section 3.2, for details. In our empirical application, we find that even a trivial model that forces the coefficient of the wage in the logistic regression to be zero would give a reasonable fit to the data.

To complete the characterization of the joint distribution of latent and observed wages, we need to characterize the form in which the minimum wage affects the joint distribution of sector and wages, both above and below the minimum wage level. To do that, we use the results from our theoretical model. For completeness, we summarize the key model implications we use to derive the likelihood function in the following remarks:

Remark 1 (Limited spillovers)**.** *Workers whose latent wages would be above* k *times the minimum wage are not affected by the policy. That is,* $W(1) = W(0)$ *and* $S(1) = S(0)$ *when* $W(0) > km$*, for a known* k *greater than or equal to one.*

This remark only restates the fact that the constraint imposed by the minimum wage in this labor market is not binding for workers that earn more than m. Thus, as a result, there will be no economic spillover effects of the minimum wage above km for any $k \geq 1$. However, this result still allows the upper part of the wage distribution to be affected by the rescaling associated with the inflows and outflows of workers that will occur as a result of the policy.

The structural model presented in Section 2.1 implies that one could choose $k = 1$, since the model predicts that $W(1) = W(0)$ whenever $W(0) < m$. However, this is a simple consequence of the assumed absence of linkages between the labor market of low-wage and high-wage workers in our model. In other words, the absence of spillovers we obtain is close to an apriori assumption of the model. In many other models (see, for example, Engbon and Moser, 2018), the minimum wage would induce spillover effects higher up on the wage distribution. Moreover, there is evidence that these effects are non-trivial (Engbon and Moser, 2018).[11] Thus, we highlight here that we only need spillovers to become small as we look further up on the wage distribution, which is an assumption justified in most if not all models in which the minimum wage generates spillover effects. In our baseline empirical results, we will assume away any spillover, which is the same as setting k to be one.

It is worth pointing out we only need to correctly establish the upper limit on spillovers if our goal is to estimate *all* of the model parameters. If our focus is only to find the effects of the minimum wage on the size of the formal sector, then *correctly specifying the spillover effects of the minimum wage turns out not to be necessary.* We show in the Appendix E that we can still obtain the effects of the minimum wage on the size of the formal sector even in the scenario in which we know that we are obtaining inconsistent estimates of the minimum wage on the wage distribution – that is, even if we misspecify the effects of the minimum wage on the upper part of the wage distribution. Correctly specifying the spillover effects – or setting the correct k – is not required to estimate that particular object.

Remark 2 (Minimum wage effects' structure (strong characterization))**.** *For wages below the minimum wage* ($W(0) < m$)*, we have the following: If* $S(0) =$ *0, then*

[11]The economic mechanisms commonly invoked to explain how the minimum wage can affect workers that would already earn more than the minimum wage are low-skill/high-skill labor substitution (Teulings, 2000)) and search externalities (Flinn, 2006), Engbom and Moser (2017)).

$S(1) = S(0)$. Additionally, with probability $\pi_d^{(0)}$, – which is potentially a function of the worker's latent wage–, the wage ($W(1) = W(0)$) continues to be observed. With the complementary probability $\pi_m^{(0)} = 1 - \pi_d^{(0)}$, the worker earns the minimum wage ($W(1) = m$).

If $S(0) = 1$, then with probability $\pi_d^{(1)}$, – which is also potentially a function of the worker's latent wage– the wage ($W(1) \approx W(0)\xi$) continues to be observed, meaning that the worker successfully transitions from the formal to the informal sector and thus will earn his corresponding informal sector's wage.[12] *In this case, the observed sector will be $S(1) = 0$, which differs from the latent sector. With probability $\pi_m^{(1)}$, the worker earns the minimum wage ($W(1) = m, S(1) = 1$). With the complementary probability ($\pi_u^{(1)} = 1 - \pi_d^{(1)} - \pi_m^{(1)}$), the worker becomes unemployed ($W(1) = \cdot, S(1) = \cdot$).*

Note that we allow here the possibility that informal workers will also "bunch" at m when the minimum wage is introduced. We abstract from these considerations in our theoretical exercise, but we accommodate for this regularity in our empirical exercise.[13]

3.2 Estimation

Collecting the results of the previous section, we obtain a tractable expression for the likelihood of the data under the minimum wage policy. This allows us to estimate the model parameters and the corresponding effects of the minimum wage using maximum likelihood.

Let $\Theta \equiv (\theta, \delta, \pi)$ be the entire vector of model parameters, that is, those governing the latent distribution of wages, the conditional probability of sector given wages and

[12]The parameter ξ that appears on the expression $W(1) \approx W(0)\xi$ measures the expected change on a formal worker's wage when he moves to the informal sector because of the minimum wage. ξ corresponds to the ratio of the means of the latent distributions in the informal and formal sectors.

[13]These probabilities are reduced-form parameters that characterize how the minimum wage affects wages and formality in the economy. These objects have, under the assumptions of the model, internal validity to assess the effects of a particular minimum wage level and the effects of small changes on it, but they are not invariant to changes in the economy, so they are subject to external validity concerns and the Lucas' critique.

minimum wage effects. Define the likelihood of observing a pair (w, s) given the minimum wage level m and model parameters Θ as $L(W(1) = w, S(1) = s|\Theta) = \Pr[S(1) = s|W(1) = w; \Theta]f(w|\Theta)$.

Given that $\log(L(W(1) = w, S(1) = s|\Theta) = \log \Pr[S(1) = s|W(1) = w; \Theta] + \log f(W(1) = w|\Theta)$, we can define the maximum likelihood estimator of the model parameters as

$$\widehat{\Theta} = \arg\max_{\Theta} \frac{1}{N} \sum_{i}^{N} \log L(w_i, s_i|\Theta)$$

.

It is useful to note that the joint likelihood of observed sector and wage is a sum of a marginal of observed wage and a conditional probability of formality given the wage and although these objects share common parameters (through π) when looking at the whole support, they have no common parameters conditional on wages above k times m. This suggests an alternative way to attack the problem that is both simpler and also more robust.

The numerical optimization of the likelihood function can be simplified by breaking the problem into two steps. First, estimate the parameters of the latent wage distribution by considering only the values above the k times the minimum wage, where km is the upper bound on how far spillover effects of the minimum wage might reach:

$$\widehat{\theta} = \arg\max_{\theta} \frac{1}{N} \sum_{i}^{N} \mathbb{1}\{w_i > km\} \log f(w_i|w_i > km; \theta).$$

The likelihood function in this first step closely resembles the likelihood of a Tobit regression model (Tobin, 1958). This procedure is simple to implement and can be performed using canned functions in standard software. It is useful to note that the parameters that index the RHS of the equation above – which are given by the vector θ – are a subset of the parameters that characterize the entire joint distribution of observed data – which are given by the vector of (θ, δ, π).

To estimate the conditional probability of the latent sector given the wages, one can run a logit regression once again in a carefully constructed sub-sample that *use only wages above the k times the minimum wage*:

$$\widehat{\delta} = \arg\max_{\beta} \frac{1}{N} \sum_{i}^{N} \mathbb{I}\{w_i > km\} \log \Pr[s_i|\ w_i; \delta].$$

Again, note that by conditioning on values above km, we characterize the conditional probability of observed formality given observed wage as a function of only the parameters that characterize the conditional probability of latent formality given the latent wage (δ). This breaks the dependency that exists in the conditional probability of observed formality given observed wage on the vector of parameters π that characterize the minimum wage effects.

This procedure yields consistent estimates because the density of wages for values above the minimum is merely a function of a subset (θ) of the parameter vector (Θ) and the fact that the true parameter θ_0 that governs the shape of the density of *latent* wages is the argument that maximizes $\mathbb{E}[\log f(W(1)|W(1) > km; \theta)]$.[14] The same holds for the conditional probability of sector given the wage: The conditional probability of observed sector given the wage, for values above the minimum wage, is only a function of δ and the true parameter δ_0 that governs the relationship between *latent* sector and wages is the argument that maximizes $\mathbb{E}[\log(\Pr[S(1)|W(1) > km; \delta])]$. In this case, estimation is simple: In the first step, one merely needs to estimate a truncated regression of wages on a constant for values above k times the minimum wage. Then, in the second step, one needs to estimate a logit regression of sector on wages, using only values above the k times the minimum wage, as before.

It is useful to note that to obtain the effects of the minimum wage on employment, formality, and the wage distribution, once these first and second steps above are performed, there is no need to estimate anything else. That is, knowing θ and δ but not π turns out to be enough to estimate the effects of the minimum wage.

[14] Strictly speaking, this holds whenever ξ is smaller than one, that is, *on average*, latent wages in the formal sector are larger than on the informal sector. In the general case, one needs to condition on wages larger than $m \times \xi$.

Together, the parameters obtained in the first and second steps fully characterize the joint distribution of formality and wages in the absence of the minimum wage policy. Since all the effects of the minimum wage are a comparison between the joint distribution that prevails under the absence of the minimum wage policy – which is characterized by θ and δ but not π and recovered by the steps above – and the joint distribution that is observed in the data, one can already obtain all the effects of interest from the comparison between the distribution that is implied by the estimates of the modified Tobit and Logit models above and the distribution that is observed in the data.

Importantly, the minimum wage effects estimated by this two-step procedure will have an important robustness property: They do not depend on the bargaining model we set to be the correct description of the labor market. They also do not depend on whether the restrictions we imposed on the shocks of the bargaining model on Assumption 2 to be valid. Also, they do not depend on whether the effects of the minimum wage on the bottom part of the wage distribution, as described by Remark 2 to be correct. All that is needed for the minimum wage effects to be right is a set of three conditions: (i) The upper bound, km, set on spillover effects in the estimation procedure to be greater than or equal to the actual bound on spillovers in the labor market; (ii) The functional form assumed for the latent distribution of wages to be correctly specified, and (iii) the conditional probability of latent formality given latent wage to be correctly specified.

In other words, although the likelihood function we derive is obtained from the bargaining model's structure combined with statistical restrictions on the model's shocks, our empirical exercise can be understood in a different light: Our likelihood coincides with a two-sector version of the reduced form Meyer and Wise's model that accommodates movements from the formal to the informal sector as a result of the minimum wage. This reduced-form model imposes only functional form assumptions on the latent marginal distribution of wages, the conditional probability of formality given the wage, and limits on spillovers. It imposes restrictions on equilibrium objects such as the latent wage distribution but places no restriction on the process that leads to these restrictions to hold, such as the behavior of workers, firms, the magnitude of amenities, and the bargaining process. The actual wage distribution that we are drawing data from can come from a labor market that is more accurately described as one that presents search frictions, as in Meghir et al. (2015), or a labor market that displays substitution between different types of labor, such as in Teulings (2000). To obtain the consistent estimates of the effects of the minimum wage, all we need is that (i) the wage distribution to be well described by a log-normal – or whichever flexible family we imposed in our estimation–, (ii) the conditional probability of formality given the wage to be well described by a logit, and (iii) a known upper bound on the limits on how far up on the wage distribution spillovers can reach to be available. We do not need the bargaining model to be correct, neither Assumptions 1, 2, and 3 above, although the bargaining model combined with these assumptions implies that these conditions hold.

Our bargaining model, combined with the restrictions we imposed on the model shocks – Assumptions 1 and 2 in the previous section – gives us guidance and justification for the choices we make in terms of spillovers and functional forms. It also makes the empirical exercise coherent with a certain model for the labor market. They also allow us to obtain more efficient estimates using the whole sample and to obtain estimates of other objects such as the vector of the reduced form probabilities π, to obtain bounds for the types of formal contracts according to our taxonomy, and welfare effects. Thus, in our preferred estimates, we will use the estimates that take the full structure of the model into account, so we can discuss welfare and the taxonomy of formal employment. In the Appendix G, Section 7.1, we show the results we obtain when using the inefficient but more robust estimates.

Taking the full model's structure into account, we can now maximize the likelihood function over the subset of parameters that remains to be estimated π using the full sample: $\widehat{\pi} = \arg\max_{\pi} \frac{1}{N} \sum_{i}^{N} \log L(w_i, s_i|\widehat{\theta}, \widehat{\delta}, \pi)$. Efficiency can then be improved by using these estimates as initial values for the maximum likelihood estimator.

In our main results, we use the three-step procedure and set k to one, which is consistent with the full structure of our bargaining model. In the Appendix G, we report the results we obtain when we use the inefficient but more robust estimates by setting a k larger than one. Furthermore, in the Appendix E, we discuss the robustness of our estimates of the size of the formal sector to misspecification of spillovers.

4 Empirical Application: The Effects of the Minimum Wage in Brazil

4.1 Descriptive Statistics

To evaluate the impact of the minimum wage in Brazil, we used the PNAD household survey pooling years from 2001 to 2005. PNAD is an acronym for the Portuguese name of the survey, which can be translated as "Nationwide Household Sample Survey". These data, which are representative of the Brazilian population, are collected yearly by the IBGE, a Brazilian statistical agency. Workers who do not report wages, workers who work in the public sector, and workers who are older than 60 years of age or younger than 18 were removed from the sample. Additionally, workers who report monthly wages above R$5000 were removed from the sample, which excludes the upper 1.15% of the wage data.

Table 1 and Figures 5 present some empirical facts concerning the joint distribution of sectors and wages for Brazilian data from the 2001–2005 period. Regarding Table 1, we observe that expected wages are higher during the later years, which are characterized by higher minimum wage levels. Wages in the formal sector are, on average, higher than wages in the informal sector. The informal sector comprises a

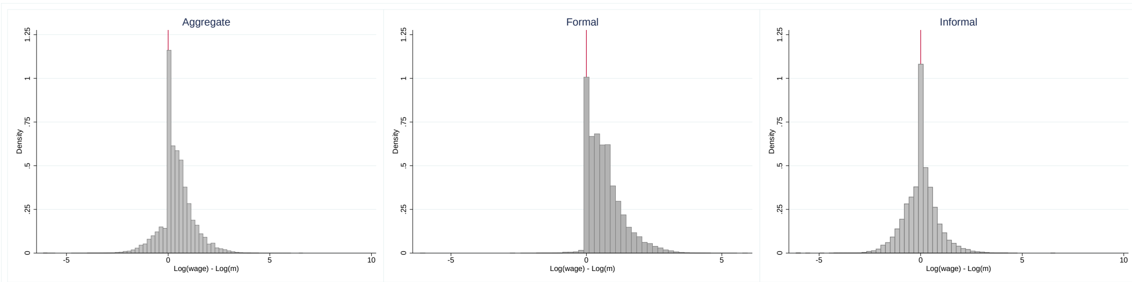
Table 1: Descriptive Statistics

Parameter	2001	2002	2003	2004	2005
$E[\log(W)]$	5.92	5.98	6.08	6.15	6.25
$Sd[\log(W)]$	0.73	0.73	0.71	0.70	0.67
Skewness$[\log(W)]$	0.37	0.31	0.22	0.19	0.19
Kurtosis$[\log(W)]$	4.28	4.52	4.66	4.76	4.75
$Pr[W = m]$	0.09	0.12	0.13	0.12	0.16
$Pr[W < m]$	0.06	0.07	0.08	0.08	0.08
$q^{80}[\log(W)]$	6.41	6.54	6.58	6.68	6.68
$q^{20}[\log(W)]$	5.30	5.38	5.48	5.56	5.70
$E[\log(W)\|S = 1]$	6.07	6.13	6.23	6.31	6.39
$E[\log(W)\|S = 0]$	5.57	5.63	5.71	5.77	5.88
$Sd[\log(W)\|S = 1]$	0.66	0.65	0.62	0.61	0.58
$Sd[\log(W)\|S = 0]$	0.77	0.78	0.77	0.75	0.74
$q^{80}[\log(W)\|S = 1]$	6.55	6.62	6.68	6.75	6.80
$q^{20}[\log(W)\|S = 1]$	5.52	5.58	5.70	5.77	5.86
$q^{80}[\log(W)\|S = 0]$	6.11	6.21	6.21	6.25	6.40
$q^{20}[\log(W)\|S = 0]$	5.19	5.19	5.30	5.30	5.30
$Pr[S = 1]$	0.70	0.70	0.71	0.71	0.72
$Pr[S = 1\|W < m]$	0.08	0.08	0.09	0.04	0.03
$Pr[S = 1\|W = m]$	0.58	0.53	0.63	0.63	0.62
$Pr[S = 1\|W > m]$	0.76	0.78	0.79	0.79	0.81
$log(m)$	5.19	5.30	5.48	5.56	5.70

Note: Nominal wages in Brazilian R$. Sample size: 573470.

