Equilibrium Under Inflation Targeting

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Abstract In this paper I study how the inflation targeting system can impact the equilibria of an infinitely repeated monetary policy game. I modify the Barro-Gordon model so that the central bank incurs a penalty (i.e., a payoff loss) whenever the actual inflation rate is not equal to its target. I assess how changes in this penalty impact the set of equilibrium outcomes that can be supported by trigger strategies that specify reversion to a one-shot Nash equilibrium. The results of this exercise show that, as argued by many authors, the inflation targeting regime can indeed help to anchor the agents' expectations about current and future inflation rates. Furthermore, if the penalty is sufficiently large, then the only element of that equilibrium set is precisely the one in which the central bank implements the target at every date.

Keywords inflation targeting; implementation; multiple equilibria; coordination

JEL classification E31; E52; E58; E61

1 Introduction

In 1990, New Zealand was the first country to implement the inflation targeting regime. Since then, many other nations have adopted this monetary policy framework. Hammond (2012) identified 27 countries following the policy regime in question in 2012. This list did not include the United States, Japan and the nations of the Euro Zone. However, as argued by Svensson (2011), the Federal Reserve, the Bank of Japan and the European Central Bank were on the path, and already close, to becoming inflation targeters. Indeed, currently each

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of these three central banks has some type of publicly announced target for the inflation rate.

Given that so many economies have embraced the inflation targeting system, it is not surprising that a large body of literature on this policy framework exists. Nevertheless, questions are still open regarding how the adoption of the inflation targeting regime (i) impacts the set of equilibrium values of the inflation rate and (ii) helps agents to coordinate on an equilibrium in which the announced target is implemented, especially in the context of an infinitely repeated game. This paper addresses precisely these two issues.

To study these questions, I take the well-kwon model of Barro and Gordon (1983a and 1983b) as the starting point to construct a suitable stage game. In its simplest form (which does not include the inflation targeting regime), the private sector sets its expectations π^e for the inflation rate, while the central bank selects the actual inflation rate π. Given these two variables, the GDP is determined according to a standard Phillips curve. The payoff of the private sector is simply the square of its inflation forecast error. Concerning the central bank, its period preferences are described by a quadratic loss function that depends on the inflation rate and the deviation of the output from a level, higher than the natural rate of production, that this player would like to achieve. As usual, this stage game has a single Nash equilibrium. In this outcome, the inflation rate is positive and equal to its expected value. For future reference, denote this equilibrium rate by $\hat{\pi}$.

To incorporate the inflation targeting regime, I add two features to the above described above stage game. First, I postulate that an exogenous target π^* exists for the inflation rate. Second, I assume that whenever the central bank implements an inflation rate $\pi \neq \pi^*$, this agent incurs a payoff loss equal to a positive constant C. Hence, this agent's payoff has a discontinuity precisely when $\pi = \pi^*$.

The equilibrium set of this stage game depends on the value of the penalty. More specifically, I show there are two critical values k_1 and k_2, where $k_2 > k_1 > 0$, for C with the properties that I describe next. If $C < k_1$, then the penalty is so small that the inflation targeting regime is irrelevant. As a consequence, the only Nash equilibrium is precisely the one of the original Barro-Gordon setup, where $\pi^e = \pi = \hat{\pi}$. If $C > k_2$, then the penalty is so large that the central bank will surely play $\pi = \pi^*$. In anticipation of this, the private sector sets $\pi^e = \pi^*$. Hence, the unique equilibrium is given by $\pi^e = \pi = \pi^*$. Finally, in the intermediate case in which $C \in [k_1, k_2]$, there are exactly two Nash equilibria: $\pi^e = \pi = \hat{\pi}$ in the first of them and $\pi^e = \pi = \pi^*$ in the other.

Next, I study the infinitely repeated version of this game. As usual, I use the one-shot Nash equilibria described above to characterize a set of subgame-perfect equilibrium outcomes for the repeated game. If $C \leq k_2$, then trigger strategies that specify reversion to the Nash equilibrium in which $\pi^e = \pi = \hat{\pi}$ can support many other equilibria. For the case in which $C > k_2$, reversion to the Nash equilibrium where $\pi^e = \pi = \pi^*$ may support other outcomes. Since those equilibrium outcomes are supported by trigger strategies and the characterization of the set depends on the value of the penalty, I denote the set in question by $\mathcal{T}(C)$.

In the context of this paper, an inflation targeting regime is constituted by a target π^* and a penalty C. Although I treat the target as an exogenous variable, one can see it as an optimal inflation rate that is a function of preferences, technology, fiscal policy, and other features of the economy. On the other hand, the parameter C can be interpreted as a concise measure of a society's ability to align the central bank's incentives with its desire to achieve a specific inflation rate. Now consider questions of the type "How large does C have to be to ensure that the target π^* can be implemented in equilibrium?", and "Is there a sufficiently large C to ensure that no deviations from π^* can happen in equilibria supported by trigger strategies?", etc. Questions of this class can be answered by studying how changes in C impact the set $\mathcal{T}(C)$, which is precisely the main exercise carried out in this paper.

This exercise provides several interesting results. Let X^* denote the outcome in which $\pi_t^e = \pi_t = \pi^*$ for all t. First, there exists a number $k_0 < k_1$ with the property that X^* belongs to $\mathcal{T}(C)$ if and only if $C \geq k_0$. Therefore, $k_0 \leq 0$, then X^* is an equilibrium outcome regardless of the value of C. Moreover, if $k_0 > 0$, the implementation of X^* with standard trigger strategies requires C to be positive. Second, while the penalty is still smaller than or equal to k_2, an increase in C will not add an outcome to $\mathcal{T}(C)$ in which $\pi_t \neq \pi^*$ for every t. Furthermore, if such an increase drops an outcome from $\mathcal{T}(C)$, then it must be the case that this outcome does not hit the target π^* sufficiently often (in a sense that can be defined precisely). Third, when C is already larger than k_2, a further increase in C will not create a new equilibrium outcome. Fourth, if $C > k_2$, then there is a neighborhood of X^* with the property that X^* is the only outcome in this neighborhood that belongs to $\mathcal{T}(C)$. Thus, X^* has a type of local-uniqueness property whenever $C > k_2$. Fifth, there is a number $k_3 > 0$ such that if $C > k_2$ and $C \geq k_3$, then X^* is the unique element of $\mathcal{T}(C)$.

It is desirable to have a brief and intuitive synthesis of the implications of the above findings. The first result establishes that the implementation of X^* with standard trigger strategies may require the introduction of the inflation targeting system. However, that is not all. Even if X^* is an equilibrium outcome in the absence of this policy regime, its adoption helps, in several ways, to achieve equilibria in which the target π^* is implemented at least on some dates. Indeed, if $C \leq k_2$, the introduction of the inflation targeting framework changes the set of equilibrium outcomes by adding sequences in which the central bank implements π^* on some dates and by removing sequences in which $\pi_t \neq \pi^*$ for all dates. If $C > k_2$, then X^* will be, in a local sense, a unique equilibrium outcome supported by strategies that specify reversion to the Nash equilibrium of the stage game. And, for C sufficiently large, this uniqueness will be global (instead of just local). Finally, if $C > k_2$, a further increase in the penalty does not create a new equilibrium outcome, so the inflation targeting regime helps to deal with the problem of equilibrium multiplicity.

A brief analysis of the related literature helps to assess the relevance of these results. Bernanke, Laubach, Mishkin, and Posen (1999, p. 11) stated that the inflation targeting system "serves as a nominal anchor for monetary policy" and "provides a focus for the expectations of financial markets and the general public." Walsh (2009, p. 200–201) argued that this type of policy "can align the public's expectations of current and future target rates with the actual goals of the central bank" and may anchor "the public's beliefs about future inflation." Svensson (2011, p. 1247) mentioned that the empirical evidence suggests that "an explicit numerical target for inflation anchors and stabilizes inflation expectations."

The previous paragraph shows a widespread view exists that the inflation targeting regime can help the economic agents to coordinate their expectations and actions. However, to my knowledge no paper has yet been published providing a theoretical foundation, in an intertemporal model, for this view. And there is at least one reason for that: equilibrium multiplicity is a prevalent feature of intertemporal games. Therefore, at a first glance, a repeated game does not seem to be a viable framework to study how the inflation targeting regime can lead to better coordination among the agents of an economy. In spite of that, this paper provides the missing rationalization for the manner in which an inflation targeting policy can contribute to the coordination of players' actions and lessen the problem of equilibrium multiplicity.

This essay has two features in common with Araujo, Berriel, and Santos (2016). First, they also studied how the inflation targeting regime can induce agents to coordinate their expectations. However, they adopted a single-period game with imperfect information. As a consequence, the problems addressed in their paper and here are not precisely the same. Second, they also assumed that the central bank incurs a payoff loss whenever it does not implement the inflation target. They interpreted the penalty as a commitment device embedded in the loss function that describes the preferences of the central bank.

There is an alternative interpretation to the penalty in question. As pointed out in Subsection 2.2.1, introducing this hypothesis is equivalent to assuming that the central bank receives a performance bonus whenever the actual inflation rate is equal to the target. Hence, this paper may be seen as a study of the effects of a specific type of compensation scheme (more specifically, a discontinuous one) for the central banker. This is in line with the findings of Mishkin and Westelius (2008), who argued that it is possible to interpret an inflation band targeting as a type of inflation contract.

