# The Effects of Public Debt with Labor-Market Frictions*

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July 27, 2023

**Abstract**

This paper studies the impact of public debt on vacancy creation, wages, and welfare using a search-and-matching model with heterogeneous agents, uninsurable idiosyncratic shocks, and wealth inequality. The trade-off between the opportunity cost of smoothing consumption and firms incentive to post vacancies in the economy shapes the public debt choice. We calibrate a general equilibrium model to match the key moments concerning unemployment, wealth and wage distributions, and the share of public debt over output to United States. First, we show that a substantial public debt hike, from 0% to 390%, has a twofold effect on labor-market outcomes. Via bilateral wage bargaining, wages decreases almost 20% while the unemployment rate increases by almost one percentage point due to the effect the interest rate hike has on firms' vacancy posting decisions. Next, we show that our welfare measure displays a laffer-curve shape, reaching the top at 240 percent of debt over output. Mainly, such a result is driven by the wealthiest agents in this economy, who are benefited from the interest rate hike while the non-wealthiest are harmed due to non-employment increase. In our model, we find that bilateral wage bargaining is more important in accommodating the public debt increase compared to labor-market frictions, which dampens firms decision on vacancy posting. When we shut wealth inequality down, we find that the unemployment rate becomes much more responsive to the interest rate hike, increasing almost 3.5 percentage points when debt over output goes from 0% to 390%, with the optimal level of public debt being 0 because of the perverse effect that public debt has on vacancy creation.

**Keywords: Optimal debt, incomplete markets, search models, labor-market**.
*J.E.L. codes: D52, E26, J24, J64, J65, O17.*

* VERY PRELIMINARY. We thank Diogo Guillen, Ellen McGrattan, Felipe Alves, Ivan Prenhaca, Jie Duan, Marco Bonomo, Marcos Sonnervig, Miguel Bandeira, Regis Barnichon, Ronald Wolthoff, Thomas Drechsel, Yvan Becard as well as participants in Asian Meeting of Econometric Scoeity 2023, Insper Brown Bag and Theories and Methods for Macro (T2M) for useful comments. We especially thank our advisor, Marcelo Santos, for all his advice and guidance in this project. All remaining errors are our responsibility. Bloise gratefully acknowledges financial support from Fapesp 2020/00371-2 and 2021/11265-1. **Caio Bloise**: Department of Economics, Insper Institute of Education and Research, Rua Quata, 300, São Paulo, SP, Brazil 04546-042. **Vitor Fancio**: Department of Economics, Insper Institute of Education and Research, Rua Quata, 300, São Paulo, SP, Brazil 04546-042.

# 1 Introduction

Public debt has been rising for many decades in developed countries to accommodate social insurance policies, and it is a useful tool that governments use to dampen shocks in bad states of nature (Yared (2019)).[1] In the Great Recession, the United States' public debt increased by roughly 26 percentage points, reaching almost 90% of output due to fiscal stimulus. During the Covid-19 pandemic crisis, it implemented the largest increase in unemployment insurance (UI) in history, spending nearly 500 billion dollars in the economy, as described by Ganong et al. (2022).

Public debt as a policy tool poses some challenges since fiscal coordination depends on many factors (how to combine public debt and taxes) and can affect heterogeneous agents differently, as noticed by Aiyagari and McGrattan (1998) and Flodén (2001). However, the effects of this increase in public debt are persistent in the long-run and provide many challenges besides this fiscal coordination. Little is known, for instance, about the effects of public debt on labor-market equilibrium, particularly on unemployment, wages, and welfare.

To illustrate the relation of public debt in the labor-market, we run a cross-country regression with data from International Labour Organization (*ILO*) and World Development Indicators (*WDI*) for 64 countries from 2010 to 2019. We find that, on average, a public debt expansion increases the unemployment rate. The result is robust to year and country-fixed effects. We show in Figure 1 the cross-country correlation between the average debt-to-GDP ratio and the average unemployment rate from 2010 to 2019. Although we find a positive and significant correlation between unemployment and debt, we lack an explanation of how the labor-market adjustment occurs to push the unemployment rate up in response to a public debt hike. To fill this gap, we study the effects of public debt on equilibrium labor-market allocations and quantify the optimal public debt in the presence of labor-market frictions. To do that, we combine two key ingredients.[2]

First, we consider a search-and-matching model as Diamond (1982), Mortensen (1982), and Pissarides (1988) (henceforth, DMP). We then study how public debt influences vacancy creation and wage bargaining between firms and workers. In this environment, employment opportunities are endogenous and depend on aggregate labor-market conditions. Second, we depart from the standard DMP setting with risk neutrality by combining the search-and-matching framework with incomplete markets as in Krusell et al.

[1] A pattern already recognized in Ball and Mankiw (1995).

[2] We use the terms debt per output, public debt, government borrowing, and debt-to-GDP interchangeably in this paper.