Source: Authors' elaboration using data from the "PNAD".

Figure 5: Wage distribution



Note: Data from years 2001 to 2005. Log-wages normalized around the minimum wage level.
Source: Authors' elaboration using the "PNAD" dataset.

large share of the aggregate economy – approximately 28% based on these data. The probability that a wage is equal to the minimum wage ($\Pr[W = m]$) ranges from 8 to 14%. The proportion of workers who receive wages below the minimum wage $\Pr[W < m]$ ranges from approximately 6 to 8%. The probability of working in the formal sector as a function of the wage is discontinuous. It is approximately 79% for values above the minimum wage, 59% at the minimum wage, and virtually zero below it.

4.2 Results

In this section, we report the results of our estimation of the effects of the minimum wage. Our preferred specification uses lognormality for the latent wage distribution and a linear in wage specification for the logistic regression of the conditional probability of formality given the wage. Further details about the estimation can be found in Appendix D.

Although our model estimates allow us to obtain estimates of the effects of the minimum wage on the wage distribution, our main interest lies in the effects of the minimum wage on the size of the formal sector. The reasons for this are twofold: First, the effects of the minimum wage on the wage distribution depend heavily on our ability to correctly characterize the spillover effects of the policy. In contrast, as we show in Appendix E, the effects of the minimum wage on the size of the formal

Table 2: Parameter Estimates

Parameter		2001	2002	2003	2004	2005
Aggregate						
π_d	Non-compliance	0.20***	0.22***	0.20***	0.21***	0.19***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
π_m	Bunching	0.24***	0.36***	0.28***	0.25***	0.37***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
π_u	Non-employment	0.56***	0.42***	0.52***	0.54***	0.44***
		(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
$Pr[W(0) < m]$	Fraction Affected	0.27***	0.27***	0.33***	0.33***	0.33***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.01)
$Pr[S(0) = 1]$	Latent size of the formal sector	0.76***	0.78***	0.78***	0.78***	0.81***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Formal Sector						
$\pi_d^{(1)}$	Sector mobility	0.04***	0.21***	0.09***	0.08***	0.19***
		(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
$\pi_m^{(1)}$	Bunching	0.19***	0.23***	0.22***	0.20***	0.26***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$\pi_u^{(1)}$	Non-employment	0.77***	0.56***	0.69***	0.71***	0.55***
		(0.02)	(0.02)	(0.02)	(0.01)	(0.02)
Informal Sector						
$\pi_d^{(0)}$	Non-compliance	0.62***	0.24***	0.56***	0.59***	0.21***
		(0.01)	(0.03)	(0.02)	(0.02)	(0.03)
$\pi_m^{(0)}$	Bunching	0.38***	0.76***	0.44***	0.41***	0.79***
		(0.01)	(0.03)	(0.02)	(0.02)	(0.03)

Note: Bootstrapped standard errors (computed using 100 replications) are given in parentheses.

Source: Authors' elaboration using the "PNAD" dataset.

sector are highly *insensitive* to misspecification of spillover effects.

Second, the labor reallocation across sectors as a result of the policy is what allows us to obtain information about the taxonomy of contract types we defined earlier in the paper. For those reasons, we focus mainly on the effects on the size of the formal sector, and we treat the other parameters such as the effects of the minimum wage on average wages – although surely of interest on their own– as a nuisance for the purposes of our exercise.

In examining the point estimates and standard errors in Table 2, we observe sizable estimates of the unemployment effects of the minimum wage. The results also indicate that the minimum wage affects wages in both the formal and informal sectors. The evidence from Table 2 suggests that sector mobility is limited. The estimates of the sector-mobility parameter ($\pi_d^{(1)}$) are approximately 12%, averaging across different years.

We estimate the latent size of the formal sector of approximately 78% of the economy (taking the year 2004 as an example). This implies that the minimum wage reduces the size of the formal sector by approximately 8%. The informal sector, on the other hand, grows approximately by 27%, from about 22% to 28% of the economy. This larger effect in the relative size of the informal sector is explained by the fact that this sector is approximately 3.5 times smaller than the formal sector in the absence of the policy. These values are similar to, although smaller than, the results obtained by Jales (2018) using a reduced-form density discontinuity framework.

Again taking the year 2004 as an example and using our welfare approximation formula, we find that the minimum wage increases the ex-ante expected welfare of formal workers affected by the policy by around 52 Brazilian Reais. This is roughly 7% of the average wage earned by formal sector workers in that year.[15] Lastly, these estimates imply that – for the subset of formal worker-firm pairs that the minimum wage is binding – the fraction of contracts of the first kind must be between 71 and 91 percent, whereas the fraction of contracts of the second kind must be between 4 and 23 percent.[16]

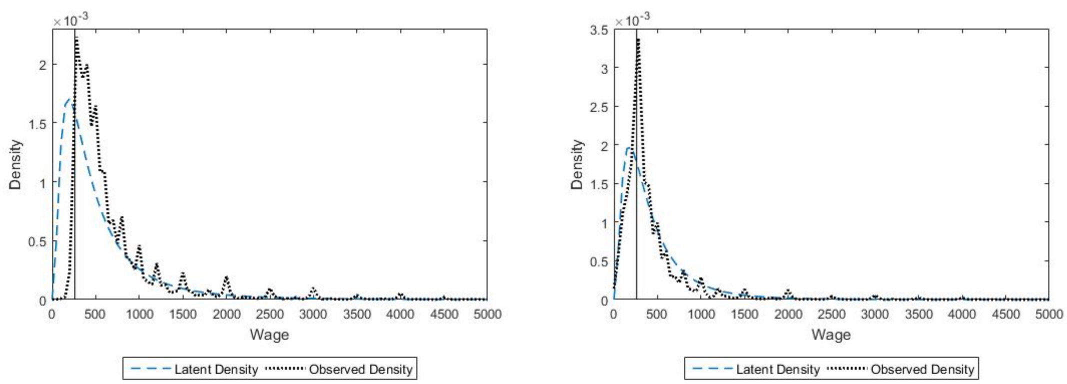
Figure 6 displays the observed and latent densities for the formal and informal sectors based on the model parameter estimates for the year 2004. The latent wage distribution tends to be below the observed distribution for the formal sector for values above the minimum wage. This is a consequence of workers moving away from the formal sector (into either unemployment or informal employment). The sector-mobility channel increases the measured density above the minimum wage due to a rescaling effect. The informal sector, as predicted by the model, behaves in the opposite way: The observed density tends to be below the latent density for values above the minimum wage. This result is due to the inflow of workers from the formal

[15] We stress that our welfare approximation is only valid for small enough minimum wage levels and it does not include any spillover or general equilibrium effect of the policy, so this particular estimate is subject to a much higher degree of uncertainty than the other estimates we report.

[16] These bounds come from the fact that the type of contract is identified provided that the worker firm pair either moves to the informal sector or ends the employment relationship. For those that bunch at the minimum wage, the minimum wage does not increase the costs of formality to an extent large enough to identify the nature of the contract.

Figure 6: Latent and Observed Densities

Formal Sector Informal Sector



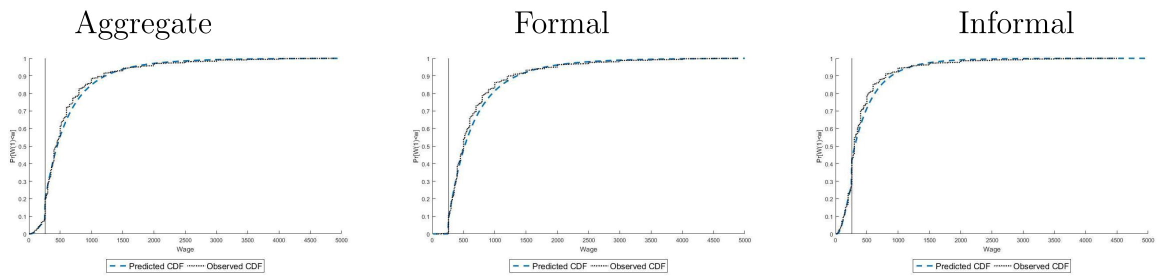
Note: Density estimates using a gaussian kernel (bandwidth = R$30). Year 2004. Source: Author's elaboration using the "PNAD" dataset.

sector, which induces a rescaling of the density and reduces its values above the minimum wage.

Figure 7 displays the fit of the model for the cumulative distribution functions of the unconditional and conditional wage distributions. The model seems to be able to capture the most important features of the joint distribution of sector and wage with a relatively small number of parameters, which gives some suggestive evidence that the functional form assumptions we use for the distribution of wages and the conditional probability of formality given the wage are not severely misspecified. We discuss in greater detail the model fit in Appendix G, Section 7.2. We find that the model can reasonably fit moments that are not directly targeted, such as the skewness and the kurtosis present in the wage distribution. In the Appendix G, we also discuss the robustness of these estimates to the restrictions we impose on spillovers and also on the form through which the minimum wage affects the bottom part of the wage distribution.

In Appendix E, we evaluate the robustness of our key parameter of interest – the effect of the minimum wage on the size of the formal sector – to various degrees of misspecification of the spillover effects of the policy. We find that our estimators have

Figure 7: Observed and Predicted CDFs



Note: Year 2004. Source: Author's elaboration using the "PNAD" dataset.

important robustness properties that prevent them from being too sensitive to errors in estimating the effects of the minimum wage on the wage distribution. As a result, even errors of orders of magnitude in the location and scale of the wage distribution do not induce large changes in our estimate effects on the size of the formal sector.

4.2.1 Marginal Effects of the Minimum Wage

In this section, we compute the effects of changes in the minimum wage level, using a plug-in approach for the terms appearing the expressions for the marginal effects. More details are discussed in Appendix G.

Table 3 displays the estimated effects of *changes* in the minimum wage level implied by the estimates of the structural parameters. We computed the effects of the minimum wage on average wages, employment, and the relative size of the formal sector. To place the numbers in perspective, we multiply the marginal effects obtained by a typical change in the value of the minimum wage observed in the analyzed period (R$20,00).

The estimates show that the minimum wage increases wages for the aggregate economy. The estimated effect is approximately R$7.80, or 40% of the change in the minimum wage. The estimated effects on average wages show a larger effect in the formal sector, of approximately R$14.90. Both of these effects are driven by the increase in the wage of low-wage workers *and* the decrease in the proportion of low-wage workers.

The results indicate an effect close to zero for wages in the informal sector. The small estimated effects of the minimum wage on average wages in the informal sector result from a combination of higher wages for some informal workers and the inflow of low-wage workers to the informal sector. This latter channel decreases the perceived effect of the policy in the informal sector.

Averaging the estimates across the period considered, the results suggest an approximately 1.6% decrease in employment following a typical (and exogenous) change in the minimum wage level. The decrease in the size of the formal sector is larger, approximately 3%. The estimates show that the informal sector experiences a 0.95% increase in employment. In terms of the relative size of the formal sector, the estimates suggest a decrease of approximately 3.97%. To place this number in perspective, the size of the formal sector in 2004 is 0.72. This estimate suggests that an exogenous increase (of R$20,00) in the minimum wage level would induce the formal sector to decrease to 0.71, that is, to decrease by one percentage point. This effect takes into account the outflow of workers from the formal sector, the inflow of workers to the informal sector, and the decrease in the size of the formal sector due to unemployment.[17]

4.3 Reduced-form evidence

Figure 8 displays levels of formality across states by year, as a function of a standard measure of the strength of the minimum wage: the distance between the median wage and the minimum wage level. If the minimum wage decreases the size of the

[17]This exercise highlights that the effect of marginal changes in the minimum wage may be too small to be detected using time-series variation, especially if minimum wage changes are more likely to happen during the boom part of business cycles. Nevertheless, the cumulative effect of all minimum wage changes on the size of the formal sector may not be inconsequential.

Table 3: Marginal Effects

Parameter	Expression	2001	2002	2003	2004	2005
Average Wage						
Aggregate	$\frac{\partial E[W(1)]}{\partial m}$	8.40***	6.72***	8.41***	8.28***	7.18***
		(0.30)	(0.29)	(0.28)	(0.23)	(0.34)
Formal Sector	$\frac{\partial E[W(1)\|S(1)=1]}{\partial m}$	16.16***	12.60***	16.54***	15.97***	13.23***
		(0.56)	(0.55)	(0.53)	(0.45)	(0.64)
Informal Sector	$\frac{\partial E[W(1)\|S(1)=0]}{\partial m}$	1.61***	0.28	1.53***	1.44***	1.19***
		(0.21)	(0.20)	(0.18)	(0.16)	(0.16)
Employment						
Aggregate (% Change)	$\frac{\partial c}{\partial m}\frac{1}{c} \times 100$	-1.96***	-1.28***	-1.71***	-1.65***	-1.13***
		(0.08)	(0.08)	(0.08)	(0.06)	(0.09)
Formal Sector (% Change)	$\frac{\partial c^{(1)}}{\partial m}\frac{1}{c^{(1)}} \times 100$	-3.62***	-2.59***	-3.37***	-3.12***	-2.38***
		(0.14)	(0.13)	(0.12)	(0.10)	(0.14)
Informal Sector (% Change)	$\frac{\partial c^{(0)}}{\partial m}\frac{1}{c^{(0)}} \times 100$	0.31***	1.60***	0.74***	0.68***	1.41***
		(0.11)	(0.10)	(0.09)	(0.09)	(0.08)
Relative size (% Change)	$\frac{\partial \log(Pr[S(1)=1]/Pr[S(1)=0])}{\partial m} \times 100$	-3.93***	-4.18***	-4.11***	-3.81***	-3.79***
		(0.07)	(0.07)	(0.07)	(0.07)	(0.08)

Note: Marginal effect estimates multiplied by a typical change (R$20.00) in the minimum wage. Bootstrapped standard errors (computed using 100 replications) are given in parentheses. Source: Author's elaboration.

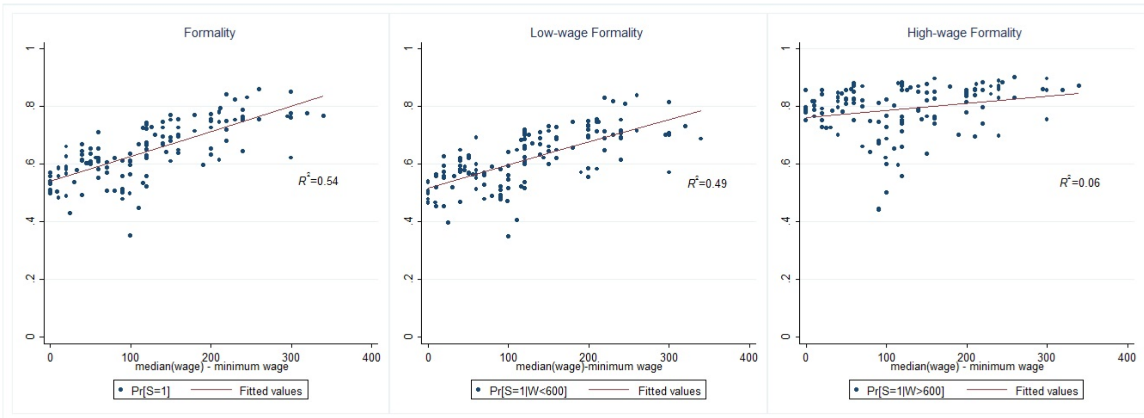
formal sector, one should expect, *ceteris paribus*, that comparable states with a larger fraction of workers affected by the policy to present smaller levels of formality. One reasonable concern, though, is that these states might not be comparable: States that are different in terms of their wage distributions are also different in terms of their latent levels of formality. We can, however, gather some evidence of these differences by looking at points in the wage distribution in which we do not expect the minimum wage to have any substantive effect on formality.