The remainder of this paper is organized as follows. Section 2 lays out the stage game and characterizes its set of Nash equilibria. Section 3 presents the infinitely repeated game and characterizes the set of equilibrium outcomes that can be supported by trigger strategies that specify reversion to a Nash equilibrium of the stage game. The effects of changes in the penalty on this last set are studied in Section 4. Section 5 concludes.

2 Preliminaries: The Stage Game

The goal of this section is to present and analyze the stage game underlying the infinitely repeated game that is the focus of this paper. To ease the exposition, I first discuss a version of the stage game without inflation targeting; this version is essentially identical to the single-period inflation bias model adopted by Kydland and Prescott (1977) and, more famously, by Barro and Gordon (1983a and 1983b). In a second step, I modify the game to incorporate the inflation targeting regime.

2.1 Without Inflation Targeting

There are two players in the game. The first is the central bank (player b). The second is the general public (player p), which encompass people, firms and other agents. Players move simultaneously. Agent p selects an value π^e for expected the inflation rate. Her/his choice must lie in the set $\Pi = [0, \pi_{\max}]$, where $\pi_{\max}$ is a positive number large enough so that it is never reached in any of the equilibria discussed in this paper. Player b selects a value $\pi \in \Pi$ for the actual inflation rate. Given those choices, the value $y \in \mathbb{R}$ of the natural logarithm of the GDP is determined according to the Phillips curve

$$y = \bar{y} + \alpha(\pi - \pi^e),$$

where $\bar{y}$ is the natural logarithm of the natural rate of output and α is a positive parameter. The payoff of p is given by

$$V(\pi^e, \pi) = -(\pi - \pi^e)^2. \tag{1}$$

The function

$$w(y, \pi) = -\{\gamma\pi^2 + [y - (1 + \mu)\bar{y}]^2\}, \tag{2}$$

where $\gamma >$ and $\mu > 0$, is the payoff of b.

I adopt the usual procedure of using the Phillips curve to express the payoff of b as function of π^e and π. That is, define the function W so that $W(\pi^e, \pi) = w(\bar{y} + \alpha(\pi - \pi^e), \pi)$.

Hence,

$$W(\pi^e, \pi) = -\{\gamma\pi^2 + [\alpha(\pi - \pi^e) - \mu\bar{y}]^2\}. \tag{3}$$

Thus, from now on I assume that the central bank's payoff is described by W. This player's problem is

$$\max_{\pi \in \Pi} W(\pi^e, \pi) \tag{4}$$

while player p solves

$$\max_{\pi^e \in \Pi} V(\pi^e, \pi). \tag{5}$$

A *Nash equilibrium* for this game is a vector $(\hat{\pi}^e, \hat{\pi})$ with the properties that: (i) given $\hat{\pi}^e$, $\hat{\pi}$ solves the problem of player b and (ii) given $\hat{\pi}$, $\hat{\pi}^e$ solves the problem of player p.

The game has a unique Nash equilibrium. Denote the partial derivatives $\partial W/\partial \pi^e$ and $\partial W/\partial \pi$ by, respectively, W_1 and W_2. It is a simple exercise to show that, for a fixed π^e, the function $W(\pi^e, \cdot)$ is strictly concave. Therefore, the solution of (4) is characterized by

$$W_2(\pi^e, f(\pi^e)) = 0, \tag{6}$$

where f is the best response function of b. It is a simple exercise to show that

$$f(\pi^e) = \frac{\alpha^2}{\gamma + \alpha^2}\pi^e + \frac{\alpha\mu}{\gamma + \alpha^2}\bar{y}. \tag{7}$$

Optimality by player p requires $\pi^e = \pi$. Thus, $\hat{\pi}^e = \hat{\pi} = f(\hat{\pi})$; as a consequence,

$$\hat{\pi} = \frac{\alpha\mu}{\gamma}\bar{y}. \tag{8}$$

It is worth to point out that $\hat{\pi}$ is the unique fixed point of f.[1]

2.2 With Inflation Targeting

In this subsection I carry out two tasks. First, I modify the previous game to incorporate the inflation targeting regime; second, I characterize the equilibrium set of the resulting new game. Each of those tasks is carried out in a separated subsection.

[1] Since $\pi = \pi^e$ in the unique Nash equilibrium, one could be tempted to define an equilibrium just as a value for π instead of a two-dimensional vector. However, in the stage game with inflation target it is not so obvious that $\pi = \pi^e$ in equilibrium. Moreover, the analysis of the repeated game requires to have a clear distinction of the actions of each player. As a consequence, had I introduced this simplification, I would have to reverse it shortly after. Hence, I opted for not adopting it.

2.2.1 Introducing Inflation Targeting

I assume that an outside agent (for instance, the legislative) carries out the task of modifying the game.[2] More specifically, this agent publicly announces that the central bank has to pursue a target $\pi^* \in (0, \hat{\pi})$ for the inflation rate. This announcement is made before the players implement their actions. By itself this change will not impact the equilibrium of the stage game. Thus, it is necessary to carry a second modification, which I discuss next.

When studying the inflation targeting regime, many authors assume that a term similar to $-\kappa(\pi - \pi^*)^2$, where κ is a positive constant, shows ups in the objective function of the central bank or in a social welfare function.[3] Following this approach here would require substituting the term $\kappa(\pi - \pi^*)^2$ for $\gamma\pi^2$ in expression (2). However, this procedure implicitly assumes that the external agent who introduces the inflation targeting regime has the ability to modify the payoff function of the central bank and that is at odds with the standard practice of taking preferences as given. Furthermore, once that is accepted that external agent is able to change the preferences of player b, the obvious question is why the external agent will not set the parameter μ equals to 0 to ensure that $\pi = \pi^*$ in equilibrium. Therefore, I take a different path. Following Obstfeld (1994), who assumes that a central bank faces a fixed penalty whenever the exchange rate does not remain constant, I suppose that the external agent assess a penalty to player b whenever the actual inflation rate differs from the target π^*. This penalty has the property that it leads to a payoff loss equals to $C > 0$. Formally, define the indicator function I so that

$$I(\pi) = \begin{cases} 1, & \text{if } \pi \neq \pi^* \\ 0, & \text{if } \pi = \pi^* \end{cases} .$$

while the function U, which is given by

$$U(\pi^e, \pi) = W(\pi^e, \pi) - I(\pi)C, \tag{9}$$

is the payoff of the central bank.

Araujo, Berriel and Santos (2016) adopted this assumption when studying inflation targeting policies in a context of imperfect information.[4] They interpret the penalty as a commitment technology. The same interpretation is valid here. Furthermore, one can assume

[2] It is possible to assume that the changes are implement by the central bank itself. I disscuss this matter at end of this subsection.

[3] For some examples, see Svensson (1997 and 1999), Drazen (2000), and Capistrán and Ramos-Francia (2010).

[4] In a previous and much less ambitious work (Cunha 2019), I used the same approach to study whether or not the existence of indexation in the Brazilian economy impacts its central bank ability to implement the inflation target.

that there is an increasing relation between the robustness of the institutions of a society and the value of C. A second interpretation consists in viewing the penalty as equivalent to a performance-based compensation contract for the central banker. Indeed, consider the function $\tilde{U}(\pi^e, \pi) = W(\pi^e, \pi) + [1 - I(\pi)]C$. This function specifies that player b has a payoff increment equals to C whenever $\pi = \pi^*$. Hence, one can associate $\tilde{U}$ to the situation in which the central banker has an extra income that allows she/him to enjoy additional C payoff units by achieving the target π^*. Since $\tilde{U}(\pi^e, \pi) = U(\pi^e, \pi) + C$, for the purposes of this paper the functions U and $\tilde{U}$ are equivalent. As a consequence, the contract interpretation can indeed be applied here.[5]

I close this subsection with a brief remark on the possibility of the above modifications being carried out by the central bank instead of an external player. The results presented in this paper do not depend on which agent introduces the changes. However, there are two points to be considered if one assumes that the central bank itself is in charge of implementing the inflation targeting regime. The first is related to the magnitude of C. As shown in the next sections, the value of this parameter impacts the set of equilibrium outcomes; the higher it is, the easier is to coordinate on an equilibrium in which the inflation rate is equal to the target π^*. Hence, one has to be concerned whether or not central bank is able to choose a C as high as it can be done by an outside institution. The second is related to the selection of the target rate. However, if the central bank is in charge of choosing π^*, then it seems reasonable to assume that this agent will attempt to maximize its payoff when carrying out the task in question. In the context of this paper, where expressions (3) and (9) determinate b's payoff, this player should set $\pi^* = 0$.

2.2.2 Equilibrium

The problem of player p is still given by (5). In her/his turn, player b solves

$$\max_{\pi \in \Pi} U(\pi^e, \pi). \tag{10}$$

A *stage Nash equilibrium with inflation targeting* is a vector (π^e, π) with the properties that: (i) given π^e, π solves the problem of player b and (ii) given π, π^e solves the problem of player p.