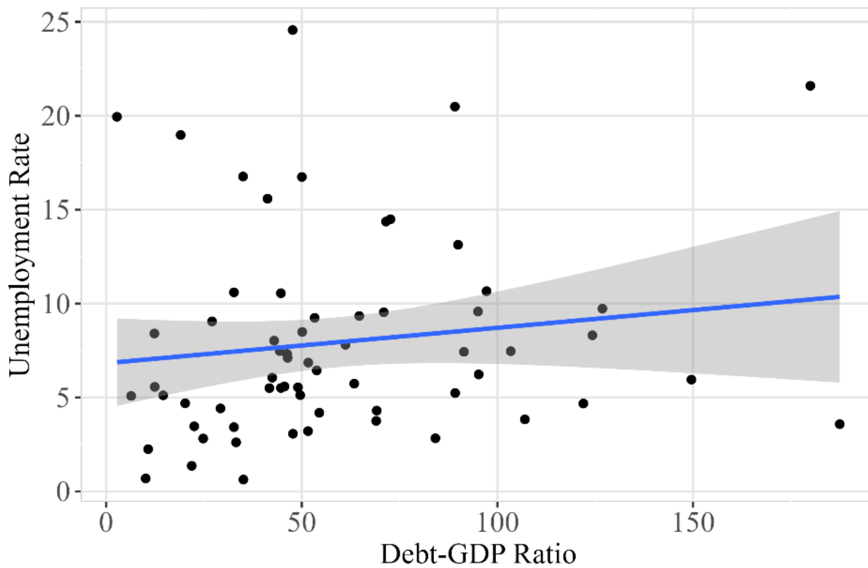


Figure 1: **Debt-to-GDP ratio and unemployment rate:** Each observation represent the average debt-to-GDP ratio and unemployment rate from 2010 to 2019 for a particular country in the dataset.

(2010) and Setty and Yedid-Levi (2021). Our motivation for doing this is to study the effect of public debt on labor-markets when workers care about consumption smoothing. In addition, wealth affects workers' reservation wages by allowing for diminishing marginal utility in consumption and imperfect insurance. Thus, cross-sectional distributions of wealth play a critical role in determining the aggregate labor responses to changes in the government budget constraint through public debt.

We combine these margins in an incomplete-markets model as Bewley (1986), Huggett (1993), and Aiyagari (1994) with labor-market frictions. The lifespan is uncertain, and agents survive from one period to the next with a given probability. Individuals may resort to self-insurance to protect themselves against the uncertainty in labor income, and thus savings will be, to some extent, motivated by precautionary reasons. As in the standard DMP model, firms enter by posting vacancies and match with workers bilaterally, with match probabilities given by an aggregate matching function. In addition, firms can direct their search by posting vacancies for agents of a given level of education.

The combination of such elements brings tension between the incomplete markets and search-and-matching frictions in shaping the optimal public debt. On the one hand, a public debt expansion increases the return on assets, thus making it less costly for individuals to postpone consumption. This is the well-known effect of liquidity that an expansion of public debt brings to agents in an incomplete markets economy. On the other hand, this effect occurs through a hike in the interest rate due to the crowding-out effect of capital, which reduces the net present profit of posting vacancies in the economy. Hence, the unemployment rate increases due to higher debt in the economy. To understand which effect excel, is crucial to have a good calibration concerning the labor-market variables and wealth inequality.

We use microdata from the U.S. to inform the model and proceed with a quantitative analysis. The model matches key facts concerning unemployment, wage and wealth distributions, as well as the level of public debt in this economy. The calibrated model allows us to conduct several counterfactual experiments to shed light on the impacts of the public debt increase or decrease on labor-market outcomes, inequality, and welfare.

We find that a public debt increase has a twofold effect on labor-market outcomes. First, it pushes the unemployment rate up and output down through the evasion of capital in the economy, i.e., the crowding-out effect. Capital evasion has consequences on the interest rate, which goes up when public debt increases, reducing the net present value of firms' profit and leading to less vacancy posting in the economy. Although capital evasion affects firms' decision to post vacancies, firms also share such losses with employed workers by decreasing wages, being this the second effect. We find that the aggregate average wage declines almost 20% when debt increases from 0% to 390%. Still, the former effect overcomes the latter and this labor price fall is not enough to compensate for the profit decline, leading to a decrease in vacancy postings. When the public debt increases from 0% to 390%, the unemployment rate goes to 5.97% from 4.90%, and output decreases to 25.06%.[3]

Since agents are ex-ante heterogeneous concerning their ability, which can be interpreted as pre-market skills such as innate ability or obtained through education, we see how heterogeneous is the public debt increase in unemployment and wages. We find that, despite the similar wage decline for the four educational groups, the increase in the unemployment rate is heterogeneous, being worst for individuals that have an education lower than high school (+1.55 p.p.) compared to individuals that have college (+0.5 p.p.).[4] We go further to understand such different results among educational groups and analyze how the expected benefit of vacancy creation is affected. We find that the expected real-

[3]Here, we do not consider any measure of search intensity. Hence, from the workers' perspective, her willingness to find a job will not affect the job-finding rate. Our model only has the supply channel, i.e., the firms' decision to post vacancies.