The graph on the left corner of Figure 8 displays the aggregate level of formality at each state by year, as a function of the distance between the median wage and the minimum wage. We see there a strong relationship: States for which the minimum wage is more binding tend to present smaller levels of formality. This relationship may or may not be related to the causal effect of the minimum wage since these differences could be attributed to general differences in the level of formality across states that happen to correlate with the minimum wage strength. The third graph, however, suggests that this should not be a serious concern. Once one looks at the levels of formality for wages above R$600, the relationship between the minimum wage and the level of formality becomes weak. For wages above R$600,00, we do not expect any difference to be attributed to minimum wage effects since this threshold rules out workers that are directly affected by the policy. Thus, the relationship observed in the right graph is a measure of how comparable the levels of formality are across different states, at least for the workers with wages higher than R$600,00. The results suggest that, although far from being a perfect control group, the bias associated with latent differences in formality levels across states should be fairly small. This is certainly true for the upper part of the wage distribution. If one believes that this is also true for the bottom part of the wage distribution, then looking at the levels of formality for low-wage groups could be informative about the effects of the minimum wage on the size of the formal sector.

The center graph in Figure 8 displays the relationship between the minimum wage and the level of formality for the bottom part of the wage distribution. In contrast with the graph on the right, there seems to be a strong relationship between how close the minimum wage is to the median wage and how small the size of the formal sector for low-wage workers is. This also shows that the differences between the levels of formality across states are driven mainly by differences in the levels of formality that prevail *at the bottom* of their wage distributions. The minimum wage stands out as the most immediate explanation for this empirical regularity.

This evidence is in line with the main results of our empirical exercise: The minimum wage seems to cause a decrease in the size of the formal sector. Once one compares states by how strongly they were affected by the minimum wage policy, these states tend to have a substantially smaller size of the formal sector for the bottom part of the wage distribution, even though they seem to have somewhat comparable levels of the size of the formal sector in the upper part of their wage distributions.

Figure 8: Formality and wages across states by year



Source: Author's elaboration using the "PNAD" dataset.

5 Conclusion

This paper uses a bargaining framework in which workers and firms engage in a bargain over the wage and the sector in which production will take place to rationalize the joint distribution of employment, sector, and wages. This framework is shown to be useful to understand the effects of policies that increase the costs of operating in the informal sector (such as increased enforcement) and policies that impose constraints on firms operating in the formal sector, such as the minimum wage. We use this framework to propose a novel taxonomy of formal employment contracts that complements the taxonomy used in this literature to discuss informality (Ulyssea, 2018).

In our empirical exercise, we evaluate the effects of the minimum wage in the Brazilian economy using a dual economy statistical model under a parametric assumption regarding the shape of the latent wage distribution. In contrast with usual applications of Bunching methods, our choice of functional forms for our joint distribution is guided by theory. The model seems to approximate most of the stylized facts concerning the joint distribution of sectors and wages using a small number of parameters and displays a close fit to un-targeted moments such as skewness and kurtosis.

We find that the minimum wage generates statistically significant non-employment effects. The policy also leads a fraction – approximately 6% – of the affected workers to move from the formal to the informal sector of the economy. This sector-mobility channel, combined with the non-employment effects, leads to a decrease of approximately 8% in the size of the formal sector when compared to the counterfactual scenario of the absence of the minimum wage. Small sector-mobility probabilities induce large changes in the relative size of the informal sector because the latent size of the formal sector is approximately 3.5 times larger than the informal sector. Our estimates suggest that the informal sector experiences a growth of around 27%, from 22% to 28% of the economy, as a result of the minimum wage policy.

Reduced-form evidence of such a sector mobility channel is in line with the model's prediction. That is, the differences in the size of the formal sector across states for low- and high-wage groups replicate the patterns predicted by the model. These results highlight the importance of accounting for the relationship between the level of formality and labor market policies, such as the minimum wage, when analyzing the effects of these policies in developing countries.

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SUPPLEMENTAL APPENDIX – MINIMUM WAGE AND INFORMALITY IN A ROY BARGAINING ECONOMY: EVIDENCE FROM A BUNCHING ESTIMATOR

Hugo Jales [1] and Zhengfei Yu [2]

CONTENTS

[1]Syracuse University, department of economics. Email: hbjales@syr.edu.
[2]University of Tsukuba, Faculty of Humanities and Social Sciences.

1. APPENDIX A: MODEL ANALYSIS

In this section, we first prove that (s^*, w^*) characterized by (2) solves the Nash bargaining problem (1) in the paper.

Recall that the worker's utility is given by $U(l, s, w,) = l \times (w - \epsilon + \eta_s)$ and the firm's profit is given by $\Pi(l, s, w) = l \times (\alpha - w - \tau_s)$. The equilibrium employment, sector, and wage is obtained by maximizing the Nash Product:

$$(l^*, s^*, s^*) = \arg\max U(l, s, w)^{\nu} \Pi(l, s, w)^{1-\nu}, \tag{A.1}$$

where ν is the worker's bargaining power and $1 - \nu$ is the firm's bargaining power. A simple way to find the maximum of the Nash product in this case is to first set s to one, and then maximize over w. Denote the Nash Product when evaluated at $l = 1$, $s = 1$ – so the worker is employed and the constract is formal – at the wage w_1 that maximizes the Nash product given $l = 1$ and $s = 1$ by $B(1, 1, w_1)$. Denote the Nash Product when $l = 1$ and $s = 0$ – so the worker is employed and the contract is informal – at the wage w_0 that maximizes the Nash Product given $l = 1$ and $s = 0$ by $B(1, 0, w_0)$, Lastly, compare the maximum obtained when s is set to one, the maximum obtained when $s = 0$ – that is, compare the values of $B(1, 1, w_1)$ and $B(1, 0, w_0)$. Given l and s, the w_s that maximizes the Nash Product can be found by taking the first order condition of $B(l, s, w)$ with respect to w. This yields:

$$w_s^* = \nu(\alpha - \tau_s) + (1 - \nu)(\epsilon - \eta_s). \tag{A.2}$$

when $\nu = 1/2$ this collapses to the wage equations we use in the simplified version of the model presented on the paper, so that $w_1 = \frac{1}{2}(\alpha + \epsilon - \eta_1 - \tau_1)$ and $w_0 = \frac{1}{2}(\alpha + \epsilon - \eta_0 - \tau_0)$. Evaluating $\Pi(l = 1, s, w)$ and $U(l = 1, s, w)$ at the optimal value of $w = w_s$, we obtain $U(1, s, w_s) = \nu r_s$ and $\Pi(1, s, w_s) = (1 - \nu) r_s$, where $r_s = \alpha - \epsilon + \eta_s - \tau_s$ is the surplus of the match when the worker is assigned to sector s. Using these expressions, we can obtain the value of the Nash Products when the worker is assigned to each sector: When $s = 1$, we have that $B(1, 1, w_1) = v^{\nu}(1 - \nu)^{1-\nu} r_1$, and when $s = 0$, we have that $B(1, 0, w_0) = v^{\nu}(1 - \nu)^{1-\nu} r_0$. This implies that $B(1, 1, w_1)$ is greater than $B(1, 0, w_0)$ only if $r_1 > r_0$. This yields the optimal sector assignment for all workers that employed workers: $s = \mathbb{1}\{r_1 > r_0\}$. Plugging in the definitions of the formal and informal sectors' match surpluses and removing the common terms, we obtain:

(A.3)

$$s^* = \mathbb{1}\{r_1 > r_0\} = \mathbb{1}\{\alpha - \epsilon + \eta_1 - \tau_1 > \alpha - \epsilon + \eta_0 - \tau_0\} = \mathbb{1}\{\eta_1 - \eta_0 > \tau_1 - \tau_0\},$$

which proves the solution (2) in the paper.

1.1. *Relationship with Rosen and Roy models*

The model combines two distinct explanations for differences in wages across sectors: Compensating differentials and (Roy-model type of) self-selection. When $\eta_0 = \eta_1$, the worker is always employed in the sector where the wage draw is larger. That is, $s^* = \mathbb{1}\{w_1 > w_0\}$, as in the standard Roy-model. When $\tau_1 = \tau_0$, workers sort themselves based on their idiosyncratic valuation of amenities, that is, $s^* = \mathbb{1}\{\eta_1 > \eta_0\}$. Note that a mechanism similar to compensating differentials is present since $\frac{\partial w_1}{\partial \eta_1} < 0$, so the worker's equilibrium wage is decreasing in η_1, the valuation of the amenities associated with working in that sector.

This section further explores the relationship between the model and the canonical models of Roy and Rosen. It is interesting to see how the equilibrium wage and sectors respond to changes in the worker's bargaining power. In the baseline setup, for simplicity, we set the worker's and firm's bargaining power to 0.5. It is interesting, however, to consider the results when the bargaining power approaches the limits of one or zero.

Proposition A1 *When the bargaining power of the worker is one, we have that:*

(a) $w_s = \alpha - \tau_s$*. This, in turn, implies that* $w_1 - w_0 = \tau_0 - \tau_1$*. Furthermore, we have that* $\frac{\partial w_s}{\partial \alpha} = 1$*,* $\frac{\partial w_s}{\partial \tau_s} = -1$*, and* $\frac{\partial w_s}{\partial \eta_s} = 0$*. Furthermore, we have that* $w_1 - w_0 = \Pi(1, 1, w) - \Pi(1, 0, w)$*, so differences in assigned wages across sectors are productivity differentials.*

(b) If, in addition, $\eta_1 = \eta_0$*, then the joint distribution of* (w_1, w_0, s^*) *is indistinguishable from the standard Roy model, in which* $s^* = \mathbb{1}\{w_1 > w_0\}$*.*

(c) Instead, if $\tau_0 - \tau_1$ *is constant across workers and* $Cov(\alpha, s^*) = 0$*, we have that* $\mathbb{E}[w_1|s^* = 1] - \mathbb{E}[w_0|s^* = 0] = \mathbb{E}[w_1 - w_0] = \tau_0 - \tau_1 = \Pi(1, 1, w) - \Pi(1, 0, w)$*. That is, differences in expected wages identify productivity differentials.*

Proposition A1 shows that in the particular case in which the workers have all the bargaining power, then wages cease to reflect Rosen's (1986) notion of compensating differentials. In this case, a worker's wage will have no relationship with the amenities associated with the job because he can extract all of the surplus associated with better amenities into higher utility levels. As a result, wages only reflect differences in the technology of production across sectors. These factors remain relevant in determining the worker's wage because they affect the participation constraint of the firm, whereas the amenities, which only show up on the worker's utility, do not.

It is well-known that in bargaining models, whenever one party has all the bargaining power, this party is able to extract all of the surplus of the match. Thus, part (a) of Proposition A1 simply re-states this result in the context of our model. What is really interesting about it is that once we recognize that the workers will have all the bargaining power, we are able to arrive at a clear *economic interpretation* for the causal effect of sector on wages (sector productivity differentials). More importantly, the second part of Proposition A1 shows that if workers do not perceive differences in amenities across sectors, then the bargaining model features a joint distribution of sector and wages that is indistinguishable from the standard Roy model, in which workers select the sector that provides them the highest wage ($s^* = \mathbb{1}\{w_1 > w_0\}$).

This result, to the best of our knowledge, is not present in usual bargaining models since bargaining models usually have only the wage, not the pair of sector and wage, as variables used in the bargaining process.

PROPOSITION A2 *When the bargaining power of the worker is zero, we have that:*
(a) $w_s = \epsilon - \eta_s$*. This, in turn, implies that* $w_1 - w_0 = \eta_0 - \eta_1$*,* $\frac{\partial w_s}{\partial \alpha} = 0$*,* $\frac{\partial w_s}{\partial \eta_s} = -1$*,* $\frac{\partial w_s}{\partial \tau_s} = 0$*. Furthermore,* $w_1 - w_0 = U(1,1,w) - U(1,0,w)$*, so difference in the assigned wages across sectors are compensating differentials.*
(b) If, in addition, $\tau_1 = \tau_0$*, then the joint distribution of* (w_1, w_0, s^*) *is indistinguishable from a modified Roy featuring only "seemingly irrational selection", in which* $s^* = \mathbb{I}\{w_1 < w_0\}$*.*
(c) Instead, if $\eta_1 - \eta_0$ *is constant across workers and* $Cov(\epsilon, s^*) = 0$*, we have that* $\mathbb{E}[w_1|s^* = 1] - \mathbb{E}[w_0|s^* = 0] = \eta_1 - \eta_0 = U(1,1,w) - U(1,0,w)$*. That is, differences in expected wages identify compensating differentials.*

Proposition A2 shows that in the particular case in which the firms have all the bargaining power, then wages cease to reflect differences in productivity across sectors. In this case, a worker's wage will have no relationship with the worker's productivity α or the sector-specific productivity shock τ_s. Just as in the previous case, the intuition for this result is simple: When the worker has no bargaining power, then the firm manages to capture any surplus that is associated with higher productivity (in the form of general productivity α or sector-specific productivity τ_s). This is the case because when the firm has all the bargaining power, then wages are only affected by the variables that show up in the worker's participation constraint, namely the disutility of work ϵ and the sector-specific amenities η_s. In this setting, wages only reflect the opportunity cost of employment ϵ, and the differences in wages across sectors reflect only Rosen's notion of compensating differentials.

We emphasize that it is well-known that the party that possesses all the bargaining power will be able to extract all of the rents of the transaction. Thus, the first part of Proposition A2 simply restates this known result in the context of our model. Using this result, we are able to again attach an economic interpretation for the coefficient of formality in a wage regression. The interpretation, when workers have no bargaining power, is the magnitude of a Rosen type of *compensating wage differential*.

Equipped with the results of part (a) of Proposition A2, we are able to consider one interesting special case. Part (b) of Proposition A2 shows the curious result that if firms do not perceive productivity differences assigning the worker to different sectors ($\tau_1 = \tau_0$), then the bargaining model features a joint distribution of sector and wages that looks quite peculiar. In this setting, workers are always employed in the sector in which they earn the *least*. There is, however, an intuitive explanation for this result: When the firm has all the bargaining power, and there are no differences in the production technology across sectors, then the sector choice that maximizes the surplus is the one in which workers are assigned to the sector that they like the most (the one with the highest value for the amenities). Since workers value the amenities, the corresponding wage that respects the worker's participation constraint is smaller. Thus, workers go to the sector that they like the most, and, because of that, they end up in the sectors where they earn the least ($s^* = \mathbb{1}\{w_1 < w_0\}$).