I now turn to the task of characterizing the equilibrium set of the stage game. This requires solving problem (10). Since U is discontinuous at $\pi = \pi^*$, this cannot be done using solely the standard tools. Fortunately, there is a simple procedure that works out in this

[5] Persson and Tabellini (2000, Sec. 17.3) provide an overview of literature on the relation between inflation targets and compensation and preferences of the central banker.

context: compare the values of $U(\pi^e, f(\pi^e))$ and $U(\pi^e, \pi^*)$ (or, equivalently, the values of $W(\pi^e, \pi^*)$ and $W(\pi^e, f(\pi^e)) - C$) and select the argument $f(\pi^e)$ or π^* that yields the higher payoff. The next lemma formalize this discussion.[6]

Lemma 1 *The following five statements are true:*
(i) $\max_{\pi\in\Pi} U(\pi^e, \pi) = \max\{U(\pi^e, \pi^*), U(\pi^e, f(\pi^e))\}$*;*
(ii) $\max\{U(\pi^e, \pi^*), U(\pi^e, f(\pi^e))\} = \max\{W(\pi^e, \pi^*), W(\pi^e, f(\pi^e)) - C\}$*.*
(iii) if $U(\pi^e, \pi^*) > U(\pi^e, f(\pi^e))$*, then* π^* *is the unique solution of (10);*
(iv) if $U(\pi^e, \pi^*) < U(\pi^e, f(\pi^e))$*, then* $f(\pi^e)$ *is the unique solution of (10);*
(v) if $U(\pi^e, \pi^*) = U(\pi^e, f(\pi^e))$*, then* π^* *and* $f(\pi^e)$ *are the only two solutions of (10).*

As one should expected, the equilibrium set depends on the value of the penalty C. Suppose that this parameter is small. In this case the inflation targeting regime should not be relevant and the only Nash equilibrium should be $(\hat{\pi}, \hat{\pi})$, which is the one identified in the game without inflation target. On the other hand, if C is large, then the central bank should play π^* regardless of the value of π^e. In anticipation of this, player p should play π^*. Hence, the only Nash equilibrium should be (π^*, π^*). For intermediate values of C, both $(\hat{\pi}, \hat{\pi})$ and (π^*, π^*) are Nash equilibria.

It turns out that the above intuitive reasoning is indeed correct. However, formalizing it requires attributing precise meanings to the notions of small, large and intermediate values of C. Fortunately, this is a feasible task. Define the parameters k_1 and k_2 according to

$$k_1 = W(\pi^*, f(\pi^*)) - W(\pi^*, \pi^*) \tag{11}$$

and

$$k_2 = W(\hat{\pi}, f(\hat{\pi})) - W(\hat{\pi}, \pi^*). \tag{12}$$

Since $f(\pi^e)$ uniquely maximizes $W(\pi^e, \cdot)$ and π^* is different from $f(\pi^*)$ and $f(\hat{\pi})$, both k_1 and k_2 are positive. Moreover, k_1 is exactly the value of C for which $U(\pi^*, f(\pi^*)) = U(\pi^*, \pi^*)$. Hence, if $C = k_1$ and player p selects $\pi^e = \pi^*$, then player b will be indifferent between the actions $f(\pi^*)$ and π^*. In a similar fashion, $U(\hat{\pi}, f(\hat{\pi})) = U(\hat{\pi}, \pi^*)$ for $C = k_2$. Thus, if the last equality holds, then b can optimally play either $f(\hat{\pi})$ or π^* as a response to $\pi^e = \hat{\pi}$.

Besides playing an important role in this subsection, the next result will be used in the other parts of this paper.

Lemma 2 *The parameters* k_1 *and* k_2 *satisfy the inequality* $k_1 < k_2$*.*

[6] All proofs, as well as two examples, are available in an appendix at the end of the paper.

It is now possible to say that the penalty is small when $C < k_1$, large when $C > k_2$, and intermediate when $C \in [k_1, k_2]$. The next proposition, which concludes this subsection, formalizes the intuitive description of the equilibrium set.

Proposition 1 *If $C < k_1$, then $(\hat{\pi}, \hat{\pi})$ is the unique stage Nash equilibrium with inflation targeting. If $C > k_2$, then (π^*, π^*) is the unique stage Nash equilibrium with inflation target. If $C \in [k_1, k_2]$, then $(\hat{\pi}, \hat{\pi})$ and (π^*, π^*) are the only stage Nash equilibria with inflation targeting.*

3 The Infinitely Repeated Game

In this part of the paper I start the study of the game constituted by the infinite repetition of the stage game with inflation targeting of the previous section. In Subsection 3.1 I carry out some basic tasks, as setting up some notation and presenting a suitable equilibrium definition. In Subsection 3.2 I spell the conditions that characterize the set of equilibrium outcomes that can be supported by trigger strategies that specify reversion to one of the Nash equilibria of the stage game.

3.1 Structure and Equilibrium Definition

Denote a vector (π_t^e, π_t) of date-t actions by x_t, while ρ and δ are the respective discount factors of players p and b. As usual, these two parameters belong to the interval $(0, 1)$.[7] Given a sequence $\{x_t\}_{t=0}^{\infty}$ of actions, the payoffs of p and b from date t onwards are given, respectively, by

$$\sum_{r=t}^{\infty} \rho^{r-t} V(\pi_r^e, \pi_r) \tag{13}$$

and

$$\sum_{r=t}^{\infty} \delta^{r-t} U(\pi_r^e, \pi_r). \tag{14}$$

Let h^t be a history $(x_0, x_1, \ldots, x_t)$ of actions. At the beginning of each date t, both players know the history h^{t-1}. Player p implements an action $\pi_t^e \in \Pi$, while b implements an action $\pi_t \in \Pi$. Denote these choices by $s_t(h^{t-1})$ and $\sigma_t(h^{t-1})$. Thus, a strategy for p is a sequence $s = \{s_t\}_{t=0}^{\infty}$, while a strategy for b is a sequence $\sigma = \{\sigma_t\}_{t=0}^{\infty}$. At each date t, given

[7] It is worth to clarify two issues concerning the discounting factors. First, the results of this paper do not depend on ρ and δ being or not different from each other. Second, I did not adopt the standard notation (β) used in the macro literature to emphasize that neither ρ nor δ has to be equal to the discount factor of a typical household of an economy subjacent to the games studied here.

the history h^{t-1} and the strategy σ of player b, p selects a continuation sequence $\{s_r\}_{r=t}^{\infty}$ to maximize (13). This player takes into consideration that the actions of b evolve according to $\pi_r = \sigma_r(h^{r-1})$. In a similar fashion, given h^{t-1}, s, and the rule $\pi_r^e = s_r(h^{r-1})$, b selects a continuation sequence $\{\sigma_r\}_{r=t}^{\infty}$ to maximize (14).

An *equilibrium with inflation targeting* is a pair of strategies (s, σ) such that, at every date t and for every every history h^{t-1}, $\{s_r\}_{r=t}^{\infty}$ and $\{\sigma_r\}_{r=t}^{\infty}$ solve the problems of the corresponding players. An *equilibrium outcome with inflation targeting* is a sequence $\{x_t\}_{t=0}^{\infty}$ with the property that there is an equilibrium (s, σ) that satisfies $\pi_t^e = s_t(x_0, x_1, \ldots, x_{t-1})$ and $\pi_t = \sigma_t(x_0, x_1, \ldots, x_{t-1})$. Clearly, an equilibrium with inflation targeting is subgame perfect.

3.2 A Set of Equilibrium Outcomes

In this subsection I enunciate sufficient conditions for a sequence $\{x_t\}_{t=0}^{\infty}$ to be an equilibrium outcome with inflation targeting. As often done, I use the stage Nash equilibria of Proposition 1 as the starting point to find other equilibrium outcomes. Given that the set of stage equilibria depends on the value of C, it is necessary to consider separately the two cases $C \le k_2$ and $C > k_2$.

Suppose that $C \le k_2$. A standard argument based on trigger strategies establishes that in this context, a sequence $\{x_t\}_{t=0}^{\infty}$ is an equilibrium outcome with inflation targeting if it satisfies

$$\pi_t = \pi_t^e \tag{15}$$

and

$$\sum_{r=t}^{\infty} \delta^{r-t} U(\pi_r^e, \pi_r) \ge \max_{\pi \in \Pi} U(\pi_t^e, \pi) + \delta \frac{U(\hat{\pi}, \hat{\pi})}{1-\delta}. \tag{16}$$

If $C > k_2$, the corresponding conditions are (15) and

$$\sum_{r=t}^{\infty} \delta^{r-t} U(\pi_r^e, \pi_r) \ge \max_{\pi \in \Pi} U(\pi_t^e, \pi) + \delta \frac{U(\pi^*, \pi^*)}{1-\delta}. \tag{17}$$

This discussion is formalized in the next proposition.

Proposition 2 *If $C \le k_2$, then every sequence $\{x_t\}_{t=0}^{\infty}$ that satisfies (15) and (16) is an equilibrium outcome. If $C > k_2$, then every sequence $\{x_t\}_{t=0}^{\infty}$ that satisfies (15) and (17) is an equilibrium outcome.*

This proposition provides a characterization of the set of all equilibrium outcomes that can be supported by trigger strategies that specify reversion to one of the stage Nash equilibria. I denote this set by $\mathcal{T}(C)$, where the $\mathcal{T}$ comes from the word *trigger* and the C is spelled out to emphasize that the set depends on the the value of the penalty.