[4]We divide the heterogeneity in ex-ante education into four groups, as in Setty and Yedid-Levi (2021): lower than high-school (LHS), high-school (HS), above high-school but not completed college (AHS) and college (C).

ization of a vacancy falls more for those with lower education (-9.06%) compared to those with college (-5.85%), which explains why the unemployment rate is higher for the first educational group.

We also ask what is the optimal public debt in this environment with search-and-matching frictions in the labor market. Our welfare measure, the consumption equivalent variation (CEV), increases by 2.97% when the debt over GDP reaches 240% of GDP but, after that, starts to decrease. Hence, the welfare curve per debt/output ratio displays a laffer-curve shape, reaching the top at a very high level of public debt and declining afterward. The welfare measures favor the policies that induce better equity among agents, and the liquidity rise in the economy reaches such effect. The uncertainty welfare component increased by 8.57% while the level one declined by 5.16%. When we decompose by labor market status, we see that employed workers benefited from such an increase (3.57%) while unemployed ones suffered a significant welfare loss (-10.71%).

When we analyze the welfare gains for ex-ante innate ability, the disagreement among different groups about the social planner policy of public debt arises. Individuals that do not have college are hurt by such large debt increases, while those with college experience a welfare increase. Workers with lower than high-school, high-school, and above high-school education experience a welfare decline of 5.53%, 7.03%, and 7.89%, respectively, while those with college see their welfare increase by 9.33%. Mainly, those individuals with college accumulate more assets to smooth consumption during their lifetime and are benefiting from the hike in the interest rate, while the other three educational subgroups do not have such capacity for precautionary savings, relying more on their wages, which declined abruptly from the capital crowding-out effect.

We conduct a decomposition exercise to quantify the importance of the different ingredients of our model in shaping the optimal public debt policy. Since 240% of output is too high, we want to see how the labor-market frictions components act to accommodate such debt increase. We show that wage rate acts to dampen capital evasion in this economy. Since wage is defined as an outcome of a bilateral bargaining problem, firms pass through wages their losses instead of abruptly reducing the vacancy posting in the economy. When the labor-market tightness is kept constant, which indicates that the job-finding probability does not change, wages go down by 10%. On the other hand, when wages are constant, and the planner allows to labor-market adjustment, unemployment sharply goes to 25.8%, leading to a welfare loss of 24.8%. Hence, taking into account two major components of search-and-matching, vacancy creation and wages, we see that the latter has a more important role in accommodating such debt increases than the former.

When the interest rate is at the same level as the benchmark situation but with a public debt much higher, the incentives to save sharply decline as well as the aggregate capital, but not affect as many labor-market outcomes as in the other exercises, reassuring the interest rate as the main channel through vacancy creation in this economy.

As our traditional welfare measure faces a trade-off between equity and efficiency, we conduct an optimal public debt exercise using a different utilitarian welfare function that the only planner concern is with efficiency. We find that shutting down public debt is optimal in such an efficiency welfare sense. The interest rate increase due to the crowding out effect of capital decline vacancy posting in the economy, leading to less matches and driving output down. When we decompose the results across types of workers in this economy, we see no disagreement among them, with employed, unemployed, high and low-productive types asking for less government borrowing in the economy.

Finally, we re-calibrate the model to eliminate all ex-ante education and, thereby, the wealth inequality in the model.[5] According to the exercise, the response of aggregate outcomes is more elastic than in the benchmark calibration. The unemployment rate increases by 3.5 percentage points, reaching roughly 11% when public debt goes from 0% to 390%. Firms are more responsive to capital evasion than in the model with ex-ante heterogeneity, leading to a large fall in output due to the inability to match formation with workers. Since most individuals do not have the same ability to accumulate assets to smooth consumption during their lifetime, the firms' decision to hold vacancies harm them. In particular, we find that the efficiency concern dominates in the welfare sense, which drives our welfare CEV to the bottom, among unemployed, employed, high, and low-productive individuals.

## 1.1 Related Literature

We depart from the classical Aiyagari and McGrattan (1998), one of the first to see the optimal public debt in a standard incomplete markets model. They find that optimal debt is the same that resembles the U.S. at that time. Flodén (2001) extends that framework asking what is the optimal combination of public debt and transfers. He finds the same result as Aiyagari and McGrattan (1998) only when transfers are low. Also, despite the low utilitarian welfare gains of modifying public debt, there are substantial gains and losses when you decompose it considering redistributional and level effects. Vogel (2014) analy-

[5] Although we eliminate ex-ante individuals' ability, the model still generates ex-post asset accumulation and wealth inequality, but not as pronounced as in the benchmark calibration.

ses the optimal combination of transfers and taxation with public debt in an overlapping generations model. He finds that government must have public savings instead of debt, which benefits those individuals that do not depend on asset returns. In a more stylized model, Peterman and Sager (2022) explore that, when considering a life-cycle model, the optimal policy is to save and not to borrow, i.e., to have public savings instead of public debt. The authors find that when the planner pushes to public savings instead of debt, the reduction on interest rate tilts consumption profile up and hours down due to lower incentives agents have to save.