We recognize that in standard bargaining models in which workers and firms bargain *only* with respect to the wage, it is well known that firms will extract all of the surplus of the match when they have all the bargaining power. However, to the best of our knowledge, these results we have here for the case of the joint bargain over both sector and wages and joint distributions we obtain in these special cases are novel in the literature. Part (b) of proposition A2 is an example of that. We end up with a model in which each and every worker is assigned to the sector that pays him the least, which is precisely the opposite of the expectations of anyone that looks at the problem of sector choice through the lenses of the Roy model.[1]

[1]When the bargaining power of the worker is υ and $Cov(\eta_1 - \eta_0, \tau_1 - \tau_0) = 0$, then $Var[w_1 - w_0] = \upsilon^2 Var(\eta_1 - \eta_0) + (1-\upsilon)^2 Var[\tau_1 - \tau_0]$. If $Var[\tau_1 - \tau_0] \approx Var[\eta_1 - \eta_0]$, then the fraction of the variance

In the following, we prove Propositions A1 and A2. Recall that both propositions have three parts. We focus on the proof of proposition 1 since the proof of proposition 2 is analogous. We start with some definitions. The compensating wage differential is the difference in the worker's utility across two distinct sector allocations when evaluated at the same wage. It measures the worker's willingness to pay for the job the (relative) amenities of the job. In our setting, the compensating wage differentials is then defined as $U(1,1,w) - U(1,0,w)$. Evaluating this expression, we find that the compensating wage differential is given by the difference in job amenities $\eta_1 - \eta_0$. Similarly, the sector productivity differential is the difference between the firm's profit when hiring the worker in the formal sector when compared to hiring the worker in an informal contract, when both of these options are evaluated at the same wage. It measures the differences in the worker's productivity (net of taxes, sector-specific costs, and distortions) when he is assigned to different types of employment. In our setting, the sector productivity differential is defined as $\Pi(1,1,w) - \Pi(1,0,w)$. Evaluating this expression in the context of our model, we find that the sector productivity differential is given by $\tau_1 - \tau_0$. Lastly, we say that a joint distribution of sector and wages is consistent with the (standard) Roy model if the joint distribution of (w_1, w_0, s) is such that $s = 1$ if, and only if, $w_1 > w_0$. That is, sector choice is based on income maximization and income maximization alone.[2]

Proof of Proposition 1: Recall from the discussion of the previous section that, for any bargaining power ν, the wage equation is given by $w_s = \nu(\alpha - \tau_s) + (1-\nu)(\epsilon - \eta_s)$. Taking $\nu = 1$ yields the particular case in which the wage equation reduces to $w_s = \alpha - \tau_s$. This establishes the first claim of part (a) of proposition A1. Taking partial

of the causal effect of sector on wage $(w_1 - w_0)$ explained by Roy's mechanism is entirely dictated by the bargaining power of worker and firms, and equal to $\frac{\upsilon^2}{\upsilon^2+(1-\upsilon)^2}$. For a paper that focuses on this type of decomposition in a different context, see Taber and Vejlin (2020).

[2] The Roy model is typically further parametrized, so the joint distribution of (w_1, w_0) is assumed to be bivariate normal, but this is not as important as the relationship between sector choice and income maximization.

derivatives of w_s with respect to τ_s, α, and η_s, yields the second result: $\frac{\partial w_s}{\partial \alpha} = 1$, $\frac{\partial w_s}{\partial \tau_s} = -1$, and $\frac{\partial w_s}{\partial \eta_s} = 0$. This yields the second claim. Finally, taking the difference between w_1 and w_0 yields the conclusion that $\Pi(1,1,w) - \Pi(1,0,w)$ is equal to $w_1 - w_0$, so the differences in wages across different sector assignments are the differences in the worker's productivity across sectors. This completes the proof of the last result in part (a) of the proposition.

To prove part (b), recall that, for any bargaining power, we have that $s = \mathbb{I}\{\eta_1 - \eta_0 > \tau_1 - \tau_0\}$. Then, if $\eta_1 = \eta_0$, then the sector choice equation collapses to $s = \mathbb{I}\{\tau_1 < \tau_0\}$, which in this case, using the simplified wage equation $w_s = \alpha - \tau_s$, leads us to conclude that $s = \mathbb{I}\{w_1 > w_0\}$. This finally yields the conclusion that when $\eta_1 = \eta_0$ and $\nu = 1$, the joint distribution of (w_1, w_0, s) is indistinguishable from the standard Roy model.

To prove part (c), we assume instead that $\tau_1 - \tau_0$ is constant across workers and that $Cov(\alpha, s^*) = 0$. Then, $\mathbb{E}[w_1|s = 1] - \mathbb{E}[w_0|s = 0] = \mathbb{E}[\alpha - \tau_1|s = 1] - \mathbb{E}[\alpha - \tau_0|s = 0] = \tau_1 - \tau_0 = \Pi(1, 1, w) - \Pi(1.0, w)$, where the first equality follows from the wage equation, the second follows from the assumption on the covariance between α and s, and the third follows from the assumption on the heterogeneity in τ across workers. In this case, differences in wages across sectors identify sector productivity differentials.

Taking $\nu = 0$ yields the particular case in which the wage equation reduces to $w_s = \epsilon - \eta_s$. Taking the difference between w_1 and w_0 yields the conclusion that $U(1,1,w) - U(1,0,w)$ is equal to $w_1 - w_0$, so the differences in wages across different sector assignments are compensating wage differentials. Now, if $\tau_1 = \tau_0$, then the sector choice equation collapses to $s = \mathbb{I}\{\eta_1 > \eta_0\}$, which in this case is identical to $s = \mathbb{I}\{w_1 < w_0\}$, which yields the conclusion that when $\eta_1 = \eta_0$ and $\nu = 1$, the joint distribution of (w_1, w_0, s) is indistinguishable from a Roy model featuring seemingly irrational selection, that is, $s = \mathbb{I}\{w_1 < w_0\}$. Lastly, instead, assume that $\eta_1 - \eta_0$ is constant across workers and $Cov(\epsilon, s^*) = 0$. Then, $\mathbb{E}[w_1|s = 1] - \mathbb{E}[w_0|s = 0] = \mathbb{E}[\epsilon - \eta_1|s = 1] - \mathbb{E}[\epsilon - \eta_0|s = 0] = \tau_1 - \tau_0 = U(1, 1, w) - U(1, 0, w)$, where the first equality follows from the wage equation, the second follows from the assumption on the covariance between ϵ and s^* and the third follows from the assumption on the heterogeneity in η across workers. In this case, differences in wages across sectors identify compensating wage differentials.

2. APPENDIX B: WELFARE

2.1. *Welfare Formulas*

From the structure imposed by the model presented in Section 2, it is possible to characterize the welfare effects of the minimum wage. Here we derive the expressions for both worker's welfare and also aggregate welfare,[3] defined as the sum of workers' utility and firm's profits. Recall that $w_s = \frac{1}{2}(\alpha + \epsilon - \eta_s - \tau_s)$ let and $\Delta = \frac{1}{2}(\eta_1 - \eta_0 - \tau_1 + \tau_0)$. Workers are employed formally whenever $\Delta > 0$. Define the aggregate welfare:

$$r^*_{s^*} \equiv \Pi(l^*, s^*, w^*) + U(l^*, s^*, w^*) \tag{A.4}$$

In the presence of the minimum wage, aggregate welfare will be given by the sum of profits and utility that prevail at the equilibrium allocation under the effects of the policy:

$$\tilde{r}_{\tilde{s}} \equiv \Pi(\tilde{l}, \tilde{s}, \tilde{w}) + U(\tilde{l}, \tilde{s}, \tilde{w}) \tag{A.5}$$

The difference between (A.5) and (A.4) corresponds to the welfare effect of the minimum wage. The expected value of that difference writes:

$$\begin{aligned} \mathbb{E}[\tilde{r}_{\tilde{s}} - r^*_{s^*}] &= -\mathbb{E}[r^*_{s^*} | w^* < m, \tilde{l} = 0] \Pr[w^* < m, l^* = 0] \\ &\quad - \mathbb{E}\left[(\eta_1 - \tau_1) - (\eta_0 - \tau_0) | w^* < m, s^* > \tilde{s}\right] \Pr[w^* < m, s^* > \tilde{s}]. \end{aligned}$$

Regarding the expression above, note that the welfare changes only for workers for which the minimum wage "bites' –that is – workers for which $w^* < m$. Also, when the

[3]Note that we do not include the tax revenues in the aggregate welfare definition.

worker-firm pair respond to the policy by setting the wage equal to m while keeping the worker in the same sector, then the welfare effect of this response is zero. This is so because, in this event, the minimum wage acts simply as a transfer of surplus from the firm to the worker. However, if the response is to destroy the match, then the welfare effect of the policy in this event is the negative of the surplus associated with that match. Lastly, when the worker moves to the informal sector, then the welfare effect of this change is given by minus $(\eta_1 - \tau_1) - (\eta_0 - \tau_0)$. This term is negative since the policy is inducing the worker to be assigned to a sub-optimal sector given his draw of η_s and τ_s. It is, thus, straightforward to see that the aggregate welfare effects of the minimum wage in this setting are negative.

Finally, it is interesting to note that the Bargain framework something that looks like a version of "efficient rationing" in the sense of Lee and Saez (2012).[4] It is possible to characterize the reactions of worker-firm pairs according to a set of inequalities associated with the difference between the worker's productivity and his outside option. If the worker moves to the informal sector, we have the following inequalities $\tau_1 - \eta_1 \leq \tau_0 - \eta_0 \leq \alpha - \epsilon$. This means that the surplus when formal is higher than the surplus when informal. However, the surplus when informal is still positive. If the worker-firm pair decides to end the match, we have the following inequalities $\tau_1 - \eta_1 \leq \alpha - \epsilon \leq \tau_0 - \eta_0$. This means that the difference between the worker's productivity and its outside option has to be bounded from above by the difference between the costs of informality and the worker's valuation of the amenities in the informal sector. Furthermore, if a worker loses the job, it must be the case that $\alpha \leq m + \tau_1$. These two inequalities limit the negative welfare effect of the minimum wage since the unemployment falls on matches with productivity α bounded from above by $m + \tau_1$

[4] We thank Adam Lavecchia for pointing out this connection to us.

and also by $\epsilon + \tau_0 - \eta_0$. We can obtain a similar expression for the welfare of workers:

$$\begin{aligned}\mathbb{E}[U(l^*, s^*, w^*) - U(\tilde{l}, \tilde{s}, \tilde{w})] &= -\mathbb{E}[U(l^*, s^*, w^*)|\tilde{l} = 0, w^* < m] \Pr[\tilde{l} = 0, w^* < m] \\ &\quad + \mathbb{E}[m - w^*|\tilde{w} = m] \Pr[\tilde{w} = m] \\ &\quad + \frac{1}{2}\mathbb{E}[(\eta_1 - \tau_1) - (\eta_0 - \tau_0)|\tilde{s} = 1, s^* = 0] \Pr[\tilde{s} = 1, s^* = 0].\end{aligned}$$

This expression states that the expected change on worker's welfare by the minimum wage policy is given by an weighted average of the loss of utility associated with the disemployment effect, the gain associated with the wage increases, and the losses associated with inducing workers to move to the informal sector, where each of these factors are weighted by the probability of these events. Differently than in the case of aggregate welfare, the effects of the minimum wage on worker's welfare are not guaranteed to be negative. The welfare of workers can increase if the term associated with the workers that bunch at the minimum wage is large enough.

2.2. *The effect of a "small" minimum wage*

Let $r_s = \frac{1}{2}(\alpha - \tau_s - \epsilon + \eta_s)$ for $s = 1, 2$. Note that r_1 and r_0 are the surplus for the worker and the firm in the formal and informal sector respectively. The Nash Product in the formal sector with $w = m$ is

$$\begin{aligned}B(1, 1, m) &= [(m - \epsilon + \eta_1)(\alpha - m - \tau_1)]^{1/2} \\ &= \left[\frac{1}{4}(\alpha - \tau_1 - \epsilon + \eta_1)^2 - \left(m - \frac{1}{2}(\alpha + \epsilon - \tau_1 - \eta_1)\right)^2\right]^{1/2} \\ &= \sqrt{r_1^2 - (m - w_1)^2}.\end{aligned}$$

On the other hand, the Nash Product for the informal sector is $B(1, 0, w_0) = r_0$. Recall that $\Delta = (\eta_1 - \eta_0) - (\tau_1 - \tau_0) = r_1 - r_0$.

Proof of Proposition 1: To simplify notations, we use "shift", "bunch," and "unemp" to denote the consequences of the minimum wage: workers shifting from formal to the informal sector, bunching at the minimum wage, and becoming unemployed. Note that these three events correspond to the conditioning events in the statement of Proposition 1.

(a). Workers shifting from formal to informal sector due to the minimum wage are characterized by $\Delta \geq 0$, $w_1 < m$, $r_1^2 - (m - w_1)^2 < r_0^2$, and $r_0 > 0$.

For arbitrary small value $c > 0$, if the minimum wage $m \leq c^2$, then

$$\begin{aligned}
\Pr\left[\Delta \leq c | shift\right] &= \frac{\Pr\left[0 \leq r_1 - r_0 \leq c, r_1^2 - (m - w_1)^2 < r_0^2, w_1 < m, r_0 > 0\right]}{\Pr\left[r_1 - r_0 \geq 0, r_1^2 - (m - w_1)^2 < r_0^2, w_1 < m, r_0 > 0\right]} \\
&= \frac{\Pr\left[r_1 - r_0 \geq 0, r_1^2 - (m - w_1)^2 < r_0^2, w_1 < m, r_0 > 0\right]}{\Pr\left[r_1 - r_0 \geq 0, r_1^2 - (m - w_1)^2 < r_0^2, w_1 < m, r_0 > 0\right]} \\
&= 1,
\end{aligned}$$

where the second equality comes from that $r_1^2 - r_0^2 < (m - w_1)^2 \leq c^2 \Rightarrow r_1 - r_0 \leq c$.

(b). Workers bunching at the minimum wage m are characterized by $\Delta \geq 0$, $w_1 < m$, $r_1^2 - (m - w_1)^2 \geq r_0^2 \geq 0$, and $r_0 \geq 0$. Then for all m, including small m, we have,

$$\begin{aligned}
\lim_{c \downarrow 0} \Pr\left[\Delta \leq c | bunch\right] &= \lim_{\epsilon \downarrow 0} \frac{\Pr\left[0 \leq r_1 - r_0 \leq c, r_1^2 - (m - w_1)^2 \geq r_0^2 > 0, w_1 < m\right]}{\Pr\left[r_1 - r_0 \geq 0, r_1^2 - (m - w_1)^2 \geq r_0^2 > 0, w_1 < m\right]} \\
&= 0.
\end{aligned}$$

(c). Workers become unemployed if and only if $r_1 > 0$, $w_1 < m$, $r_1 < m - w_1$, and $r_0 \leq 0$. Then for all m, including small m, we have

$$\begin{aligned}
\lim_{c \downarrow 0} \Pr\left[\Delta \leq c | unemp\right] &= \lim_{c \downarrow 0} \frac{\Pr\left[r_1 - r_0 \leq c, r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]}{\Pr\left[r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]} \\
&= \lim_{c \downarrow 0} \frac{\Pr\left[0 < r_1 - r_0 \leq c, r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]}{\Pr\left[r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]} \\
&= 0.
\end{aligned}$$

(d). For any small value of c, if $m \leq c$, then $r_1 < m - w_1$ implies $r_1 \leq c$. Therefore, for $m \leq c$, we have

$$\begin{aligned}
\Pr\left[r_1 \leq c | unemp\right] &= \frac{\Pr\left[r_1 \leq c, r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]}{\Pr\left[r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]} \\
&= \frac{\Pr\left[r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]}{\Pr\left[r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]} \\
&= 1.
\end{aligned}$$

3. APPENDIX C: DISTRIBUTIONAL APPROXIMATIONS

In this section, we derive the results concerning the functional form for the joint distribution of sector and wages that prevails in this economy both in the presence and also in the absence of the minimum wage. These results allow us to arrive at a tractable structure that can be used to estimate the model parameters. Here we prove the results discussed in Section 3.1 of the paper.

3.1. *Latent wage distributions*

For $s \in \{0,1\}$, consider $w_s = \frac{1}{2}(\alpha + \epsilon - \eta_s - \tau_s)$. Decompose $\epsilon - \eta_s - \tau_s$ into two parts: a part predicted by α and the remaining part.

$$\epsilon - \eta_s - \tau_s = \beta_{0s} + \beta_{1s}\alpha + u_s,$$

where β_{0s} and β_{1s} are constants and $\mathbb{E}[\alpha u_s] = \mathbb{E}[u_s] = 0$. Hence

$$w_s = \frac{1}{2}(\alpha + \beta_{0s} + \beta_{1s}\alpha + u_s) = \frac{1}{2}\alpha(1+\beta_{1s})\left(1 + \frac{u_s}{\alpha(1+\beta_{1s})}\right) + \frac{1}{2}\beta_{0s}.$$

We restate Assumptions 1 to 3 in more specific forms as follows.