4 The Penalty and the Equilibrium Outcomes

In this section I study how the set $\mathcal{T}(C)$ is affected by changes in C, as well as under which conditions playing the target π^* at every date is an equilibrium action for both players and some other related issues.

At this point, I need to introduce some new notation. I denote a generic sequence $\{x_t\}_{t=0}^{\infty}$ by X and the sequence in which $x_t = (\pi^*, \pi^*)$ for all t by X^*. Similarly, $\hat{X}$ is the sequence with the property that $x_t = (\hat{\pi}, \hat{\pi})$ for all t . The set of all sequences in $\Pi \times \Pi$ is denoted by $\mathbb{X}$. The subset of $\mathbb{X}$ containing all sequences that have the property that $\pi_t = \pi^*$ for some t is denoted by $\mathbb{X}^*$, while $\tilde{\mathbb{X}}^*$ is the subset of $\mathbb{X}$ containing all sequences with the property that $\pi_t \neq \pi^*$ for every t (i.e., the complement of $\mathbb{X}^*$ with respect to $\mathbb{X}$). I also have to clarify a minor point concerning the usage of the symbols $\subseteq$ and $\subset$. The latter requires the inclusion to be a proper one, while the former allows for the two sets in analysis to be equal.

4.1 The Sustainability of X^*

In this subsection I provide a necessary and sufficient condition for X^* to be an element of $\mathcal{T}(C)$. Define the parameter k_0 so that

$$k_0 = (1 - \delta)k_1 - \delta[W(\pi^*, \pi^*) - W(\hat{\pi}, \hat{\pi})].$$

It is worth to point out that that $k_0 < k_1$.

Proposition 3 *The sequence X^* belongs to $\mathcal{T}(C)$ if and only if $C \geq k_0$.*

It may be the case that $k_0 \leq 0$. If that happens, then X^* will be an equilibrium outcome regardless of the value of C (including the limit case of $C = 0$). On the other hand, the implementation of X^* with trigger strategies requires C to be positive whenever $k_0 > 0$, which happens if and only if $\delta < k_1/\{k_1 + [W(\pi^*, \pi^*) - W(\hat{\pi}, \hat{\pi})]\}$.

4.2 The Impacts of Changes in C

In this part of the paper I study how the set $\mathcal{T}(C)$ evolves as C increases. I provide several results on this matter. These findings are split into three propositions.

Suppose that $C \leq k_2$. Then, many equilibrium outcomes can be supported by the strategy of reverting to $\hat{X}$ after a deviation. Indeed, let X' be any sequence such that $\pi_r \neq \pi^*$ for every date r. Now, take a generic date t. For dates larger than t the value of C is irrelevant (under the point of view of player b) when comparing the payoffs of X' and $\hat{X}$, since the penalty will be assessed at every date under both sequences. However, by deviating to π^* at date t, player b can avoid the penalty in this date. Therefore, the larger the value of C, the larger will be the incentive of player b to deviate from X'. Thus, as C increases, sequences belonging to $\tilde{\mathbb{X}}^*$ tend to be dropped out of $\mathcal{T}(C)$.

Proposition 4 *If* $C_1 < C_2 \leq k_2$, *then* $\mathcal{T}(C_2) \cap \tilde{\mathbb{X}}^* \subseteq \mathcal{T}(C_1) \cap \tilde{\mathbb{X}}^*$.

It is worth two mention two points about this proposition. First, provided that C does not become higher than k_2, if an increase in the penalty adds a sequence X to the set $\mathcal{T}(C)$, then X must belong to $\mathbb{X}^*$. Second, as illustrated by Example 1 in the Appendix, the sets $\mathcal{T}(C_2) \cap \tilde{\mathbb{X}}^*$ and $\mathcal{T}(C_1) \cap \tilde{\mathbb{X}}^*$ do not have to be equal.

As a consequence of Proposition 3, if X^* is an equilibrium outcome for a given penalty, then it will also be for a higher one. At a first glance, it may appear that this result can be extended to any sequence belonging to $\mathbb{X}^*$. However, an additional condition is required. Indeed, consider two penalties C_1 and C_2 such that $C_1 < C_2 \leq k_2$. Now, let $X \neq X^*$ be an element of $\mathcal{T}(C_1) \cap \mathbb{X}^*$ and t be any date in which $\pi_t \neq \pi^*$. It may happen that an increase in C may induce the central bank to deviate to π^* at date t to avoid incurring in a higher penalty. However, if in future dates X hits π^* sufficiently often, such a deviation will not be an optimal action for player b. Indeed, consider the inequality

$$\sum_{r=t+1}^{\infty} \delta^{r-t}[1 - I(\pi_r)] \geq I(\pi_t). \tag{18}$$

The sum in the left-hand side is simply a discounted counting of the number of times that, after date t, X hits the target π^*. Since $I(\pi^*) = 0$, this inequality will surely hold if $\pi_t = \pi^*$. If $\pi_t \neq \pi^*$, the sum will have to be no less than 1 for the condition to hold. Denote the subset of $\mathbb{X}^*$ containing all sequences that satisfies (18) for all t by $\mathbb{X}^*_\delta$. In the next proposition I show that if an element of $\mathbb{X}^*_\delta$ belongs to $\mathcal{T}(C_1)$, then it will also belong to $\mathcal{T}(C_2)$.

Proposition 5 *If* $C_1 < C_2 \leq k_2$, *then* $\mathcal{T}(C_1) \cap \mathbb{X}^*_\delta \subseteq \mathcal{T}(C_2) \cap \mathbb{X}^*_\delta$.

This proposition implies that while C remains smaller than or equal to k_2, if a sequence is dropped from to set of equilibrium outcomes as a consequence of an increase in the penalty, then it must be the case that X does not hit π^* sufficiently often as required by (18).

Moreover, Example 2 in the Appendix illustrates that one should not assume that the sets $\mathcal{T}(C_1) \cap \mathbb{X}_\delta^*$ and $\mathcal{T}(C_2) \cap \mathbb{X}_\delta^*$ are equal.

As established in Proposition 2, the conditions that characterize the set of equilibrium outcomes that can be supported by reverting to a Nash equilibria of the stage game depend on whether or not $C > k_2$. Thus, it is important to understand what happens to $\mathcal{T}(C)$ when the value of C changes from k_2 to a higher one. An intuitive analysis of the those conditions suggests that for a penalty C slightly higher than k_2, $\mathcal{T}(C)$ should be a proper subset of $\mathcal{T}(k_2)$. Indeed, for a given penalty, every sequence that satisfies (17) will also satisfy (16). Hence, exactly when the penalty shifts from k_2 to a slightly higher value, every sequence that satisfies (16) but does not satisfy (17) will be dropped from the set of equilibrium outcomes. Furthermore, for $C > k_2$, given a sequence $X \neq X^*$ that respect (17), a further increase in C will impact the inequality in question in such a way that X may fail to satisfy it. Hence, whenever $C_2 > C_1 > k_2$, $\mathcal{T}(C_2)$ should be a subset of $\mathcal{T}(C_1)$. This discussion is formalized in the next proposition.

Proposition 6 *If $C_2 > C_1 > k_2$, then $\mathcal{T}(C_2) \subseteq \mathcal{T}(C_1) \subset \mathcal{T}(k_2)$.*

An obvious implication of this last result is that if C is already larger than k_2, a further increase in the penalty will not add an element to $\mathcal{T}(C)$. Furthermore, the sets $\mathcal{T}(C_1)$ and $\mathcal{T}(C_2)$ may be equal. Indeed, as a consequence of the forthcoming Proposition 8, both $\mathcal{T}(C_1)$ and $\mathcal{T}(C_2)$ will be equal to $\{X^*\}$ whenever C_1 is sufficiently large.

4.3 The Local and Global Uniqueness of X^*

I present two main results in this subsection. First, in Proposition 7 I show that provided that $C > k_2$, a sequence different from but sufficiently close to X^* cannot be an element of $\mathcal{T}(C)$. Second, in Proposition 8 I prove that X^* is the unique element of $\mathcal{T}(C)$ whenever C is sufficiently large. Furthermore, I provide some complementary findings in Propositions 9 and 10.

To grasp the intuition behind the first of these results, assume that $C > k_2$. Thus, (π^*, π^*) is the stage Nash equilibrium used to support equilibrium outcomes of the repeated game. Now, take a sequence $X \neq X^*$. At any date t in which $\pi_t \neq \pi^*$, a deviation from π_t to π^* will allow b to avoid the penalty C at the date in question. Moreover, if X is such that π_r is sufficiently close to π^* for all $r > t$, the current gain from dodging the penalty will more than offset any conceivable losses in future dates. Therefore, X cannot belong to $\mathcal{T}(C)$.

Proposition 7 *For every $C > k_2$, there exists a positive number ε (that does not depend on δ) such that if a sequence $X \neq X^*$ has the property that $|\pi_t - \pi^*| < \varepsilon$ for all t, then $X \notin \mathcal{T}(C)$.*

Concerning the second result, its underlying reasoning is relatively simple. As C becomes larger, eventually the central bank incentives to avoid the penalty will be strong enough to prevent it from ever implementing an inflation rate different from π^*. That being said, the real problem consists in obtaining the desired result without requiring C to be needless large.