Taking into consideration transitional dynamics, Desbonnet and Weitzenblum (2012) extend Flodén (2001) paper to see the public debt effects in the short-run, mainly concerned about the impact of it on those individuals that are constrained. Rohrs and Winter (2017) add wealth inequality to the transitional dynamics of debt, showing that even reducing it can be costly due to distributional conflict and to whom such tax burden arises in the transition from one steady state to the other.

Our paper is related to the literature that tries to see the effects of fiscal policy on labor-markets. Lama and Medina (2019) analyzes how austerity affects aggregate outcomes, such as unemployment and wages, in a business cycle model with labor-market frictions. Yum (2018) abstracts from labor-market frictions but analyzes how transfers and taxation affect aggregate labor supply in an incomplete markets model.

We also contribute to the literature that combines fiscal policy and taxation - Chamley (1986), Aiyagari (1995) - and more recently, Açikgoz et al. (2018) and Dyrda and Pedroni (2022), adding labor-market frictions as an important outcome of fiscal policy. Of course, we contribute to the literature that combines labor-market frictions with incomplete markets, as described by Krusell et al. (2010), Eeckhout and Sepahsalari (2021), Setty and Yedid-Levi (2021), and Santos and Rauh (2022), analyzing how public debt affects the firm's decisions of posting vacancies. Our paper is also related with Kehoe et al. (2019), which inspects how labor market adjusts when credit tightness occurs. They find that credit tightness increases shadow price, having impact on the cost of posting vacancies in the economy, in a similar firms behavior that occurs in our paper through interest rate. To the best of our knowledge, this is the first paper to study the role of public debt in a machinery that displays a frictional environment characterized by search-and-matching, calibrated to generate a plausible wage and wealth inequality.

This paper is organized as follows. In Section 2, we present the empirical exercise. In Section 3, we present the model economy. In Section 4 we describe the parameterization and calibration of the benchmark economy as well how labor-market frictions are estimated. Section 5 discusses the property of benchmark economy while Section 6 present the main findings of our quantitative experiments. Section 7 concludes.

# 2 Evidence

We explore the relationship between the unemployment rate and the debt-to-GDP ratio. We use data from the International Labour Organization (*ILO*) and the World Development Indicators (*WDI*). ILO provides information about the unemployment rate. WDI includes information about the debt-to-GDP. We consider the ILO dataset, which comprises surveys that include agents that are 15 years old or older. Debt-to-GDP ratio corresponds to central government debt per GDP. We focus the analysis on the period from 2010 to 2019 based on the data availability.

We report in Table 1 the regression results of the unemployment rate on the debt-to-GDP ratio for different specifications. Column (1) shows the result when we do not consider any control. From Column (2) to Column (4), we consider year-fixed effect, country-fixed effect, and both year and country-fixed effects, respectively. The relation between debt-to-GDP and the unemployment rate is positive in all specifications and only statistically insignificant in the third column. Since the ILO dataset may not provide precise comparability between the unemployment rate measures, Column (4) is our preferred specification by including country-fixed effect. In this case, a one percentage point increase in the debt-to-GDP ratio leads to a rise of 0.0487 percentage points in the unemployment rate.

Table 1: Regression Coefficients of Unemployment Rate on Debt-to-GDP

| | Unemployment rate | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Debt-to-GDP | 0.022*** | 0.023** | 0.027 | 0.0487*** |
| Country fixed effect | No | No | Yes | Yes |
| Year fixed effect | No | Yes | No | Yes |
| Observations | 469 | 469 | 469 | 469 |
| Number of countries | 64 | 64 | 64 | 64 |
| Adjusted R-squared | 0.029 | 0.034 | 0.020 | 0.869 |

Notes: Regression coefficients from regressions of the unemployment rate on the debt-to-GDP ratio. Constant not reported. *** indicates significance at 0.1% level, ** indicates significance at 1% level, and * indicates significance at 5% level.

## 3 The Environment

**Preferences**

Time is discrete and runs forever. There is a unity measure of agents in this economy. Workers are risk-averse and firms are risk-neutral. Agents have preferences over random streams of consumption, $c_t$, according to

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta_e^t\left(\prod_{s=0}^{t}\psi\right)u(c_t)\right] \tag{1}$$

where the utility function takes the following functional form.

$$u(c_t) = \frac{c_t^{1-\mu}}{1-\mu} \tag{2}$$

We also denote the variable $\psi$ as the survival parameter, indicating that new individuals may born and enter the economy unemployed, with zero assets and no bequest. The parameter $\beta_e$ represents the discount factor for each ex-ante educational level.