Assumption A1 $\log \alpha$ *has a normal distribution* $N(\mu, \sigma^2)$.

Assumption A2 *There are small positive numbers* t *and* κ *such that* $\Pr\left[|\frac{u_s}{\alpha(1+\beta_{1s})}| > t\right] \le \kappa$.

Assumption A3 *In the absence of the minimum wage, the support of* w_s *is* $\mathbb{R}^+$ *for* $s \in \{1,0\}$.

Proof of Proposition 3: Proposition 3 immediately follows from Proposition A3 below. Note that $S(0) = \mathbb{1}\{\Delta > 0\}$.

Proposition A3 *(i). Under Assumptions A1 to A3,* w_s *approximately has a log-normal distribution. That is,* $\log w_s$ *has a normal distribution with mean* $\mu - \log((1+\beta_{1s})/2)$ *and standard error* σ.
(ii). Further assume that $(\log w_s, \Delta)'$ *approximately have a joint normal distribution with mean* $(\mu_s, \mu_\delta)'$ *and covariance* $\begin{bmatrix} \sigma^2 & \rho_s\sigma\sigma_\delta \\ \rho_s\sigma\sigma_\Delta & \sigma_\Delta^2 \end{bmatrix}$. *Then the conditional log wage*

densities $f_{\log w_1}(x|\Delta \geq 0)$ and $f_{\log w_0}(x|\Delta < 0)$ approximately have the following forms:

$$f_{\log w_1}(x|\Delta \geq 0) \approx \frac{1}{\sigma}\phi\left(\frac{x-\mu_1}{\sigma}\right)\Phi\left(\frac{\frac{\mu_\Delta}{\sigma_\Delta}+\rho_1\frac{x-\mu_1}{\sigma}}{\sqrt{1-\rho_1^2}}\right)/\Phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right), \tag{A.6}$$

$$f_{\log w_0}(x|\Delta < 0) \approx \frac{1}{\sigma}\phi\left(\frac{x-\mu_0}{\sigma}\right)\Phi\left(\frac{-\frac{\mu_\Delta}{\sigma_\Delta}-\rho_0\frac{x-\mu_0}{\sigma}}{\sqrt{1-\rho_0^2}}\right)/\Phi\left(-\frac{\mu_\Delta}{\sigma_\Delta}\right). \tag{A.7}$$

(iii). The density of the latent wage w is approximately

$$f_{\log w}(x) \approx \frac{1}{\sigma}\left[\phi\left(\frac{x-\mu_1}{\sigma}\right)\Phi\left(\frac{\frac{\mu_\Delta}{\sigma_\Delta}+\rho_1\frac{x-\mu_1}{\sigma}}{\sqrt{1-\rho_1^2}}\right)+\phi\left(\frac{x-\mu_0}{\sigma}\right)\Phi\left(\frac{-\frac{\mu_\Delta}{\sigma_\Delta}-\rho_0\frac{x-\mu_0}{\sigma}}{\sqrt{1-\rho_0^2}}\right)\right].$$

PROOF: (i).

$$\Pr(w_s \leq x) = \Pr\left(\log\alpha + \log((1+\beta_{1s})/2) + \log\left(1+\frac{u_s}{\alpha(1+\beta_{1s})}\right) \leq \log(x - \beta_{0s}/2)\right)$$

By Assumption A2,

$$\Pr\left(\log\left(1+\frac{u_s}{\alpha(1+\beta_{1s})}\right) > \log(1+t)\right) \leq \kappa.$$

Since t is a small number, $\log(1+t) \approx t$. Then there is a small number $t' > t$ such that

$$\Pr\left(\log\left(1+\frac{u_s}{\alpha(1+\beta_{1s})}\right) > t'\right) \leq \kappa, \text{ and } \Pr\left(\log\left(1+\frac{u_s}{\alpha(1+\beta_{1s})}\right) < -t'\right) \leq \kappa.$$

Applying Lemma A1 (to be stated below) with $X = \log\alpha + \log((1+\beta_{1s})/2)$ and $Y = \log\left(1+\frac{u_s}{\alpha(1+\beta_{1s})}\right)$, we have

$$\Pr(w_s \leq x) \approx \Pr(\log\alpha + \log((1+\beta_{1s})/2) \leq \log(x-\beta_{0s}/2)). \tag{A.8}$$

By Assumption A1, $\log\alpha + \log((1+\beta_{1s})/2)$ has a normal distribution with mean $\mu - \log((1+\beta_{1s})/2)$ and standard error σ.

The support of w_s is $(\beta_{0s}/2, +\infty)$. By Assumption A3, $\beta_{0s} = 0$. Therefore, $\log w_s$ has a normal distribution with mean $\mu - \log((1+\beta_{1s})/2)$ and standard error σ.

(ii). We focus on $f_{\log w_1}(x|\Delta>0)$. $f_{\log w_0}(x|\Delta<0)$ can be shown in the same way.

$$\begin{aligned}
f_{\log w_1}(x|\Delta>0) &= \int_0^\infty f_{\log w_1,\Delta}(x,\delta)d\delta/\Pr(\Delta>0) \\
&= f_{\log w_1}(x)\int_0^\infty f_{\Delta|\log w_1}(\delta|x)d\delta/\Pr(\Delta>0) \\
&= f_{\log w_1}(x)\Pr(\Delta>0|\log w_1=x)/\Pr(\Delta>0) \qquad \text{(A.9)}
\end{aligned}$$

By the joint normality assumption in (ii), $\Delta|\log w_1 = x$ approximately has the normal distribution with mean $\mu_\Delta+\rho_1\sigma_\Delta/\sigma$ and variance $(1-\rho_1^2)\sigma_\Delta^2$. Therefore,

$$\Pr(\Delta>0|\log w_1=x)=\Phi\left(\frac{\frac{\mu_\Delta}{\sigma_\Delta}+\rho_1\frac{x-\mu_1}{\sigma}}{\sqrt{1-\rho_1^2}}\right).$$

Meanwhile, by the log-normality of w_1 and the normality of Δ, we have

$$f_{\log w_1}(x)=\frac{1}{\sigma}\phi\left(\frac{x-\mu_1}{\sigma}\right),\quad \Pr(\Delta>0)=\Phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right)$$

The desired result immediately follows. Note that the right hand side of (A.6) is a little bit more general than the density function of skew-normal. It reduces to the skew-normal if $\mu_\Delta=0$.

If Δ is independent of α, then $f_{\log w_1}(x|\Delta>0)=f_{\log w_1}(x)$, which has a normal distribution by the result of part (i).

(iii). Observe that

$$\begin{aligned}
f_{\log w}(x) &= \Pr(\Delta\geq 0)\, f_{\log w}(x|\Delta\geq 0)+\Pr(\Delta<0)\, f_{\log w}(x|\Delta<0) \\
&= \Pr(\Delta\geq 0)\, f_{\log w_1}(x|\Delta\geq 0)+\Pr(\Delta<0)\, f_{\log w_0}(x|\Delta<0),
\end{aligned}$$

where $f_{\log w_1}(x|\Delta>0)$ and $f_{\log w_0}(x|\Delta<0)$ are given in part (ii).

If $w_1=w_0$, then $f_{\log w}(x)\approx\frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)$, that is, w approximately has a log-normal distribution.

Q.E.D.

The following lemma is used in the proof of Proposition A3.

Lemma A1 *Consider random variables X and Y. Assume that (i). $X \sim N(\mu, \sigma)$; (ii). there are small positive numbers t and κ such that $\Pr(|Y| > t) \leq \kappa$. Then* $\Pr(X + Y \leq z) \approx \Pr(X \leq z) = \Phi\left(\frac{z-\mu}{\sigma}\right)$.

Proof: For any $z \in \mathbb{R}$, observe that

$$\begin{aligned} \Pr(X + Y \leq z) &\leq \Pr(X \leq z + t) + \Pr(Y \leq -t) \\ \Pr(X \leq z - t) &\leq \Pr(X + Y \leq z) + \Pr(-Y \leq -t), \end{aligned}$$

Combining the inequalities yields

$$\Pr(X \leq z - t) - \kappa \leq \Pr(X + Y \leq z) \leq \Pr(X \leq z + t) + \kappa.$$

Since X is normal and t, κ is small by assumption, $\Pr(X \leq z - t)$, $\Pr(X \leq z)$ and $\Pr(X \leq z + t)$ are close to one another. Hence, $\Pr(X + Y \leq z) \approx \Pr(X \leq z)$. *Q.E.D.*

3.2. *Log odds ratio of conditional probability*

In this section, we show precisely the extent to which the Logit approximation is valid for the conditional probability of latent formality, $S(0)$, given the latent wage, $W(0)$, which is the statement of Proposition 4 in the paper. In the setting of Proposition A3, the log odds ratio

$$\begin{aligned}
&\log\left(\frac{\Pr[S(0) = 1|W(0) = w]}{1 - \Pr[S(0) = 1|W(0) = w]}\right) \\
&= \log f_{w_1}(w|\Delta \geq 0) - \log f_{w_0}(w|\Delta < 0) + \log(p/(1-p)) \\
&\approx \log\phi\left(\frac{\log w - \mu_1}{\sigma}\right) - \log\phi\left(\frac{\log w - \mu_0}{\sigma}\right) + \log\Phi\left(\frac{\frac{\mu_\Delta}{\sigma_\Delta} + \rho\frac{\log w - \mu_1}{\sigma}}{\sqrt{1-\rho^2}}\right) \\
&- \log\Phi\left(\frac{-\frac{\mu_\Delta}{\sigma_\Delta} - \rho\frac{\log w - \mu_0}{\sigma}}{\sqrt{1-\rho^2}}\right) + \log(p/(1-p)) \\
&= \log\Phi\left(\frac{\frac{\mu_\Delta}{\sigma_\Delta} + \rho\frac{\log w - \mu_1}{\sigma}}{\sqrt{1-\rho^2}}\right) - \log\Phi\left(\frac{-\frac{\mu_\Delta}{\sigma_\Delta} - \rho\frac{\log w - \mu_0}{\sigma}}{\sqrt{1-\rho^2}}\right) + \frac{\mu_1 - \mu_0}{\sigma^2}\log w + \frac{-\mu_1^2 + \mu_0^2}{2\sigma^2} \\
&+ \log(p/(1-p)).
\end{aligned}$$

One can approximate $\Phi(x)$ by Polya's formula $\Phi(x) \approx 0.5\left(1+\sqrt{1-\exp(-2x^2/\pi)}\right)$, which has a maximum error of 0.003 when $x = 1.6$. Overall the log adds ratio can be approximated by a function of $\log w$ with unknown parameters.

3.3. *In the presence of minimum wage*

This section characterizes the distribution of wages in the presence of minimum wage. The results here will be used in Appendix F. In the setting of Proposition A3, $w_0 \approx w_1\xi$ and $r_1 \approx Kw_1 + c$ for some positive constants ξ and K. For a worker previously in the formal sector, i.e., $S(0) = 1$, his/her wage in the presence of the minimum wage can be characterized by

$$W(1) = \begin{cases} w_1, & \text{if } w \geq m, \\ m, & \text{if } \frac{m-c}{K+1} \leq w < m \text{ and } \Delta \geq b(w), \\ w_0, & \text{if } \frac{m-c}{K+1} \leq w < m \text{ and } 0 \leq \Delta < b(w), \text{or } w < \frac{m-c}{K+1} \text{ and } 0 \leq \Delta \leq Kw + c, \\ \cdot & \text{if } w < \frac{m-c}{K+1} \text{ and } \Delta > Kw + c. \text{ (unemployment)} \end{cases}, \tag{A.10}$$

where $b(w) = Kw + c - \sqrt{(Kw+c)^2 - (m-w)^2}$. Let $c^{(1)} \equiv \Pr[S(1) = 1]$, one has

$$c^{(1)} = \int_m^\infty \int_0^\infty f_{w_1,\Delta}(w,\delta) d\delta dw + \int_{\frac{m-c}{K+1}}^m \int_{b(w)}^\infty f_{w_1,\Delta}(w,\delta) d\delta dw.$$

For the workers in the formal sector,

$$\begin{aligned} f_{W(1)}(w|S(1)=1) &= \frac{\mathbb{I}\{w > m\}}{c^{(1)}\sigma w}\phi\left(\frac{\log w - \mu_1}{\sigma}\right)\Phi\left(\frac{\frac{\mu_\Delta}{\sigma_\Delta} + \rho_1\frac{\log w - \mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right) \\ &\quad + \frac{\mathbb{D}(w-m)}{c^{(1)}\sigma}\int \frac{1}{w}\phi\left(\frac{\log w - \mu_1}{\sigma}\right)\pi_m^{(1)}(w) dw, \end{aligned} \tag{A.11}$$

where

$$\pi_m^{(1)}(w) = \mathbb{I}\{\frac{m-c}{K+1} \leq w < m\}\left[1 - \Phi\left(\frac{\frac{b(w)-\mu_\Delta}{\sigma_\Delta} - \rho_1\frac{\log w - \mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right)\right].$$

Therefore, the expected wage in the formal sector is

$$\begin{aligned}
\mathbb{E}\left[W(1)|S(1)=1\right] &= \int_0^\infty w f_{W(1)}(w|S(1)=1)dw \\
&= \frac{1}{c^{(1)}}\int_m^\infty w\int_0^\infty f_{w_1,\Delta}(w,\delta)d\delta dw + \int_{\frac{m-c}{K+1}}^m \int_{b(w)}^\infty f_{w_1,\Delta}(w,\delta)d\delta dw.
\end{aligned}$$

Now consider the informal sector. Let $c^{(0)} \equiv \Pr\left[S(1)=1\right]$.

$$\begin{aligned}
c^{(0)} &= \frac{1}{\xi}\int_{\frac{(m-c)\xi}{K+1}}^{m\xi}\int_0^{b(w/\xi)} f_{w_1,\Delta}(w/\xi,\delta)d\delta dw + \frac{1}{\xi}\int_{\max\{0,-c/K\}}^{\frac{(m-c)\xi}{K+1}}\int_0^{Kw/\xi+c} f_{w_1,\Delta}(w/\xi,\delta)d\delta dw \\
&\quad + \frac{1}{\xi}\int_0^\infty\int_{-\infty}^{\min\{0,Kw+c\}} f_{w_1,\Delta}(w/\xi,\delta)d\delta dw.
\end{aligned}$$

The observed wage density in the informal sector can be written as

$$f_{W(1)}(w|S(1)=0) = \frac{f_{w_1}(w)}{c^{(0)}\xi}\pi_d^{(1)}(w) + \frac{f_{w_1}(w)}{c^{(0)}\xi}\Phi\left(\frac{\frac{\min\{0,Kw+c\}-\mu_\Delta}{\sigma_\Delta} - \rho_1\frac{\log w-\mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right),$$

where

$$\begin{aligned}
\pi_d^{(1)}(w) &= \mathbb{I}\{\frac{(m-c)\xi}{K+1} \le w < m\xi\}\left[\Phi\left(\frac{\frac{b(w)-\mu_\Delta}{\sigma_\Delta} - \rho_1\frac{\log w-\mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right) - \Phi\left(\frac{\frac{-\mu_\Delta}{\sigma_\Delta} - \rho_1\frac{\log w-\mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right)\right] \\
&\quad + \mathbb{I}\{\max\{0,\frac{-c\xi}{K}\} < w < \frac{(m-c)\xi}{K+1}\}\left[\Phi\left(\frac{\frac{Kw+c-\mu_\Delta}{\sigma_\Delta} - \rho_1\frac{\log w-\mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right) - \Phi\left(\frac{\frac{-\mu_\Delta}{\sigma_\Delta} - \rho_1\frac{\log w-\mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right)\right.
\end{aligned}$$

As a result, the expected wage in the informal sector writes

$$\begin{aligned}
\mathbb{E}\left[W(1)|S(1)=0\right] &= \frac{1}{c^{(0)}\xi}\int_{\frac{(m-c)\xi}{K+1}}^{m\xi} w\int_0^{b(w/\xi)} f_{w_1,\Delta}(w/\xi,\delta)d\delta dw \\
&\quad + \frac{1}{c^{(0)}\xi}\int_{\max\{0,-c/K\}}^{\frac{(m-c)\xi}{K+1}} w\int_0^{Kw/\xi+c} f_{w_1,\Delta}(w/\xi,\delta)d\delta dw \\
&\quad + \frac{1}{c^{(0)}\xi}\int_0^\infty w\int_{-\infty}^{\min\{0,Kw+c\}} f_{w_1,\Delta}(w/\xi,\delta)d\delta dw.
\end{aligned}$$

The density of the observed wage is

$$f_{W(1)}(w) = \frac{c^{(1)}f_{W(1)}(w|S(1)=1) + c^{(0)}f_{W(1)}(w|S(1)=0)}{c^{(1)}+c^{(0)}},$$

and the expected wage is

$$\mathbb{E}[W(1)] = \frac{c^{(1)}}{c^{(1)}+c^{(0)}}\mathbb{E}\left[W(1)|S(1)=1\right] + \frac{c^{(1)}}{c^{(1)}+c^{(0)}}\mathbb{E}\left[W(1)|S(1)=0\right].$$

4. APPENDIX D: ESTIMATION

4.1. *Likelihood function*

Collecting the results of the previous section, we obtain a tractable expression for the likelihood of the data under the minimum wage policy. Let $\Theta \equiv (\theta, \delta, \pi)$ be the entire vector of model parameters, that is, those governing the latent distribution of wages, the conditional probability of sector given wages, and minimum wage effects. Define the likelihood of observing a pair (w, s) given the minimum wage level m and model parameters Θ as $L(W(1) = w, S(1) = s|\Theta) = \Pr[S(1) = s|W(1) = w; \Theta]f(w|\Theta)$.