Since $\hat{X} \in \mathcal{T}(C)$ for all $C \leq k_2$, the aforementioned uniqueness requires $C > k_2$. Next, define k_3 according to

$$k_3 = W(0,0) - W(\pi^*, \pi^*). \tag{19}$$

It is established in the next proposition that $C \geq k_3$ and $C > k_2$ are sufficient conditions for X^* to be the sole element of $\mathcal{T}(C)$. It is worth to point out that it is not the case that k_3 must be larger than k_2. Indeed, it is possible to show that $k_2 = (\alpha^2 + \gamma)(\hat{\pi} - \pi^*)^2$ and $k_3 = \gamma(\pi^*)^2$. Thus,

$$k_3 > k_2 \Leftrightarrow \pi^* > \frac{(\alpha^2 + \gamma)^{0.5}}{(\alpha^2 + \gamma)^{0.5} + \gamma^{0.5}} \hat{\pi}.$$

Proposition 8 *If $C > k_2$ and $C \geq k_3$, then $\mathcal{T}(C) = \{X^*\}$.*

Next I investigate whether the converse of the last proposition is true. That is, I study whether the equality $\mathcal{T}(C) = \{X^*\}$ implies that $C > k_2$ and $C \geq k_3$. However, this is equivalent to study if $C \leq k_2$ or $C < k_3$ implies that $\mathcal{T}(C) \neq \{X^*\}$. Since $\hat{X}$ will belong to $\mathcal{T}(C)$ whenever $C \leq k_2$, it remains to consider what happens when $k_2 < C < k_3$.

Suppose that $k_2 < k_3$ and take any C in the interval (k_2, k_3). Hence, (19) implies that $W(0,0) - W(\pi^*, \pi^*) - C > 0$. Now, let π_C be any inflation rate in the interval $(0, \pi^*)$ with the property that

$$W(\pi_C, \pi_C) - W(\pi^*, \pi^*) - C > 0 \tag{20}$$

and $\mathbb{X}_C$ be the subset of all sequences in $\mathbb{X}$ such that $\pi_t^e = \pi_t$ and $\pi_t \in [0, \pi_C]$ for all t.

Proposition 9 *Suppose that $k_2 < k_3$. Hence, for every $C \in (k_2, k_3)$ there is a number $\delta_C \in (0,1)$ with the property that if $\delta \in [\delta_C, 1)$, then $\mathbb{X}_C \subseteq \mathcal{T}(C)$.*

One may wonder if the assumption on δ can be dispensed with. As shown in the next proposition, the answer is no.

Proposition 10 *Suppose that $k_2 < k_3$. Hence, for every $C \in (k_2, k_3)$ there is a number $\delta'_C \in (0,1)$ with the property that if $\delta \in (0, \delta'_C)$, then $\mathcal{T}(C) = \{X^*\}$.*

I end this subsection with two brief comments. First, Proposition 9 makes clear that k_3 is not a unnecessarily high lower bound. Second, the critical discount rates δ_C and δ'_C are uniform over $\mathbb{X}$ (i.e., they do not depend on the sequences).

4.4 Summing Up

I close this section with a summary of its findings. I have studied under which conditions X^* is an element of $\mathcal{T}(C)$. It turns out that there is a number k_0 with the property that $X^* \in \mathcal{T}(C)$ if and only if $C \geq k_0$. It may that happens that $k_0 \leq 0$; if so, then X^* will be an equilibrium outcome with inflation targeting regardless of the value of the penalty (including the limit case in which $C = 0$). If $k_0 > 0$, then X^* will be an element of $\mathcal{T}(C)$ only if $C > 0$.

I have also assessed how changes in C impact $\mathcal{T}(C)$. Recall that the characterization of this set depends on whether or not $C \leq k_2$. I showed that while this inequality holds, an increase in C will not add to $\mathcal{T}(C)$ a sequence X in which $\pi_t \neq \pi^*$ for all t and it will not drop from the set in question a sequence X that hits the target π^* sufficiently often. If $C > k_2$ then $\mathcal{T}(C)$ is proper subset of $\mathcal{T}(k_2)$ and an increase in C will not lead to an enlargement of the set $\mathcal{T}(C)$.

Finally, I have studied under which conditions X^* is the unique element of $\mathcal{T}(C)$. For such a uniqueness to happen, it is necessary that $C > k_2$. When this inequality is satisfied, then X^* has a local uniqueness property. That is, for every $C > k_2$ there is a neighborhood of X^* such that no sequence in this neighborhood will belong to $\mathcal{T}(C)$. Moreover, there is a number $k_3 > 0$ such that if $C \geq k_3$, then X^* is the only element of $\mathcal{T}(C)$.

5 Concluding Remarks

Several researchers have argued that a main feature of the inflation targeting system consists of aligning the beliefs of economic agents with the goals of the central bank and helping to stabilize the expectations about future inflation rates. Despite being popular, it appears that so far no paper has been published providing theoretical underpinnings for such a feature in the context of an intertemporal model.

This paper's main goal consists of filling that gap. Therefore, I have studied the mechanics of the inflation targeting system in an infinitely repeated game. Its stage game is a variant of the well-known Barro-Gordon model. Compared with its parent, this modified version has just two additional features: (i) there is an exogenous target for the inflation rate, and (ii) the central bank incurs a penalty (i.e., a payoff loss) whenever it fails to achieve that target. This penalty can be interpreted as a concise measure of a society's ability to induce the central bank to pursue the target in question.

I have assessed how changes in the penalty impact the set of equilibrium outcomes that can be supported by trigger strategies that specify reversion to a Nash equilibrium of the stage game. This exercise led to five major results. First, for a sufficiently large penalty, the outcome in which the inflation target is implemented at every date belongs to that set. Second, as the penalty increases, outcomes in which the target is never implemented tend to be dropped from the set in question, while outcomes in which the target is implemented with sufficiently high frequency are never dropped. Third, when the penalty is already larger than a critical value, a further increase of it will not enlarge the set in question. Fourth, if the penalty is larger than that critical value, the equilibrium outcome in which the target is implemented on every date is locally unique. Fifth, this outcome is the only element of that equilibrium set for a sufficiently large penalty. Hence, the inflation targeting system does have, from a theoretical viewpoint, the ability to help players to align their expectations and coordinate on an equilibrium in which the target is implemented.

Appendix: Proofs and Examples

Proof of Lemma 1. I start by statement (i). Let π^e be any element of Π. Then,

$$\pi \neq \pi^* \Rightarrow U(\pi^e, \pi) = W(\pi^e, \pi) - C \leq W(\pi^e, f(\pi^e)) - C \leq U(\pi^e, f(\pi^e)).$$

Hence,

$$\pi \neq \pi^* \Rightarrow U(\pi^e, \pi) \leq \max\{U(\pi^e, \pi^*), U(\pi^e, f(\pi^e))\}. \tag{21}$$

Since $U(\pi^e, \pi^*) \leq \max\{U(\pi^e, \pi^*), U(\pi^e, f(\pi^e))\}$, the inequality in (21) holds for all $\pi \in \Pi$; therefore, (i) is true. Since $U(\pi^e, \pi^*) = W(\pi^e, \pi^*)$ and

$$f(\pi^e) \neq \pi^* \Rightarrow U(\pi^e, f(\pi^e)) = W(\pi^e, f(\pi^e)) - C,$$

the equality in (ii) holds if $f(\pi^e) \neq \pi^*$. If $f(\pi^e) = \pi^*$, then both sides of the equality in question will be equal to $W(\pi^e, \pi^*)$. Hence, statement (ii) is true. The last three statements follow directly from (i). ∎

Proof of Lemma 2. Define the function $\psi(\pi^e)$ according to

$$\psi(\pi^e) = W(\pi^e, f(\pi^e)) - W(\pi^e, \pi^*). \tag{22}$$

Thus, $\psi'(\pi^e) = W_1(\pi^e, f(\pi^e)) + W_2(\pi^e, f(\pi^e))f'(\pi^e) - W_1(\pi^e, \pi^*)$. Together, the last equality and (6) imply that $\psi'(\pi^e) = W_1(\pi^e, f(\pi^e)) - W_1(\pi^e, \pi^*)$. Since $W_1(\pi^e, \pi) = 2\alpha[\alpha(\pi - \pi^e) - \mu\bar{y}]$, $\psi'(\pi^e) = 2\alpha^2[f(\pi^e) - \pi^*]$. On the other hand, (7) and (8) imply that

$$f(\pi^*) - \pi^* = \frac{\gamma}{\gamma + \alpha^2}(\hat{\pi} - \pi^*) > 0.$$

Given that f is strictly increasing, $f(\pi^e) - \pi^* \geq f(\pi^*) - \pi^* > 0$ for every $\pi^e \geq \pi^*$. Thus,

$$\psi'(\pi^e) > 0, \forall \pi^e \geq \pi^*. \tag{23}$$

Therefore, $\psi(\pi^*) < \psi(\hat{\pi})$. Since $\psi(\pi^*) = k_1$ and $\psi(\hat{\pi}) = k_2$, $k_1 < k_2$. ∎