**Asset markets**

Consumers face uninsurable idiosyncratic shocks. Because markets are incomplete, they cannot perfectly smooth consumption. Thus, savings may be precautionary and allow partial insurance against shocks. Agents can accumulate three kinds of tangible assets: physical capital, $k$, which is used as input for production, bonds $b$, and equity $x$, which is a claim for the aggregate profit. Let $r$ be the return to capital and $d$ be the dividend paid to equity holders. The total amount of equities is normalized to one. There is no aggregate risk, so the equity price remains in equilibrium. The equity price $p$ has to satisfy a standard no-arbitrage condition, which implies that the returns on holding capital and equity are equal:

$$p = \frac{d+p}{1+r-\gamma} \tag{3}$$

where $\gamma$ is the depreciation of capital.

Since capital, bonds and equity are riskless, provide the same return, and therefore are the same from the consumer's viewpoint, we do not have to keep track of the asset composition of the consumers. In the following, we define total financial resources:

$$a = (1+r-\gamma)k + (p+d)x + (1+r)b \tag{4}$$

We use $a$ as the state variable for the consumer. Since we are in an incomplete market framework, the individual can self-insure with $a$, although we do not allow borrowing ($\underline{a}$

= 0). Firms seek to maximize the expected value of the sum profits streams, discounted by $\varphi = \frac{1}{1+r}$. We follow Krueger et al. (2017) and Setty and Yedid-Levi (2021) in assuming that the deceased's assets pay extra returns to the survivor, as the effective discount is $\varphi\psi$.

**Labor-Markets Frictions**

Firms are ex-ante identical and small in the sense that the match is one firm - one worker.

The encounter between a firm and a worker is via search, where this search is random, i.e., does not depend on the specific characteristics of firms/workers. We assume the unemployed one always seeks a job. If not successful, the worker will stay unemployed. As usual in this framework, the technology that puts together firm and worker and results on the number of jobs match per unit of time is given by $M(u, v_e)$, which is assumed to be a constant-return-to-scale function, where $u$ is the number of unemployed workers in this economy and $v_e$ the number of vacancies, where $e$ denotes the educational level.[6] Denoting $\theta_e = \dfrac{v_e}{u}$ as the tightness of the labor-market structure, the job seekers meet the prospective employers with probability $m(1, \theta_e) = \chi_e(\theta_e)^{1-\zeta_e}$ while the other way around occurs with probability $q(\theta_e^{-1}, 1) = \chi_e(\theta_e)^{-\zeta_e}$. The job finding rate is given by $m(1, \theta_e)$ .[7]

There is also the possibility of exogenous separation, given by $\delta_e$, where flow upon unemployment will depend on the individual educational level.

**Production Technology**

The production of the firm occurs when a match is formed in a decentralized way and is given by $f(\tilde{k}(e,z)) = \tilde{k}(e,z)^{\alpha}$, where $\tilde{k}(e,z) = \frac{k(e,z)}{1-u(e,z)}$, capital per labor ratio, $\alpha$ is the share parameter of capital and $u(e,z)$ is the share of unemployed workers by educational level and idiosyncratic shock. Capital depreciates at rate $\gamma$. The aggregate level of capital will be determined depending on the aggregate asset level accumulated by the individuals, i.e., by market clearing. Since we are assuming a frictionless capital market, all firms pay the same rental rate $r$, which will be set as the first-order condition of the production function. Hence, this rental rate is endogenous in the model.

**Idiosyncratic productivity shock**: An employed worker's idiosyncratic productivity shock evolves according to the AR(1) process:

$$\log z' = \rho \log z + \epsilon' \tag{5}$$

[6]Here, we are following most of the search-and-matching literature assuming that $M(u, v_e) = \chi_e u^{\zeta_e} v_e^{1-\zeta_e}$, i.e., a Cobb-Douglas matching function.

[7]Here, we denote the probability of finding a job and the job finding rate interchangeably due to our model calibration. See the calibration section below.

Take note that $\epsilon \sim N(0, \sigma_\epsilon^2)$. Here, $\rho$ denotes the persistence of innovation. We assume that a new $\epsilon$ is drawn every employment period and when transitioning from unemployment to employment. A worker who transits out of a job maintains her lever of productivity $z$ throughout the unemployment spell, while newborns draw initial productivity from the invariant distribution. The continuous-valued autoregressive process is usually replaced by a discrete state-space Markov chain $G(z, z')$ using the approximation method proposed by Rouwenhorst (1995).