Three functional forms must be specified to obtain the exact expression for the likelihood function: A functional form for the unconditional distribution of latent wages, one for the conditional probability of latent sector given the latent wage, and, lastly, one functional form for the probability of non-compliance as a function of latent wage. We use lognormality for the latent wage distribution, so $f_0(w) = \frac{1}{w\sigma}\phi(\frac{\log(w)-\mu}{\sigma})$, where ϕ is the density of the standard normal distribution. For the conditional probability of latent sector given the latent wage, $\Pr[S(0) = 1|W(0) = w]$, we use a linear logit specification, so $\Pr[S(0) = 1|W(0) = w] \equiv \Lambda(w) = \frac{e^{\delta_0+\delta_1 w}}{1+e^{\delta_0+\delta_1 w}}$. Finally, for the probability of non-compliance as a function of the wage, we use a constant specification, so $\pi_d^s(w) = \pi_d^{(s)}$ for all w and for $s \in \{0, 1\}$. [5]

Let $\psi(\Theta) \equiv \frac{\pi_m^{(1)} \int^m f_0(w|\theta)\Lambda(w|\delta)dw}{\int^m \pi_m(w)f_0(w|\theta)dw}$. For the first term appearing in the log-likelihood, we have:

$$\begin{aligned}\log \Pr[S(1) = s|W = w; \Theta] \quad = \quad & \mathbb{1}\{w = m\}\left[\mathbb{1}\{s = 1\}\log\psi(\Theta) + \mathbb{1}\{s = 0\}\log(1 - \psi(\Theta)\right] \\ & + \mathbb{1}\{w > m\}\left[\mathbb{1}\{s = 1\}\log\Lambda(w|\delta) + \mathbb{1}\{s = 0\}\log(1 - \Lambda(w|\delta))\right]. \end{aligned} \tag{A.12}$$

[5] Our main results – namely that the minimum wage generates noticeable disemployment effects, that the size of the formal sector is reduced, and that the likelihood of a formal worker moves to the informal sector as an effect of the minimum wage policy is small, or around 10%– are unchanged when different specifications of these functional forms are used. We tried flexible polynomials for the non-compliance probability, higher-order terms for the logit specification, and more flexible functional forms for the marginal distribution of latent wages. In particular, if anything, we find that the probability of moving to the informal sector is even smaller (and the corresponding probability of unemployment and bunching are larger) when different specifications are used. So we believe that our main qualitative results are robust to the functional forms we used.

For the second term in the log-likelihood, we have:

$$\begin{aligned}\log f(w|\Theta) &= 1\!\!1\{w<m\}\log(\pi_d(w)f_0(w|\theta)) \\ &\quad +1\!\!1\{w=m\}\log\Big(\int^m \pi_m(u)f_0(u|\theta)du\Big) \\ &\quad +1\!\!1\{w>m\}\log f_0(w|\theta)-\log c(\Theta), \end{aligned} \tag{A.13}$$

where $\pi_d(w) = \Lambda(w|\delta)\pi_d^{(1)} + (1 - \Lambda(w|\delta))\pi_d^{(0)}$, and $\pi_m(w) = \Lambda(w|\delta)\pi_m^{(1)} + (1 - \Lambda(w|\delta))\pi_m^{(0)}$, and $c(\Theta) \equiv 1 - \int^m \pi_u^{(1)}\Lambda(w)f_0(w)dw$. The parameter $c(\Theta)$ can be interpreted as the ratio of employment before and after the introduction of the minimum wage.

Given that $\log L(W(1) = w, S(1) = s|\Theta) = \log \Pr[S(1) = s|W(1) = w;\Theta] + \log f(W(1) = w|\Theta)$, we can define the maximum likelihood estimator of the model parameters as $\widehat{\Theta} = \arg\max_\Theta \frac{1}{N}\sum_i^N \log L(w_i, s_i|\Theta)$.

5. APPENDIX E: THE ROBUSTNESS OF THE FORMAL SECTOR ESTIMATES

In this section, we show that estimates of the effects of the minimum wage on the size of the formal sector based on the two-step procedure is robust to misspecification of the spillover effects of the minimum wage. Misspecification of spillovers may bias our estimate of the latent wage distribution and the parameters of the latent conditional probability function $Pr[S(0) = 1|W(0) = w]$ defined in Proposition 4 of the main text. In the following, we first investigate the effect brought by the biases in the latent wage distribution and then take into account the effect brought by the biases in the parameters of the latent conditional probability function. In both cases, we found that the implied biases brought by misspecification of the spillover effects to be very small.

5.1. *Robustness to the biases in the latent wage distribution*

Recall from Proposition 4 in the main text that the latent conditional probability $Pr[S(0) = 1|W(0) = w] \equiv \Lambda(w;\delta_0)$. To focus on the main reason for the robustness of our estimates to biases in the latent wage distribution, consider a linear form of Λ

function: $\Lambda(w;\delta_0) = \delta_{00} + \delta_{01}w$. The linearity is assumed for convenience. We discuss later how to approach more general cases by including a quadratic term, for example. In this setup, we have

$$Pr[S(0) = 1] = \delta_{00} + \delta_{01}E[W(0)].$$

That is, assuming we know the intercept and slopes for the moment, the only thing we need to know to obtain the latent size of the formal sector is the latent mean of wages. Now, we can obtain estimates of δ_{00} and δ_{01} simply by regressing formality on wages for wages above the minimum wage. Location-shift spillovers might move the intercept, but they can not affect the key object of interest, which is the estimated slope – and the slope is the object that dictates how much our estimates should change when we consider other values for the latent mean of wages–, so it is fine to treat these parameters as objects we can estimate from the data.

Now, the linear specification for the Λ function yields the convenient result that – provided that we know δ_{00} and δ_{01}, for us to know the latent size of the formal sector, we simply need to know the right value for $E[W(0)]$. We have estimates of this object from our maximum-likelihood procedure, but they are highly sensitive to misspecification of spillovers.[6] The discussion earlier in this section suggests, however, that if δ_{01} is small, then finding the exact value of $E[W(0)]$ may not be that important. Table I shows that this is, indeed, the case.

To produce the results in Table I, we first regress observed formality on wages, using only data 40 Brazilian Reais above the minimum wage level. We use this to obtain estimated values for δ_{00} and δ_{01}. As opposed to the rest of the paper, we use wages in levels because it is easier to contrast them to baseline values, but the results are identical if we run everything in log-wages as well.

[6]In this section, we restrict the effect of the misspecification of spillovers to the estimate of the latent wage distribution. The possible effect on the estimation of parameters for function Λ will be discussed in the next section.

Now, in possession of the parameters of the conditional probability function, the question is: How sensitive are our estimated effects of the minimum wage on the size of the formal sector to the precise knowledge of the mean of the latent wage distribution $E[W(0)]$? The answer is not at all. In Table II, we report the implied latent size of the formal sector if the true mean of $W(0)$ is set at an arbitrary level. We use a range that is orders of magnitude larger than any spillovers ever estimated in the literature: We start by assuming that the $E[W(0)]$ was zero in the absence of the minimum wage – which is implicitly the same as saying that spillovers are incredibly large, so large that the *mean* of the wage distribution would be zero without a minimum wage policy. Then we progressively move the value we set for the mean of $W(0)$, up to one thousand Brazilian Reais, which about twice the average wages we observe in 2001, and about 3.3 times the minimum wage level in 2005. This is the same as saying that if $E[W(0)]$ is truly one thousand Brazilian Reais, then minimum wage has very large and quite negative spillover effects, so the average wage that we observed in the presence of the policy is about one half of what would be observed without the policy.

The point of using such a considerable range is to be sure that it nearly certain contains the mean of the Brazilian wage distribution if the minimum wage policy were not in place. We can only be wrong using such a range if the true mean, in the absence of the minimum wage, was either negative or if minimum wage generates sizable *negative* spillover effects, so average wages substantially decrease when the policy is in place. Since none of these scenarios is reasonable, we are quite confident that the mean of the latent wage distribution is within the range we have chosen.

Now, how much the estimated latent size of the formal sector changes when we consider the possibility that we got the wrong number for $E[W(0)]$ when we consider the possibility that we misspecified spillovers? The answer is not much. Taking the year 2001 as an example, the implied latent size of the formal sector that we would obtain if the mean of wages were indeed zero is 0.75, whereas if the true mean of $W(0)$ were instead one thousand Brazilian Reais, the value would be then 0.80. Our preferred

Table I: Latent Size of the Formal Sector as a function of Latent Mean of Wages

Year	2001	2002	2003	2004	2005
Latent mean of wages:	Implied Latent Size of the Formal Sector:				
$E[W(0)] = 0$	0.75***	0.75***	0.76***	0.79***	0.79***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$E[W(0)] = 250$	0.77***	0.76***	0.77***	0.80***	0.79***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$E[W(0)] = 500$	0.78***	0.77***	0.79***	0.81***	0.80***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$E[W(0)] = 750$	0.79***	0.79***	0.80***	0.82***	0.81***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$E[W(0)] = 1000$	0.80***	0.80***	0.81***	0.82***	0.82***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Conditional Probability coefficients ($\Lambda(w)$):					
$Pr[S = 1 \mid W = w]$	*Linear Approximation Coefficients:*				
Intercept (δ_{00})	0.75***	0.75***	0.76***	0.79***	0.79***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Wage ($\delta_{01} \times 10^3$)	0.05***	0.05***	0.05***	0.03***	0.03***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.010$.
Standard errors in parenthesis. Conditional probability estimates from a linear regression of observed formality on wages, using data of wages 40 Brazilian Reais above the minimum wage. Slope coefficients reported multiplied by 10^3. See text for details.

estimate for this object reported in the paper is 0.76.

The observed size of the formal sector that prevails in 2001 is 0.7. Thus, our estimates of the minimum wage on the size of the formal sector would be somewhere between 5 percentage points to 10 percentage points. And this range is obtained when we entertain spillovers that are, again, orders of magnitude larger than anything that any researcher would consider reasonable.

Why do these estimates not seem to respond to the latent wage distribution? The reason is that the slope coefficient of observed formality on observed wages above the minimum wage is minuscule. As a result, Our (lack of) ability to correctly pin down the latent distribution of wages does not hinder our ability to pin down the latent size of the formal sector, so even large errors in the marginal distribution of latent wages do not translate into large error in the estimated latent size of the formal sector.

It is also useful to point out that these results are not driven by the linear form we chose for the conditional probability function. The linearity simplifies the exercise because it only forces us to make guesses for the center, so the latent size of the formal sector is immediately pinned down once we know one key parameter. For example, consider a quadratic form of Λ function: $\Lambda(w;\delta_0) = \delta_{00} + \delta_{01}w + \delta_{02}w^2$. Notice that $Pr[S(0) = 1] = \int \Lambda(w;\delta_0) f_0(w;\mu,\sigma)dw$. When the identified latent size of formal sector is considered, the effect of using a "wrong" latent distribution f instead of the true one f_0 (with mean μ_1 and the second moment μ_2) can be evaluated as follows:

$$\begin{aligned}&\int \Lambda(w;\delta_0) f(w)dw - \int \Lambda(w;\delta_0) f_0(w)dw \\ &= \delta_{01}\left(E(W) - \mu_1\right) + \delta_{02}\left(E(W^2) - \mu_2\right),\end{aligned} \tag{A.14}$$

where $E(W)$ and $E(W^2)$ are the first and second moments of the "wrong" latent distribution f. Equation (A.14) shows that the sensitivity of the identified $Pr[S(0) = 1]$ depends fundamentally on the slope the curvature of the conditional probability function Λ. In our empirical application, this function is approximately flat, therefore, the coefficient δ_{02} is *even smaller* than δ_{01}. Furthermore, if anything, there is evidence that minimum wage induces wage compression (lower variance), so the results could make or estimates even more stable than the ones that we get when we ignore the quadratic term.

5.2. *Taking into account the biases in the conditional probability function*

We now show another dimension of the insensitivity of our estimates of the latent size of the formal sector to the misspecification of spillovers. This one is conceptually distinct from the discussion in the previous section, which focused on the role of the small slope coefficients for the conditional probability function that are observed in the Brazilian labor market. In this part, we do not rely on the assumption that the coefficients δ_{00} and δ_{01} in the Λ function can be consistently estimated. Even in this case, the latent size of the former sector, $Pr[S(0) = 1]$, is invariant to certain unknown forms of spillover effect on the wage brought by the minimum wage.

Let W_1 be the spillover-ridden wage that captures the spillover effect of the minimum wage but ignores other effects. Note that, by definition, what we observe in the data above m is the distribution of spillover-ridden wages, that is, the distribution of W_1. In the absence of spillover effects, W_1 coincides with $W(0)$ above the minimum wage, and we are back to our baseline version of the model. Note, also, that we do not observe the full distribution of W_1 neither because below and at m, workers face the usual direct effects of the minimum wage in the bottom part of the wage distribution, so some workers bunch at m, other lose their jobs, and others move to informality, as we discussed in the paper. Thus, it is useful to think of the observed wage distribution consisting of two pieces: The bottom part, where we see the direct effects of the minimum wage, and the upper part, above m, where we observe spillover-ridden minimum wages.