Proof of Proposition 1. Suppose that $C < k_1$. Combine this inequality with Lemma 2 to conclude that $C < k_2$. Then, use (12) to show that $W(\hat{\pi}, \pi^*) < W(\hat{\pi}, f(\hat{\pi})) - C$, which in its turn implies that $U(\hat{\pi}, \pi^*) < U(\hat{\pi}, f(\hat{\pi}))$. Thus, if p plays $\pi^e = \hat{\pi}$, then the best action for the central bank consists in playing $\pi = f(\hat{\pi})$. Since $f(\hat{\pi}) = \hat{\pi}$, $(\hat{\pi}, \hat{\pi})$ is a stage Nash equilibrium with inflation targeting. Concerning uniqueness, optimality by player p implies that $(\hat{\pi}, \pi)$ is not an equilibrium for any $\pi \neq \hat{\pi}$. Consider now what happens when p implements an action $\pi^e = \pi' \neq \hat{\pi}$. Suppose that $\pi' = \pi^*$. Then, use the fact that $C < k_1$ and equality (11) to show that $U(\pi^*, \pi^*) < U(\pi^*, f(\pi^*))$. Hence, player b should set $\pi = f(\pi^*)$. Since $f(\pi^*) \neq \pi^*$, $\pi^e = \pi^*$ is not a best response to $\pi = f(\pi^*)$. For the case in which π' is different from both π^* and $\hat{\pi}$, recall that the optimal choice for b is (i) $\pi = \pi^*$ or (ii) $\pi = f(\pi')$. If (i) is true, then the assumption that $\pi' \neq \pi^*$ implies that such a π' does not solve the problem of player p. If (ii) holds, then the fact that $\hat{\pi}$ is the only fixed point of f implies that $\pi' \neq f(\pi')$. Again, π' cannot be an optimal strategy for p.

Now, assume that $C > k_2$. Apply Lemma 2 again to show that $C > k_1$. Hence, (11) implies that $U(\pi^*, \pi^*) > U(\pi^*, f(\pi^*))$. Thus, if p implements the action $\pi^e = \pi^*$, the central bank best response is $\pi = \pi^*$. Thus, (π^*, π^*) is a stage Nash equilibrium with inflation targeting. To show that there is no other equilibrium, observe that optimality by p implies that (π^*, π) is not an equilibrium for any $\pi \neq \pi^*$. Next, assume that p plays $\pi^e = \pi' \neq \pi^*$. If $\pi' = \hat{\pi}$, the inequality $C > k_2$ implies that $U(\hat{\pi}, \pi^*) > U(\hat{\pi}, f(\hat{\pi}))$. Thus, b should play π^* and, as a consequence, $\hat{\pi}$ is not an optimal action for player p. If π' is different from both π^* and $\hat{\pi}$, the optimal action for the central bank will be π^* or $f(\pi')$. Again, the facts that $\pi' \neq \pi^*$ and $\pi' \neq f(\pi')$ imply that such a π' is not an optimal strategy for player p.

Finally, consider the case in which $k_1 \leq C \leq k_2$. The inequality $C \geq k_1$ and (11) imply that $U(\pi^*, \pi^*) \geq U(\pi^*, f(\pi^*))$. Thus, if p plays π^*, then b can optimally play π^*. Therefore, (π^*, π^*) is a stage Nash equilibrium with inflation targeting. Similarly, the inequality $C \leq k_2$ and (12) imply that $U(\hat{\pi}, f(\hat{\pi})) \geq U(\hat{\pi}, \pi^*)$. Hence, $(\hat{\pi}, \hat{\pi})$ is also a stage Nash equilibrium with inflation targeting. To establish that there is no other equilibria, observe that optimality by player p implies that (π^*, π) is not an equilibrium for any $\pi \neq \pi^*$, while $(\hat{\pi}, \pi)$ is not for any $\pi \neq \hat{\pi}$. Moreover, if p plays $\pi' \notin \{\pi^*, \hat{\pi}\}$, then the optimal response for b is π^* or $f(\pi')$. Since $\pi' \neq \pi^*$ and $\pi' \neq f(\pi')$, π' cannot be an equilibrium strategy for p. ∎

Proof of Proposition 2. Suppose that $C \leq k_2$. Thus, $(\hat{\pi}, \hat{\pi})$ is a stage Nash equilibrium with inflation targeting. Therefore, standard trigger strategies that specify reversion to $(\hat{\pi}, \hat{\pi})$ can support any sequence $\{x_t\}_{t=0}^{\infty}$ that satisfies (15) and (16) as an equilibrium outcome of the repeated game. A similar argument can be applied to the case in which $C > k_2$.

Proof of Proposition 3. I start with the "if part". Suppose that $C \geq k_0$. If $C \geq k_1$, then (π^*, π^*) is a stage Nash equilibrium with inflation targeting and, as a consequence, X^* is an equilibrium outcome of the infinitely repeated game. If $C < k_1$, the fact that $C \geq k_0$ implies that

$$
\begin{aligned}
&C \geq (1-\delta)k_1 - \delta[W(\pi^*, \pi^*) - W(\hat{\pi}, \hat{\pi})] \Rightarrow \\
&\qquad (1-\delta)C \geq (1-\delta)k_1 - \delta\{W(\pi^*, \pi^*) - [W(\hat{\pi}, \hat{\pi}) - C]\} \Rightarrow \\
&\qquad\qquad C \geq k_1 - \frac{\delta}{1-\delta}\{U(\pi^*, \pi^*) - U(\hat{\pi}, \hat{\pi})\}.
\end{aligned}
$$

Combine the last inequality with (11) to conclude that

$$
\begin{aligned}
&C \geq W(\pi^*, f(\pi^*)) - W(\pi^*, \pi^*) - \frac{\delta}{1-\delta}\{U(\pi^*, \pi^*) - U(\hat{\pi}, \hat{\pi})\} \Rightarrow \\
&\qquad W(\pi^*, \pi^*) + \frac{\delta}{1-\delta}U(\pi^*, \pi^*) \geq W(\pi^*, f(\pi^*)) - C + \frac{\delta}{1-\delta}U(\hat{\pi}, \hat{\pi}).
\end{aligned}
$$

However, $f(\pi^*) \neq \pi^*$. Therefore,

$$
U(\pi^*, \pi^*) + \frac{\delta}{1-\delta}U(\pi^*, \pi^*) \geq U(\pi^*, f(\pi^*)) + \frac{\delta}{1-\delta}U(\hat{\pi}, \hat{\pi}). \tag{24}
$$

Now, observe that the inequality $C < k_1$ implies that $U(\pi^*, \pi^*) < U(\pi^*, f(\pi^*))$. Thus, it is possible to apply Lemma 1 to conclude that $\max_{\pi \in \Pi} U(\pi^*, \pi) = U(\pi^*, f(\pi^*))$. Combine this equality with (24) to conclude X^* satisfies (16). Hence, X^* is an equilibrium outcome with inflation targeting.

Concerning the "only if part", it suffices to show that if $C < k_0$, then $X^* \notin \mathcal{T}(C)$. A reasoning similar to the one used to obtain (24) establishes that this inequality does not hold if $C < k_0$. Thus, player b can enhance her/his payoff by deviating to $f(\pi^*)$; as a consequence, $X^* \notin \mathcal{T}(C)$. ∎

Proof of Proposition 4. Let X be any element of $\mathcal{T}(C_2) \cap \tilde{\mathbb{X}}^*$. Thus, X satisfies (15). Furthermore, for a $X \in \tilde{\mathbb{X}}^*$, inequality (16) is equivalent to

$$\sum_{r=t+1}^{\infty} \delta^{r-t}[W(\pi_r^e, \pi_r) - W(\hat{\pi}, \hat{\pi})] \geq$$
$$\max \{W(\pi_t^e, \pi^*) + C, W(\pi_t^e, f(\pi_t^e)) + [1 - I(f(\pi_t^e))]C\} - W(\pi_t^e, \pi_t).$$

Its right-hand side is non-decreasing on C, while the left-hand side does not depend on the variable in question. Given that $C_1 < C_2$ and X satisfies this condition for $C = C_2$, it must be the case that the same is true for $C = C_1$. Therefore, $X \in \mathcal{T}(C_1) \cap \tilde{\mathbb{X}}^*$. ∎

Example 1 It is not the case that the sets $\mathcal{T}(C_1) \cap \tilde{\mathbb{X}}^*$ and $\mathcal{T}(C_2) \cap \tilde{\mathbb{X}}^*$ in Proposition 4 have to be equal. Let π' be an inflation rate belonging to the interval $(\pi^*, \hat{\pi})$ and X' the sequence in which $\pi_t^e = \pi_t = \pi'$ for all t . Since $\pi^* < \pi' < \hat{\pi}$, (11), (12), (22), and (23) imply that

$$k_1 = \psi(\pi^*) < \psi(\pi') < \psi(\hat{\pi}) = k_2.$$

Furthermore, if $C > \psi(\pi')$, then $W(\pi', f(\pi')) - C < W(\pi', \pi^*)$. Hence, for any such C,

$$\max_{\pi \in \Pi}[W(\pi', \pi) - I(\pi)C] = W(\pi', \pi^*)$$

and

$$W(\pi', \pi^*) - W(\pi', \pi') + C > W(\pi', f(\pi')) - W(\pi', \pi') > 0.$$

Therefore,

$$\max_{\pi \in \Pi}[W(\pi', \pi) - I(\pi)C] - [W(\pi', \pi') - C] = W(\pi', \pi^*) - W(\pi', \pi') + C > 0. \tag{25}$$