**Government Policy**

The government levies taxes on labor, capital income, and consumption. We assume that consumption is taxed at a flat rate $\tau_c$. In addition, the government collects a non-linear and progressive tax schedule on labor income plus capital income according to the tax function suggested by Bénabou (2002) and more recently used by Heathcote et al. (2017).

$$T(y) = \max\{y - \lambda y^{1-\tau}, 0\} \tag{6}$$

where $y$ is the individual's income and $T(y)$ is tax paid. Parameters $\lambda$ and $\tau$ regulate the level and progressivity of taxation, respectively. For instance, if $\tau = 0$ then the tax rate is flat at 1 - $\lambda$, and the system is progressive $\tau > 0$.

Government revenue is used to finance a stream of exogenously given government consumption, $G_c$, and a public debt $B$. In addition, there is only one government-run program in the economy. Newly unemployed workers are eligible to receive unemployment insurance (UI) benefits. The government provides the benefit $b(e, z)$. The benefit value is calculated as the $\max\{\varrho \times w(e, a, z), \bar{b}\}$, where $\bar{b}$ represents the cap of the benefit, following the U.S. unemployment insurance legislation on states. The aggregate UI expenses is given by $UI$

Each period, the government has a debt balance of $B$ and saves or borrows (denoted $B'$) at the real interest rate $r$. If the government borrows, then $B' < 0$ and the government repays $rB'$ next period. If the government saves, then $B' > 0$ and the government collects asset income $rB'$ next period. The planner also issues tax on consumption $\tau_c$ to help finance expenditures in this economy. The resulting government budget constraint is[8]

$$G_c + UI + (1 + r)B = B' + T(y) + \tau_c C \tag{7}$$

[8]We describe the effects of public saving as an example. In our counterfactuals, we do not allow to have public savings (i.e., $B' < 0$).

## 3.1 Bellman Equations

To formulate the workers' decision recursively, let's denote by $V_s$ the value functions of workers, $s$ indicates if the worker is unemployed ($U$) or employed ($E$). Hence, $V_U(e,a,z)$ denotes the value function of an unemployed agent with ex-ante education level $e$, uninsurable idiosyncratic shock $z$ and who owns asset $a$. Idiosyncratic productivity shock is given by the transition matrix $G(z,z')$. To save notation, we substitute this stochastic matrix by the condition expectation $\mathbb{E}$ on $z$. As standard in the dynamic programming approach, we denote prime ($\prime$) as an indicator of the next period.

The value function of an unemployed worker can be stated as follows:

$$V_U(e,a,z) = \underset{c,a'}{max}\Big\{u(c) + \beta_e\psi\Big[m(\theta_e)\,\mathbb{E}\Big[\max\{V_E(e,a',z'),V_U(e,a',z')\}\Big|z\Big] + (1-m(\theta_e))V_U(e,a',z)\Big]\Big\} \tag{8}$$

Where is subject to the following budget constraint:

$$(1+\tau_c)c + a' = (1+r)a + b(e,z) + T(b(e,z)+ra) \tag{9}$$

The value function of an employed worker is the following:

$$V_E(e,a,z) = \underset{c,a'}{max}\Big\{u(c) + \beta_e\psi\Big[(1-\delta_e)\,\mathbb{E}\Big[\max\{V_E(e,a',z'),V_U(e,a',z')\}\Big|z\Big] + \delta_e V_U(e,a',z)\Big]\Big\} \tag{10}$$

Where is subject to the following budget constraint:

$$(1+\tau_c)c + a' = (1+r)a + w(e,a,z) + T(w(e,a,z)+ra) \tag{11}$$

**Firms**

On the other side of the market, there is a continuum of risk-neutral firms. First, starting from unfilled vacancies. Let's denote $\kappa_e$ as the flow cost of posting a vacancy. The value function for an unfilled vacancy is given by:

$$V_e^{unf} = -\kappa_e + \varphi\psi q(\theta_e)\,\mathbb{E}\Big[\max\{J(e,a',z'),0\}\Big|z\Big]\frac{\mathbb{S}_U(e,a,z)}{u} \tag{12}$$

Where $\mathbb{S}_U(e,a,z)$ is the measure of non-employed workers at the beginning of the period. A firm with a vacancy does not know what worker type it will meet next period. The firm does know, nonetheless, the distribution of worker types among the unemployed. Take $\frac{\mathbb{S}_U(e,a,z)}{u}$ as the conditional density function. As standard in search and matching literature, each firm has one job only, and profit maximization is equivalent to a zero-profit condition for firm entry.

The free entry conditions above pin down the labor-market tightness for any given educational level ($\theta_e$). No vacancies are created in submarket $e$ if the value of expected profits conditional on matching is sufficiently low in that submarket.

$J$ denotes the firms' value functions of filled jobs. Let's characterize the value functions of filled vacancies. In particular, even though firms do not accumulate assets for themselves, the worker may collect, which affects the wage this worker receives from the firm through Nash Bargaining. For this reason, the present-discounted value of expected profit from an occupied job depends on the worker's asset level.