Although not required, assume, for concreteness, that the spillover takes the form of a location shift and a rescale, that is, $W_1 = \rho W(0) + d + V$, where ρ and d are known constants, $W(0)$ denotes the latent wage, and V is an independent idiosyncratic error. Without loss of generality, $E(V) = 0$. We have $W(1) = W_1$ for $w > km$. This spillover form is designed to capture the fact that in many models, the minimum wage will push up the wages of workers that earn considerably more than the minimum wage while also induce some degree of wage compression (lower variance). Although we can derive the next result in more general conditions, this simplified spillover specification is rich enough to allow for these effects. As in the previous part, we maintain the linear form of the Λ function. $\Lambda(w;\delta_0) = \delta_{00} + \delta_{01}w$, where δ_{00} and δ_{01} are unknown parameters and cannot be directly estimated as $W(0)$ is not observed. (Note that in the presence of the spillover, $W(0)$ is not observed even for the upper part of wage $w > km$.) Observe that for $m > km$,

$$\begin{aligned}
Pr[S(0) = 1|W(1) = w] &= E\left[E[S(0) = 1|V, W_1 = w]|W_1 = w\right] \\
&= E\left[E[S(0) = 1|W(0) = (w - d - V)/\rho]\right] \\
&= \delta_{00} - \delta_{01}[d + E(V)]/\rho + \delta_{01}w/\rho \\
&= \delta_{00} - \delta_{01}d/\rho + \delta_{01}w/\rho \equiv \Lambda(w;\tilde{\delta}).
\end{aligned}$$

As a result, we can estimate the intercept $(\delta_{00} - \delta_{01}d)/\rho$ and the slope δ_{01}/ρ, although we cannot recover the true parameters δ_{00} and δ_{01}. In other words, when the data we observe above the minimum wage consists of spillover-ridden wages, W_1, as opposed to latent wages $W(0)$, the coefficients we obtain for the probability function capture the conditional mean of formality given W_1, the spillover-ridden wage, and not the conditional mean of formality given $W(0)$. However, we are about to show that although spillovers create a big difficulty in finding the effect of the minimum wage on the wage distribution, the same is not true for the latent size of the formal sector. The reason is that the integral of the conditional probability of formality given spillover-ridden wages W_1, when integrated over the marginal of the density of spillover-ridden wages, yield the same value as the integral of the conditional probability of formality given latent wages. The bias we get in the parameters of the conditional probability function by conditioning in spillover-ridden wages is canceled by the fact that we are integrating over spillover-ridden wages as well. Recall that

$$Pr[S(0) = 1] = \int \Lambda(w; \delta_0) f_0(w) dw. \tag{A.15}$$

We investigate how (A.15) will change if we plug in $\tilde{\delta}$ and f_1 instead of δ_0 and and f_0. (f_0 and f_1 are the density functions of $W(0)$ and $W(1)$ respectively.)

Thus, to the extent that we are able to obtain estimates of the entire distribution of W_1 – and this is an object that is observed above m – and the conditional probability function of formality given W_1, which is another object that we observe above m, then we are going to obtain exactly the correct estimated latent size of the formal sector that we would if spillovers were in fact, absent. Notice that $E(W_1) = \rho E[W(0)] + d$, one can verify that

$$\begin{aligned} &\int \Lambda(w; \tilde{\delta}) f_1(w) dw - \int \Lambda(w; \delta_0) f_0(w) dw \\ &= (\delta_{00} - \delta_{01} d/\rho) - \delta_{00} + \delta_{01} E(W_1)/\rho - \delta_{01} E[W(0)] = 0. \end{aligned} \tag{A.16}$$

The take-away message from this exercise are the following: To consistently estimate *all* of our parameters of interest – namely the effects of the minimum wage on average wages, wage inequality, employment, welfare, and the size of the formal sector – we must correctly specify how the minimum wage affects the wages above m. That is, if we assume that spillovers are absent, then we need spillovers to be absent to obtain consistent estimates. If we assume an upper-bound on spillover effects, then it must be the case that above our specified upper bound, wages are not affected by the minimum wage. However, when we focus on the effect of the minimum wage on the size of the formal sector, a very different picture arises. Our estimates are highly insensitive to our ability to correctly pin down the mean of latent wages. This property arises from a feature of the Brazilian labor market, namely the fact that the conditional probability function of formality given wages display a minuscule slope above the minimum wage. Furthermore, even in settings in which this "small slope" condition is not verified, we showed that the latent size of the formal sector can still be consistently estimated even in the worst-case scenario in which we incorrectly assume the absence of spillovers when they are in fact, present. In this case, as we have shown above, we are going to incorrectly estimate the effects of the minimum wage on average wages, on wage inequality, but we will still recover the correct latent size of the formal sector.

6. APPENDIX G: MARGINAL EFFECTS OF THE MINIMUM WAGE

Using the results in Appendix C, Section 3.3, we obtain the following equations that describe the effects of the minimum wage on expected wages,

$$\text{(A.17)} \quad \frac{\partial \mathbb{E}\left[W(1)\right]}{\partial m} = \Pr[W(1)=m] - \frac{1}{c^{(1)}+c^{(0)}}\left(\frac{\partial c^{(1)}}{\partial m} + \frac{\partial c^{(0)}}{\partial m}\right)\mathbb{E}\left[W(1)\right] + \frac{1}{c^{(1)}+c^{(0)}}\left(\frac{\partial c^{(1)}}{\partial m} + \frac{\partial c^{(0)}}{\partial m}\xi\right)m.$$

$$\text{(A.18)} \quad \frac{\partial \mathbb{E}\left[W(1)|S(1)=1\right]}{\partial m} = \Pr\left[W(1)=m|S(1)=1\right] - \frac{1}{c^{(1)}}\frac{\partial c^{(1)}}{\partial m}\left\{\mathbb{E}\left[W(1)|S(1)=1\right]-m\right\}.$$

$$\text{(A.19)} \quad \frac{\partial \mathbb{E}\left[W(1)|S(1)=0\right]}{\partial m} = \Pr[W(1)=m|S(1)=0] - \frac{1}{c^{(0)}}\frac{\partial c^{(0)}}{\partial m}\left\{\mathbb{E}\left[W(1)|S(1)=0\right]-\xi m\right\}.$$

$$\text{(A.20)} \quad \frac{\partial c}{\partial m} \approx -\frac{1-c}{m}.$$

$$\text{(A.21)} \quad \frac{\partial c^{(1)}}{\partial m} \approx -\frac{1-c^{(1)}}{m}.$$

$$\text{(A.22)} \quad \frac{\partial c^{(0)}}{\partial m} \approx \frac{c^{(0)}-1}{m}.$$

$$\text{(A.23)} \quad \frac{\partial \log(\Pr[S(1)=1]/\Pr[S(1)=0])}{\partial m} = -\frac{1-c^{(1)}}{c^{(1)}}\frac{1}{m} - \frac{c^{(0)}-1}{c^{(0)}}\frac{1}{m}.$$

where $c^{(1)} = 1-\int^m(1-\pi_m^{(1)})f_0(u|S(0)=1)du$. The effect of the minimum wage on the average wages of the employed has three key components. The first is the "bite", that is, the proportion of workers who receive the minimum wage. This is the mechanical effect of pushing up the wages of minimum wage workers. The second component concerns unemployment effects: As long as the minimum wage is smaller than the expectation of the observed wages, unemployment will increase the perceived effect of the minimum wage on average wages for those who remain employed. This effect is due to the removal of certain observations at the left tail of the distribution, which contributes to increasing the average wage for those that remain employed. The third effect is related to reallocation of labor across sectors: Some workers will migrate from the formal to the informal sector, and, to the extent that this movement induces changes in their wages, this will contribute to a change in the average.

The model predicts heterogeneous effects of the minimum wage across sectors. The marginal effects of the minimum wage conditional on the sector are a function of the "bite" *and* the coefficients that govern the movements into or out of that sector in response to the policy. The term multiplying $\frac{dc^{(1)}}{dm}$ in (A.18) measures the effect that workers moving out of formality have on expected wages in the sector. The term multiplying $\frac{dc^{(0)}}{dm}$ in (A.19) captures the effect that the entry of workers from the formal sector has on expected wages in the informal sector.[7]

In contrast to the formal sector, the minimum wage has an ambiguous impact on average wages in the informal sector. The minimum wage policy induces an inflow of low-wage workers from the formal sector to the informal sector. This mechanism, depending on the size of the model parameters, can be sufficient to induce an overall reduction of average wages in that sector. In terms of the size of each sector, as long as $\pi_d^{(1)}$ or $\pi_u^{(1)}$ is greater than zero, the minimum wage will reduce the total number of workers employed in the formal sector, and as long as $\pi_d^{(1)}$ is greater than zero, the opposite will be true for the informal sector.

7. APPENDIX G: ADDITIONAL EMPIRICAL RESULTS

7.1. *Relaxing some assumptions*

In this section, we discuss the results we obtain when we estimate the model relaxing some of the assumptions of the model. First, we discuss the results we obtain when we relax the structure imposed by Remark 2 of the paper. There, we specified the effects of the minimum wage on the bottom part of the wage distribution. An important restriction of this assumption is that the wages of workers in the informal sector are not altered by the inflow of formal workers to that sector. Additionally, we also impose that formal workers will earn a wage in the informal sector approximately equal to ξ times the wage that they would earn if they were employed in the formal sector. Under Assumptions 1, 2, and 3, it is possible to estimate the model without imposing these restrictions. To do that, we only need to impose the following weaker condition:

[7] Assumptions 3 and 4 exclude general equilibrium effects. The entry of workers into the formal sector is assumed not to change the wages of workers in the informal sector.

Assumption A4 (Minimum wage effects' structure (weak characterization))

$$Pr[W(1) > km|W(0) < km] = 0 \ , \ \ \textit{for some known } k \geq 1.$$

This assumption does not specify, and, by doing so, it leaves unrestricted the way that the wages of formal sector workers will change when they move to the informal sector, as long as they do not show up too far up in the right tail of the informal sector wage distribution. Thus, it imposes a much weaker restriction on the structure of minimum wage effects at the bottom of the wage distribution than the condition imposed by Remark 2 in the paper. It also allows the minimum wage to affect the wages of workers that earn more than the minimum wage, within the range of wages between m and km, where km is a tuning parameter, a user-specified upper bound for the spillover effect. Note that even if we set k to be one, Assumption A.4 is still weaker than Remark 2 in the paper because it leaves unrestricted the wage that the minimum wage affects the bottom part of the wage distribution.

The baseline results we present in the main text also assume that the minimum wage has no effect on wages above m. This restriction is implied by the structure of the bargaining model. However, this assumption would be violated in several different descriptions of the environment. For example, the absence of spillovers will not hold if the technology of production displays complementarity or substitution between workers across different levels of skill. The absence of spillovers will also not hold if workers are matched to firms according to a matching technology that does not differentiate workers by skill level (thus, a change in the number of low wage workers searching for jobs generate congestion externalities for high wage workers, affecting their reservation wages). For all these reasons, it is important to ask whether the results obtained in our preferred specification are substantially changed when we allow for spillover effects. Below we discuss our findings when we allow for more flexible effects of the minimum wage in the bottom part of the wage distribution and when we allow for spillover effects of the minimum wage.

Table II: Robustness - Unrestricted effects on the bottom part of the wage distribution

Parameter		2001	2002	2003	2004	2005
Aggregate						
π_d	Non-compliance	0.19***	0.23***	0.20***	0.21***	0.21***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
π_m	Bunching	0.26***	0.41***	0.30***	0.27***	0.43***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
π_u	Non-employment	0.55***	0.36***	0.50***	0.52***	0.36***
		(0.01)	(0.02)	(0.02)	(0.01)	(0.02)
$Pr[W(0) < m]$	Fraction Affected	0.26***	0.24***	0.31***	0.31***	0.29***
		(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
$Pr[S(0) = 1]$	Latent size of the formal sector	0.76***	0.77***	0.78***	0.78***	0.81***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Formal Sector						
$\pi_d^{(1)}$	Sector mobility	0.03**	0.23***	0.08***	0.08***	0.22***
		(0.01)	(0.02)	(0.02)	(0.01)	(0.02)
$\pi_m^{(1)}$	Bunching	0.21***	0.28***	0.25***	0.23***	0.33***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$\pi_u^{(1)}$	Non-employment	0.76***	0.48***	0.67***	0.69***	0.46***
		(0.02)	(0.03)	(0.02)	(0.02)	(0.02)
Informal Sector						
$\pi_d^{(0)}$	Non-compliance	0.61***	0.22***	0.54***	0.58***	0.19***
		(0.01)	(0.03)	(0.02)	(0.02)	(0.03)
$\pi_m^{(0)}$	Bunching	0.39***	0.78***	0.46***	0.42***	0.81***
		(0.01)	(0.03)	(0.02)	(0.02)	(0.03)

Note: Bootstrapped standard errors (computed using 100 replications) are given in parentheses.

Table II displays the estimates of the model parameters when we relax the effect structure in the bottom part of the wage distribution (Remark 2). In this specification, the wages of formal workers are allowed to change in a quite flexible manner when they move to the informal sector. Regarding Table II, the qualitative implications of our empirical exercise remain roughly unaltered. The policy generates sizable unemployment effects, the aggregate probability of non-compliance with the policy is approximately 20%, and the latent size of the formal sector is approximately .78.

Table III displays the estimates of the model parameters when we relax the absence of spillovers. In this specification, we allow the minimum wage to have effects on wages up to R\$40.00 above the minimum wage.[8] The qualitative results we obtained when we allow for spillover effects are similar to the ones in our preferred specification.

[8] This is roughly two times the average change in the real value minimum wage during the analyzed period. That means, for example, that in the year 2001, we are allowing the minimum wage to have an effect on wages roughly up to the real level of the minimum wage that prevails in the year 2003.

Table III: Robustness to Spillovers

Parameter		2001	2002	2003	2004	2005
Aggregate						
π_d	Non-compliance	0.18***	0.21***	0.20***	0.20***	0.18***
		(0.00)	(0.01)	(0.01)	(0.01)	(0.02)
π_m	Bunching	0.24***	0.37***	0.28***	0.26***	0.37***
		(0.01)	(0.02)	(0.01)	(0.01)	(0.03)
π_u	Non-employment	0.58***	0.42***	0.52***	0.54***	0.44***
		(0.01)	(0.03)	(0.01)	(0.02)	(0.05)
$Pr[W(0) < m]$	Fraction Affected	0.28***	0.27***	0.33***	0.33***	0.33***
		(0.00)	(0.01)	(0.01)	(0.01)	(0.02)
$Pr[S(0) = 1]$	Latent size of the formal sector	0.78***	0.78***	0.78***	0.79***	0.81***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.01)
Formal Sector						
$\pi_d^{(1)}$	Sector mobility	0.04***	0.20***	0.08***	0.08***	0.18***
		(0.00)	(0.03)	(0.01)	(0.01)	(0.05)
$\pi_m^{(1)}$	Bunching	0.19***	0.24***	0.23***	0.21***	0.27***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
$\pi_u^{(1)}$	Non-employment	0.78***	0.56***	0.69***	0.71***	0.55***
		(0.01)	(0.05)	(0.02)	(0.02)	(0.07)
Informal Sector						
$\pi_d^{(0)}$	Non-compliance	0.62***	0.25***	0.56***	0.59***	0.21**
		(0.01)	(0.07)	(0.01)	(0.02)	(0.10)
$\pi_m^{(0)}$	Bunching	0.38***	0.75***	0.44***	0.41***	0.79***
		(0.01)	(0.07)	(0.01)	(0.02)	(0.10)

Note: Bootstrapped standard errors (computed using 100 replications) are given in parentheses.

7.2. *Additional Results*

In this section, we report additional results based on our preferred specification and comment on the ability of the model to fit the most important features of the joint distribution of sector and wage. First, we display the estimated parameters of the latent wage distribution and the conditional probability of formality with respect to the wage. These parameters are not of particular interest on their own but are needed to obtain the marginal effects of the minimum wage and the estimates of the probabilities π.

Figure 1 shows the estimates of the latent wage distributions for the formal and informal sectors. We can see that the informal sector wage distribution tends to have a higher density for low wages relative to the formal sector. This follows from the positive estimated slope coefficient on the relationship between latent sectors and wage (δ_1).