Now, take a penalty $C_1 \in (\psi(\pi'), k_2)$ and a penalty $C_2 \in (C_1, k_2)$. Thus, the equality in (25) holds for $C = C_1$ and $C = C_2$. Next, define δ' so that

$$\frac{\delta'}{1 - \delta'}[W(\pi', \pi') - W(\hat{\pi}, \hat{\pi})] = W(\pi', \pi^*) - W(\pi', \pi') + C_1. \tag{26}$$

Combine the inequality $W(\pi', \pi') > W(\hat{\pi}, \hat{\pi})$ with (25) to conclude that δ' is well defined and lies in the interval $(0, 1)$. Hence, if $\delta = \delta'$ and $C = C_1$, then X' satisfies (16) as equality and this implies that $X' \in \mathcal{T}(C_1) \cap \tilde{\mathbb{X}}^*$. On the other hand,

$$W(\pi', \pi^*) - W(\pi', \pi') + C_1 < W(\pi', \pi^*) - W(\pi', \pi') + C_2.$$

Combine this inequality with (26) to conclude that X' does not satisfy (16) for $\delta = \delta'$ and $C = C_2$. As a consequence, $X' \notin \mathcal{T}(C_2) \cap \tilde{\mathbb{X}}^*$. ∎

Proof of Proposition 5. Take a sequence X belonging to $\mathcal{T}(C_1) \cap \mathbb{X}^*_\delta$. Clearly, X satisfies (15). Condition (16) can be written as

$$\sum_{r=t+1}^{\infty} \delta^{r-t}[W(\pi^e_r, \pi_r) - W(\hat{\pi}, \hat{\pi})] + \left\{ \sum_{r=t+1}^{\infty} \delta^{r-t}[1 - I(\pi_r)] - I(\pi_t) \right\} C \geq$$
$$\max \{W(\pi^e_t, \pi^*), W(\pi^e_t, f(\pi^e_t)) - I(f(\pi^e_t))C\} - W(\pi^e_t, \pi_t).$$

Since $X \in \mathbb{X}^*_\delta$, (18) implies that the expression inside the curly brackets is non-negative. Therefore, the left-hand side is non-decreasing in C, while the other side is non-increasing in C. Since X satisfies the above inequality for $C = C_1$, it will also satisfies for $C = C_2$. Hence, $X \in \mathcal{T}(C_2) \cap \mathbb{X}^*_\delta$. ∎

Example 2 The sets $\mathcal{T}(C_1) \cap X^*_\delta$ and $\mathcal{T}(C_2) \cap X^*_\delta$ in Proposition 5 do not need to be equal. For instance, let X' be the sequence in which $\pi^e_t = \pi_t = \hat{\pi}$ for t even and $\pi^e_t = \pi_t = \pi^*$ for t odd. Assume that $\delta = 0.62$. Therefore, $\delta/(1 - \delta^2) \cong 1.0071 > 1$. Thus, (18) holds, which implies that $X' \in X^*_\delta$. Consider the expression

$$\sum_{r=t+1}^{\infty} \delta^{r-t} \{W(\pi^e_r, \pi_r) - W(\hat{\pi}, \hat{\pi}) + [1 - I(\pi_r)]C\} \geq \qquad (27)$$
$$\max \{W(\pi^e_t, \pi^*), W(\pi^e_t, f(\pi^e_t)) - I(f(\pi^e_t))C\} - W(\pi^e_t, \pi_t) + I(\pi_t)C,$$

which is equivalent to (16). The term inside the curly brackets in the left-hand side is positive for r odd and equal to 0 for r even. Therefore, the left-hand side of the inequality is positive. Concerning the right-hand side, the inequality $C \leq k_2$ implies that it is equal to 0 for t even. Thus, (27) holds for all even dates. For t odd, the right-hand side is equal to $\max\{0, k_1 - C\}$. Next, assume that $\alpha = 0.5$, $\gamma = \mu = \bar{y} = 1$, and $\pi^* = 0.01$. Hence, $\hat{\pi} = 0.5$ and $k_1 = 0.19208$. The table below contains the results of the numerical evaluation, for odd date, of both sides of (27) for two distinct values of C.

Numerical evaluation of the left- and
the right-hand sides of expression (27) for t odd

C	left-hand side	right-hand side
0.02	0.16853	0.17208
0.10	0.21849	0.09208

Thus, $X' \notin \mathcal{T}(0.02)$ and $X' \in \mathcal{T}(0.10)$; as a consequence, $\mathcal{T}(0.02) \cap \mathbb{X}_\delta^* \neq \mathcal{T}(0.10) \cap \mathbb{X}_\delta^*$. ∎

Lemma 3 *There exists a real number $\theta > 0$ with the property that $\mathcal{T}(C) \subset \mathcal{T}(k_2)$ for every $C \in (k_2, k_2 + \theta]$.*

Proof. Define θ so that

$$\theta = \frac{\delta}{1-\delta}[W(\pi^*, \pi^*) - W(\hat{\pi}, \hat{\pi}) + k_2]. \tag{28}$$

Now, suppose that C belongs to $(k_2, k_2 + \theta]$ and let X be any element of $\mathcal{T}(C)$. Thus, (17) implies that

$$\sum_{r=t}^{\infty} \delta^{r-t}[W(\pi_r^e, \pi_r) - I(\pi_r)C] \geq \max_{\pi \in \Pi}[W(\pi_t^e, \pi) - I(\pi)C] + \delta \frac{W(\pi^*, \pi^*)}{1-\delta}.$$

As a consequence,

$$\sum_{r=t}^{\infty} \delta^{r-t}[W(\pi_r^e, \pi_r) - I(\pi_r)k_2] \geq \max_{\pi \in \Pi}[W(\pi_t^e, \pi) - I(\pi)(k_2 + \theta)] + \delta \frac{W(\pi^*, \pi^*)}{1-\delta}.$$

Combine this inequality with (28) to conclude that

$$\sum_{r=t}^{\infty} \delta^{r-t}[W(\pi_r^e, \pi_r) - I(\pi_r)k_2] \geq \max_{\pi \in \Pi}[W(\pi_t^e, \pi) - I(\pi)(k_2 + \theta)] + \theta + \delta \frac{W(\hat{\pi}, \hat{\pi}) - k_2}{1-\delta}. \tag{29}$$

On the other hand,

$$\begin{aligned} \max_{\pi \in \Pi}[W(\pi_t^e, \pi) - I(\pi)(k_2 + \theta)] + \theta &= \max_{\pi \in \Pi}\{W(\pi_t^e, \pi) - I(\pi)k_2 + [1 - I(\pi)]\theta\} \Rightarrow \\ \max_{\pi \in \Pi}[W(\pi_t^e, \pi) - I(\pi)(k_2 + \theta)] + \theta &\geq \max_{\pi \in \Pi}[W(\pi_t^e, \pi) - I(\pi)k_2]. \end{aligned}$$

Together, the last inequality and (29) imply that

$$\sum_{r=t}^{\infty} \delta^{r-t}[W(\pi_r^e, \pi_r) - I(\pi_r)k_2] \geq \max_{\pi \in \Pi}[W(\pi_t^e, \pi) - I(\pi)k_2] + \delta \frac{W(\hat{\pi}, \hat{\pi}) - k_2}{1-\delta}.$$

Therefore, X satisfies (16) when the penalty is equal to k_2. Since $X \in \mathcal{T}(C)$, X also satisfies (15). Therefore, $X \in \mathcal{T}(k_2)$, from which follows that $\mathcal{T}(C) \subseteq \mathcal{T}(k_2)$.

It remains to show that $\mathcal{T}(C) \neq \mathcal{T}(k_2)$. Clearly, $\hat{X} \in \mathcal{T}(k_2)$. Moreover, the inequality

$C > k_2$ implies that $W(\hat{\pi}, \hat{\pi}) - C < W(\hat{\pi}, \pi^*)$. Thus,

$$\frac{W(\hat{\pi}, \hat{\pi}) - C}{1-\delta} = W(\hat{\pi}, \hat{\pi}) - C + \delta \frac{W(\hat{\pi}, \hat{\pi}) - C}{1-\delta} < W(\hat{\pi}, \pi^*) + \delta \frac{W(\pi^*, \pi^*)}{1-\delta}.$$

Therefore, $\hat{X}$ does not satisfy (17). As a consequence, $\hat{X} \notin \mathcal{T}(C)$. ∎

For future reference, define the function $R(\pi^e, \pi, C)$ according to

$$R(\pi^e, \pi, C) = \max\{W(\pi^e, \pi^*), W(\pi^e, f(\pi^e)) - C\} - [W(\pi^e, \pi) - I(\pi)C]. \tag{30}$$

Therefore, for sequences in which $\pi_t^e = \pi_t$ for all t, inequality (17) is equivalent to

$$\sum_{r=t+1}^{\infty} \delta^{r-t}\{[W(\pi_r, \pi_r) - I(\pi_r)C] - W(\pi^*, \pi^*)\} \geq R(\pi_t, \pi_t, C). \tag{31}$$

Lemma 4 *Suppose that $C > k_2$. Thus, $R(\pi^*, \pi^*, C) = 0$, $R(\pi, \pi, C) > 0$ for $\pi \neq \pi^*$, and $R(\pi, \pi, C)$ is non-decreasing in C.*