Hence, the value of a filled vacancy is given by:

$$J(e,a,z) = \max_{\tilde{k}} \left\{ \phi_e z f(\tilde{k}(e,z)) - r\tilde{k}(e,z) - w(e,a,z) + \varphi\psi(1-\delta_e)\,\mathbb{E}\left[ \max\{J(e,a',z'),0\} \middle| z \right] \right\} \tag{13}$$

## 3.2 Nash Bargaining

Wages are determined, period by period and without commitment, using a Nash Bargaining within each worker-firm pair. The worker's outside option depends on the ex-ante education level $e$, level of assets $a$, and uninsurable idiosyncratic shock $z$, which governs the individual level of productivity. The wage for someone employed is:

$$w(e,a,z) = \arg\max_{w} \left\{ [V_E(e,a,z) - V_U(e,a,z)]^{\eta}\, J(e,a,z)^{1-\eta} \right\} \tag{14}$$

Similarly to Krusell et al. (2010) and Setty and Yedid-Levi (2021), the Nash solution generates, on average, a wage that is increasing in a worker's assets, reflecting that being unemployed is less painful for a worker with a large quantity of assets.

### 3.3 Welfare measure

The government is a benevolent Ramsey planner that fully commits to fiscal policy. The planner maximizes social welfare by choosing a feasible budget level of transfers subject to allocations being an equilibrium. We consider a Utilitarian social welfare criterion that evaluates the ex-ante expected utility across all agents in the economy:

$$\mathbb{U} = \int \mathbb{E}\left[\sum_{t=0}^{\infty} \beta_e^t \left(\prod_{s=0}^{t} \psi\right) u(c_t)\right] d\mathbb{S}(e, a, z) \tag{15}$$

Note that $\mathbb{E}$ denotes the unconditional expectation operator concerning all possible permanent types and histories. This welfare criterion considers the policy maker's concern for redistribution and insurance against idiosyncratic shocks, as well as distortions that changes in the public debt level impose on job creation.

We compute the welfare change, $\nu$, as the amount of consumption one would have to remove or add to make the utilitarian welfare criterion equal between the public debt benchmark and some alternative policy. The welfare variation (CEV, consumption equivalent variation) is calculated as follows: Let $\mathbb{U}(s)^0$ denote the utilitarian welfare function to an agent of the economy who enters in the economy with state $s$ under the benchmark public debt. The CEV measure can be calculated as follows:

$$CEV = \left[\frac{\mathbb{U}(s)^1}{\mathbb{U}(s)^0}\right]^{\frac{1}{1-\mu}} - 1 \tag{16}$$

where $\mathbb{U}(s)^1$ denotes the utilitarian welfare function to an agent of the economy who enters the economy with state $s$ under the counterfactual public debt.

**CEV Decomposition**: Aiming at understanding the source of welfare gains, we decompose the CEV in variations due to improved insurance and those due to more efficient use of aggregate resources. The main mechanism underlying the improved insurance is through uncertainty reduction, while the efficient use of aggregate resources can be understood as how individuals appropriate of the level of consumption in the counterfactual economy. Let $C^0$ denote the average consumption at the benchmark and $C^1$ the same but at the counterfactual public debt system. We may, in this case, implicitly define $CEV_{lev}$ through:

$$CEV_{lev} = \left[\frac{C^1}{C^0}\right] - 1 \tag{17}$$

Using the same consumption measures as described above, implicitly define $\rho_0$ and $\rho_1$, through:

$$\rho_0 = 1 - \frac{(1-\mu)[\mathbb{U}(s)^0]^{\frac{1}{1-\mu}}}{C^0} \tag{18}$$

$$\rho_1 = 1 - \frac{(1-\mu)[\mathbb{U}(s)^1]^{\frac{1}{1-\mu}}}{C^1} \tag{19}$$

In both expressions, $\mathbb{E}$ denotes the unconditional expectation operator for all possible permanent types and histories. Then,

$$CEV_{unc} = \frac{1-\rho_1}{1-\rho_0} - 1 \tag{20}$$

Hence, the two components approximately sum to the total welfare effect, as also established by Heathcote et al. (2008), i.e, $CEV \approx CEV_{lev} + CEV_{unc}$.

## 3.4 Stationary Equilibrium

A stationary equilibrium is a list of value functions ($V_E(e,a,z)$, $V_U(e,a,z)$, $J(e,a,z)$), decision rules for asset holding ($a_E(e,a,z)$, $a_U(e,a,z)$), a wage function $w(e,a,z)$, a population distribution across possible individuals' states $\mathbb{S}_U(e,a,z)$ and $\mathbb{S}_E(e,a,z)$, a value of type-specific labor-market tightness $\theta_e$, and a tax rate $\tau$ such that:

(i) Given the aggregate variable $\theta_e$, the wage function $w(e,a,z)$ and the policy parameters, households solve the maximization problem in Equation 8 and Equation 10.