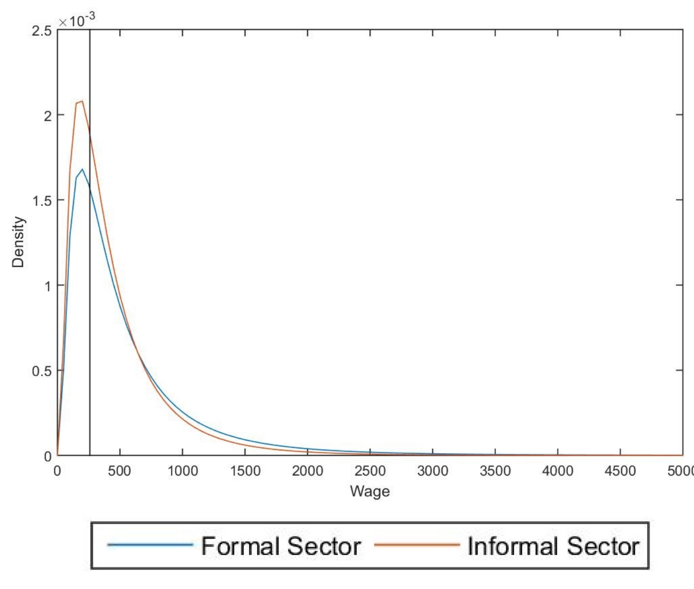
Figure 2 shows kernel density estimates of the wage distributions in the formal and

Table IV: Parameter Estimates

Parameter	2001	2002	2003	2004	2005
Latent wage distribution:					
μ	5.7432***	5.8388***	5.8713***	5.9415***	6.0661***
	(0.0095)	(0.0098)	(0.0089)	(0.0081)	(0.0120)
$\log(\sigma)$	-0.1344***	-0.1459***	-0.1493***	-0.1724***	-0.2038***
	(0.0071)	(0.0076)	(0.0067)	(0.0055)	(0.0077)
Conditional probability of sector given the wage:					
δ_0	0.9066***	1.0692***	1.0441***	1.0554***	1.3348***
	(0.0265)	(0.0229)	(0.0356)	(0.0360)	(0.0382)
δ_1	0.0006***	0.0004***	0.0005***	0.0005***	0.0002***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

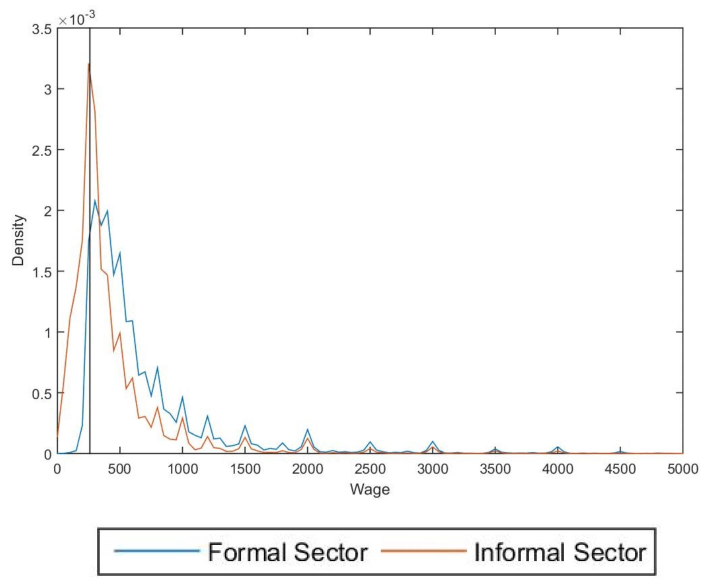
Note: Bootstrapped standard errors (computed using 100 replications) are given in parentheses.

Figure 1: Latent Densities



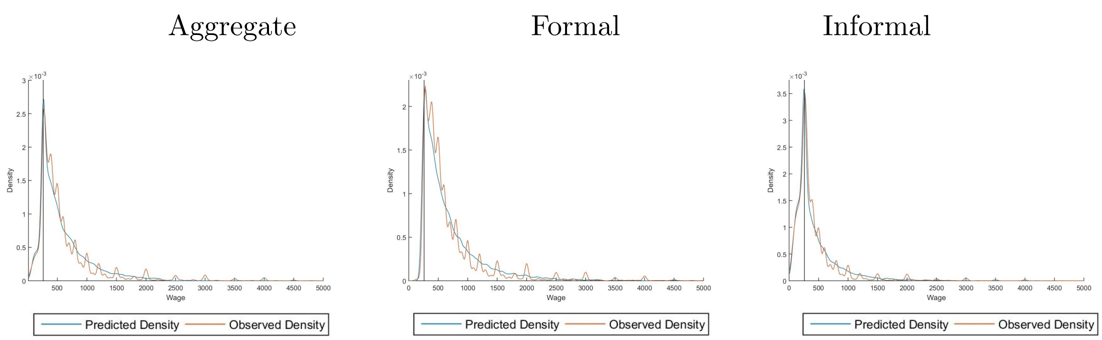
Note: Year 2004.

Figure 2: Observed Densities



Note: Density estimates using a gaussian kernel (bandwidth = R$30). Year 2004.

Figure 3: Model Fit



Note: Density estimates based on a gaussian kernel (bandwidth = R$30). Year 2004.

informal sectors for the year 2004. We observe substantial differences between the observed wage distributions in the formal and informal sectors. The formal sector wage distribution presents almost no density below the minimum wage level, whereas the informal sector exhibits considerable mass in that range. Above the minimum wage level, the formal sector density tends to be higher than the informal sector density.

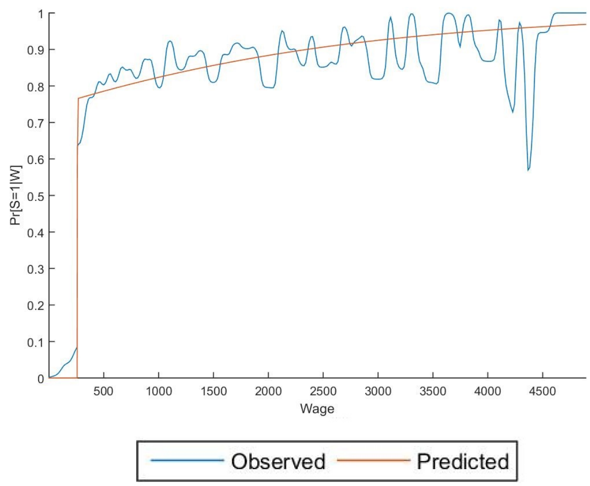
Table V: Model Fit

Parameter	2001		2002		2003		2004		2005	
	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted
$E[W]$	532.14	520.12	562.16	550.86	606.15	594.30	640.93	628.66	691.47	680.72
$Sd[W]$	538.98	464.06	565.13	487.55	573.33	502.67	587.34	514.75	600.73	531.65
Skewness[W]	3.41	3.19	3.29	3.23	3.12	3.09	3.08	2.98	2.94	2.97
Kurtosis[W]	18.06	17.41	16.70	17.35	15.31	16.05	15.00	15.07	13.91	14.54
$Pr[W=m]$	0.08	0.08	0.10	0.11	0.11	0.11	0.10	0.10	0.14	0.14
$Pr[W<m]$	0.06	0.06	0.06	0.07	0.08	0.08	0.08	0.08	0.07	0.08
$q^{20}[W]$	220.00	244.38	240.00	267.23	254.00	301.61	290.00	321.69	300.00	374.15
$E[W\|S=1]$	592.12	567.30	628.27	601.66	675.54	643.80	716.31	682.01	760.71	720.95
$E[W\|S=0]$	380.92	357.95	402.04	360.72	428.16	384.52	446.52	404.88	502.87	425.45
$Sd[W\|S=1]$	546.56	498.12	572.75	520.83	580.69	534.73	595.35	545.62	601.61	556.42
$Sd[W\|S=0]$	405.13	335.61	434.38	375.07	429.99	373.40	442.91	380.81	478.16	438.38
$q^{20}[W\|S=1]$	260.00	264.06	280.00	297.83	300.00	341.77	350.00	361.85	380.00	414.51
$q^{20}[W\|S=0]$	180.00	175.50	196.00	185.62	200.00	191.16	200.00	201.20	240.00	243.00
$Pr[S=1]$	0.72	0.71	0.71	0.70	0.72	0.71	0.72	0.71	0.73	0.72
$Pr[S=1\|W<m]$	0.08	0.00	0.09	0.00	0.09	0.00	0.05	0.00	0.04	0.00
$Pr[S=1\|W=m]$	0.58	0.57	0.51	0.48	0.62	0.60	0.62	0.60	0.60	0.56
$Pr[S=1\|W>m]$	0.77	0.77	0.78	0.78	0.79	0.79	0.80	0.80	0.82	0.82

Table V presents a comparison between certain moments of the data and those implied by the model parameters. In examining this table, we can see that the model can capture most of the features of the joint distribution of sector and wage. It predicts the discontinuous shape of $\Pr[S(1)=1|W(1)]$ observed in the data. It fits the probabilities of observing wages at and below the minimum wage level and explains the differences observed between the formal and informal sector distributions. Interestingly, the model can match higher moments of the wage distribution, such as skewness and kurtosis. This need not be the case in general, especially if the parametric family for the wage distribution is severely misspecified.

Figure 4 shows the observed and predicted conditional probabilities of formality given the wage. Figure 3 displays the predicted and observed densities of the aggregate wage distribution and the formal and informal sector distributions. By examining these figures, we again see that the model matches most of the prominent features of the data, except the “heaping” observed at round numbers. It is nevertheless interesting to note the resemblance between the predicted and observed curves in the empirical cumulative distribution for the formal and informal sectors, particularly at and below the minimum wage level.

Figure 4: Conditional Probability of Sector Given the Wage



Note: Observed conditional probabilities based on a non-parametric local constant (Nadaraya-Watson) estimator using a gaussian kernel (bandwidth = R\$30).Year 2004.

7.2.1. *Decomposing the Differences in the Wage Distributions Across Sectors – The Role of the Minimum Wage*

Figure 5 displays the empirical CDF for the formal and informal sector for wages above the minimum wage. These estimates are, by construction, invariant to the minimum wage effects on the formal and informal sectors for values below the minimum wage. Note that we observe a substantially smaller difference in the CDFs across sectors in Figure 5 than in Figure 6 in the main article. This exercise suggests that the differences across sectors observed in the upper part of the wage distribution across sectors are also a consequence of the effects of the minimum wage in the bottom part of the wage distribution.

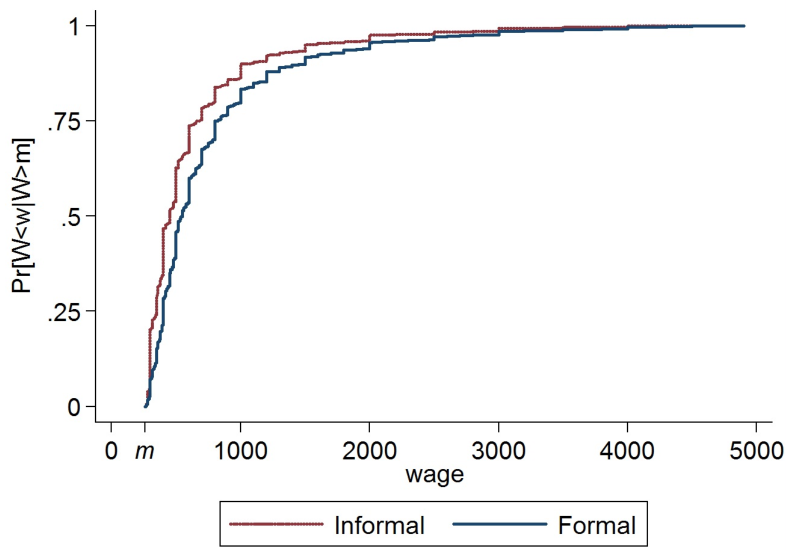
The estimates of the model parameters allow us to understand the differences between the wage distributions in the formal and informal sectors. Let $D_1 \equiv f(w|S(1) = 1) - f(w|S(1) = 0)$. That is, D_1 is defined as the observed difference in the density of wages between the formal and informal sectors. Let $D_m \equiv [f(w|S(1) = 1) - f_0(w|S(0) = 1)] - [f(w|S(1) = 0) - f_0(w|S(0) = 0)]$. That is, D_m is defined as the difference in the effects of the minimum wage between the formal and informal sectors. Define

$D_0 \equiv f_0(w|S(0) = 1) - f_0(w|S(0) = 0)$. That is, D_0 is defined as the difference between the *latent* wage densities between formal and informal sectors. From the definitions, we have $D_1 = D_m + D_0$. Given the estimates of the model parameters, it is possible to compute D_1, D_m, and D_0. By comparing these estimates, we can infer the extent to which the differences in the wage densities between the formal and informal sectors is due to the minimum wage versus differences that would be present regardless of the minimum wage policy. This decomposition can be performed at every point of the wage distribution.

Figure 6 displays the differences in the density of wages between the formal and informal sectors. Given that the observed wage distribution in the formal sector stochastically dominates that in the informal sector, we observe a negative difference between the formal and informal wage densities for low wages and positive for high wages. We observe a similar pattern for the latent density as well. Figure 6 also displays the differences between the effects of the minimum wage at each point of the wage distribution. The estimates of the differences in the minimum wage effect tend to closely follow the differences in the observed wage density across sectors. If we decompose the differences in the observed wage distributions between the formal and informal sectors in differences in latent wage distributions and differences in minimum wage effects, my estimates suggest a larger role for the latter. For example, at the 5th quantile of the wage distribution, the minimum wage policy accounts for 82% of the differences in the wage density across sectors.

A decomposition of the difference in quantiles across sectors can be performed in a similar way. The minimum wage accounts for 93% of the differences in the 10th percentile of the wage distribution across sectors. Our estimates imply that 72% of the differences in the median of the wage distribution between the formal and the informal sectors is due to the minimum wage. At higher percentiles, such as the 90th percentile, the minimum wage still accounts for 68% of the differences observed between the formal and informal sectors. The minimum wage accounts for a substantial part of the

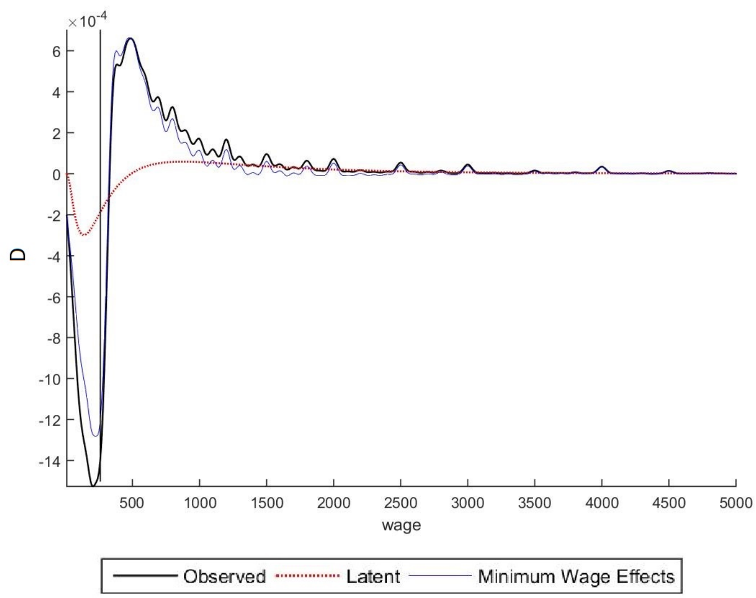
Figure 5: Empirical CDFs Above the Minimum Wage by Sector



differences in the wage distribution across sectors for low *and* high wages, even when wages above the minimum wage are, by assumption, not affected by the policy. The reason for the substantial role of the minimum wage in explaining the differences in the formal and informal sector wage densities above the minimum wage is the opposite way that the wage densities are rescaled across sectors due to the inflow/outflow of workers as a result of the minimum wage policy.

Figure 6: Decomposition of the Differences of the Densities Across Sectors

Formal minus Informal



Note: Density estimates based on a gaussian kernel (bandwidth = R$30). Year 2004.