Proof. Take any $C > k_2$. Since $C > k_1$, $W(\pi^*, \pi^*) > W(\pi^*, f(\pi^*)) - C$. Combine this inequality with (30) to conclude that $R(\pi^*, \pi^*, C) = 0$. Next, observe that

$$\pi \neq \pi^* \Rightarrow R(\pi, \pi, C) = \max\{W(\pi, \pi^*) - W(\pi, \pi) + C, W(\pi, f(\pi)) - W(\pi, \pi)\}. \tag{32}$$

Since $W(\hat{\pi}, \pi^*) - W(\hat{\pi}, \hat{\pi}) + C = C - k_2$, $R(\hat{\pi}, \hat{\pi}, C) \geq C - k_2 > 0$. Therefore, $R(\pi, \pi, C) > 0$ if $\pi = \hat{\pi}$. On the other hand, if π is different from both $\hat{\pi}$ and π^*, then $f(\pi) \neq \pi$ and, as consequence, $W(\pi, f(\pi)) - W(\pi, \pi) > 0$. I conclude again that $R(\pi, \pi, C) > 0$. Finally, an inspection of the equality in (32) establishes that $R(\pi, \pi, C)$ is non-decreasing in C whenever $\pi \neq \pi^*$. Since $R(\pi^*, \pi^*, C) = 0$ for all C, it must be the case that $R(\pi, \pi, C)$ is non-decreasing in C for all π. ∎

Proof of Proposition 6. Take two penalties C_1 and C_2 such that $C_2 > C_1 > k_2$. Let X be an element of $\mathcal{T}(C_2)$. Thus, X satisfies (31) for $C = C_2$. Now, observe that the left-hand side of (31) is non-increasing in C, while Lemma 4 implies that its right-hand side is non-decreasing in C. Therefore, X also satisfies (31) for $C = C_1$. Hence, $X \in \mathcal{T}(C_1)$; as a consequence, $\mathcal{T}(C_2) \subseteq \mathcal{T}(C_1)$.

It remains to show that $\mathcal{T}(C_1) \subset \mathcal{T}(k_2)$. The previous conclusion implies that $\mathcal{T}(C_1) \subseteq \mathcal{T}(C_0)$ for every $C_0 \in (k_2, C_1)$. By making C_0 sufficiently close to k_2, it is possible to apply Lemma 3 to conclude that $\mathcal{T}(C_0) \subset \mathcal{T}(k_2)$. Thus, $\mathcal{T}(C_1) \subset \mathcal{T}(k_2)$. ∎

Proof of Proposition 7. The continuity of W implies that for every $C > 0$ there exists a

$\varepsilon > 0$ such that

$$|\pi - \pi^*| < \varepsilon \Rightarrow |W(\pi, \pi) - W(\pi^*, \pi^*)| < C. \tag{33}$$

Clearly, ε does not depend on δ. Now, observe that

$$|W(\pi, \pi) - W(\pi^*, \pi^*)| < C \Rightarrow W(\pi, \pi) - C - W(\pi^*, \pi^*) < 0 \Rightarrow$$
$$[W(\pi, \pi) - I(\pi)C] - W(\pi^*, \pi^*) \leq 0. \tag{34}$$

Next, take a sequence $X \neq X^*$ with the property $|\pi_r - \pi^*| < \varepsilon$ for all r and let t be the first date in which $\pi_t \neq \pi^*$. Inequality (34) implies the sum in the left-hand side of (31) is smaller than or equal to 0. On the other hand, the fact that $\pi_t \neq \pi^*$ combined with Lemma 4 implies that $R(\pi_t, \pi_t, C) > 0$. Thus, X does not satisfy (31); hence, $X \notin \mathcal{T}(C)$. ∎

Proof of Proposition 8. Take any C that satisfies the stated conditions. Since $C > k_2$, $X^* \in \mathcal{T}(C)$. Next, take a sequence $X \neq X^*$ that satisfies (15). It suffices to show that X does not satisfy (31). Let t be the first date in which $\pi_t \neq \pi^*$. Apply Lemma 4 to conclude that $R(\pi_t, \pi_t, C) > 0$. Consider now the left-hand side of (31). If $\pi_r = \pi^*$, then the term inside the curly brackets is equal to 0. Suppose now that $\pi_r \neq \pi^*$. Since $W(0, 0) \geq W(\pi_r, \pi_r)$,

$$W(\pi_r, \pi_r) - W(\pi^*, \pi^*) \leq W(0, 0) - W(\pi^*, \pi^*) = k_3 \Rightarrow W(\pi_r, \pi_r) - W(\pi^*, \pi^*) \leq C.$$

Hence, the term inside the curly brackets is nonpositive if $\pi_r \neq \pi^*$. As a consequence, the sum in the left-hand side of (31) is smaller than or equal to 0. Since $R(\pi_t, \pi_t, C) > 0$, it follows that X does not satisfy (31). ∎

Proof of Proposition 9. Define $\bar{R}$ according to

$$\bar{R} = \max_{\pi \in \Pi} \left[\max\{W(\pi, \pi^*) - W(\pi, \pi) + C, W(\pi, f(\pi)) - W(\pi, \pi)\} \right]. \tag{35}$$

Since the objective function is continuous and Π is compact, $\bar{R}$ is well defined. Furthermore, $W(\pi^*, f(\pi^*)) - W(\pi^*, \pi^*) > 0$; thus, $\bar{R} > 0$. Next, define δ_C so that

$$\delta_C = \frac{\bar{R}}{[W(\pi_C, \pi_C) - C - W(\pi^*, \pi^*)] + \bar{R}}.$$

Combine (20) with the fact $\bar{R} > 0$ to conclude that $\delta_C \in (0, 1)$. Now, take an $X \in \mathbb{X}_C$ and let t be any date. Since $\pi_r \leq \pi_C < \pi^*$, $W(\pi_r, \pi_r) \geq W(\pi_C, \pi_C)$ and $I(\pi_r)C = C$. Hence,

$$W(\pi_r, \pi_r) - I(\pi_r)C - W(\pi^*, \pi^*) \geq W(\pi_C, \pi_C) - C - W(\pi^*, \pi^*).$$

Thus, if $\delta \in [\delta_C, 1)$, then

$$\sum_{r=t+1}^{\infty} \delta^{r-t}[W(\pi_r, \pi_r) - I(\pi_r)C - W(\pi^*, \pi^*)] \geq \frac{\delta_C}{1-\delta_C}[W(\pi_C, \pi_C) - C - W(\pi^*, \pi^*)] = \bar{R}. \quad (36)$$

Now, compare the equality in (32) with the objective function in (35). Since $\pi_t \neq \pi^*$, it must be the case that $\bar{R} \geq R(\pi_t, \pi_t, C)$. Together with (36), this last inequality implies that X satisfies (31); as a consequence, $X \in \mathcal{T}(C)$. ∎

Proof of Proposition 10. Given a penalty $C \in (k_2, k_3)$, take an ε as in the statement of Proposition 7 and let Π_ε be the set $\{\pi \in \Pi : |\pi - \pi^*| \geq \varepsilon\}$. Since $C < k_3$, (19) and (33) together imply that $0 \in \Pi_\varepsilon$. Therefore, Π_ε is not empty. Next, consider the problem of selecting $\pi \in \Pi_\varepsilon$ to minimize $R(\pi, \pi, C)$. Since $\pi^* \notin \Pi_\varepsilon$, the objective function is continuous in Π_ε; thus, the compactness of this set implies that there is a solution π_ε. Clearly, $\pi_\varepsilon \neq \pi^*$; thus, Lemma 4 implies that $R(\pi_\varepsilon, \pi_\varepsilon, C) > 0$. Now, define δ'_C according to

$$\delta'_C = \frac{R(\pi_\varepsilon, \pi_\varepsilon, C)}{(k_3 - C) + R(\pi_\varepsilon, \pi_\varepsilon, C)}.$$

Observe that $\delta'_C \in (0, 1)$. Now, take a sequence $X \neq X^*$ that satisfies (15). Apply Proposition 7 to conclude that if $|\pi_t - \pi^*| < \varepsilon$ for all t, then $X \notin \mathcal{T}(C)$. Suppose now that $|\pi_t - \pi^*| \geq \varepsilon$ for some date t. Since $W(\pi_r, \pi_r) \leq W(0, 0)$,

$$W(\pi_r, \pi_r) - C - W(\pi^*, \pi^*) \leq W(0, 0) - C - W(\pi^*, \pi^*) = k_3 - C \Rightarrow$$
$$W(\pi_r, \pi_r) - I(\pi_r)C - W(\pi^*, \pi^*) \leq k_3 - C.$$

Hence, if $\delta \in (0, \delta'_C)$, then

$$\sum_{r=t+1}^{\infty} \delta^{r-t}[W(\pi_r, \pi_r) - I(\pi_r)C - W(\pi^*, \pi^*)] < \frac{\delta'_C}{1-\delta'_C}(k_3 - C) = R(\pi_\varepsilon, \pi_\varepsilon, C).$$

Furthermore, $\pi_t \in \Pi_\varepsilon$; thus, $R(\pi_\varepsilon, \pi_\varepsilon, C) \leq R(\pi_t, \pi_t, C)$. Therefore, X does not satisfy (31) and this implies that $X \notin \mathcal{T}(C)$. ∎

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