(ii) Given the wage schedule $w(e,a,z)$ and the workers asset-holding decision rule, $a_E(e,a,z)$, the value of a filled vacancy $J(e,a,z)$ satisfies Equation 13.

(iii) Given the filled vacancy $J(e,a,z)$, the asset-holding decision rule, $a_U(e,a,z)$, the measure of unemployed workers $\mathbb{S}_U(e,a,z)$, the number of vacancies posted in each submarket $e$ is consistent with equation Equation 12.

(iv) The wage function $w(e,a,z)$ is determined through Nash Bargaining between the firms and the workers according to Equation 14.

(v) Equilibrium distributions, $\mathbb{S}_E(e,a,z)$ and $\mathbb{S}_U(e,a,z)$, satisfy the equilibrium stockflow equations across the different states of the economy implied by the sets of decision rules as well the idiosyncratic shocks described above.

(vi) The tax parameter $\tau$ is such that the aggregate government's budget constraint

$$T(y) + \tau_c C = G_c + rB + UI \tag{21}$$

is satisfied every period, where $B$ represents the debt per output, $C$ represents aggregate consumption, $UI$ represents the aggregate transfers with unemployment insurance, $G_c$ unproductive government spending, and $T(y)$ represents the aggregate revenue with a generic income $y$.

(vii) The dividend paid to equity owners every period is the sum of flow profits ($\pi$) from all matches, net of the expenditures on vacancies

$$d = \sum_e \Big[ \int \pi(e, a, z) d\mathbb{S}(e, a, z) - \kappa_e u_e \theta_e \Big] \tag{22}$$

(viii) Aggregate capital satisfies the asset market clearing condition:

$$\bar{k} = \frac{A}{1 + r} - (1 + r)B - p \tag{23}$$

where $p$ can be computed from Equation 3.

# 4 Quantitative Analysis

## 4.1 Calibration and estimation

The model presented before does not have a closed-form solution, so we must solve it numerically. To facilitate comparison, we assume one model period corresponds to six weeks, as most of the search-and-matching literature with incomplete markets do. Agents in our model represent the household head and we divide the labor force into four productivity types according to levels of education: Less than high-school, high-school graduates, some college and college degree. The population share of each group is denoted by $\omega_e$ and, according to CPS data, corresponds to $\omega_1 = 0.11$, $\omega_2 = 0.31$, $\omega_3 = 0.26$ and, $\omega_4 = 0.32$, respectively.

We separate the parameters into two groups: the exogenously given and parameters calibrated through the simulated method of moments. As is customary, we associate the parameters with the target that provides the most intuition for its value, but all parameters are determined jointly. The parameters' value and source/target are shown in Table 2 and Table 3.

We separate the parameters into two groups: those in the first are determined exogenously, and those in the second group are calibrated internally.

**Preference parameters**: We set the risk aversion as equal to 2. The survival rate $\psi$ is $1 - 1/320$, implying that workers have been in the market for an average of 40 years. The discount factor $\beta_e$ is defined heterogeneously, as in Krusell and Smith (1998) according to the educational level, which generates a plausible wealth distribution. The fixed-effect permanent productivity parameter $\phi_e$ is calibrated to generate a skill-wage premium according to the data, as in Setty and Yedid-Levi (2021).

**Uninsurable idiosyncratic shock**: The parameters that characterize the stochastic component of uninsurable idiosyncratic shocks are: $\rho$ and $\sigma_\epsilon$. For computational reasons, we use the algorithm described in Rouwenhorst (1995) to approximate these stochastic processes for each sector by a first-order Markov chain with 3 points. We rely on Mincer regressions estimates from Setty and Yedid-Levi (2021) and set $\sigma_\epsilon = 0.0978$.

**Technology:** The capital share $\alpha$ is equal to 0.3 as described by Krusell et al. (2010) and Setty and Yedid-Levi (2021) and is standard in the literature. The depreciation rate $\gamma$, in turn, is obtained internally. We target such rate to be equal to 0.11%, which corresponds to an investment-output ratio ($\frac{I}{Y}$) of 20%.

**Search and matching block:** About the job separation rate, the transition to unemployment parameters $\delta_e$ are calculated to generate the level of the unemployment rate for each educational cohort provided by the data. The resulting values are reported on **??**. Of course, the cost of posting vacancies in each educational group will be set such as the vacancy-unemployment ratio in the model's steady state is equal to 1, with no further consequences in other model variables. Thus, as a consequence of $\theta_e$ normalization, the scale parameters $\xi_s$ will reflect the job-finding rate of each submarket. Although individuals may have different job-finding probabilities for educational level, the differences among them are small, as shown by Santos and Rauh (2022). Since we abstract from search effort and we focus on the aggregate effects of public debt, we follow Setty and Yedid-Levi (2021) and define that every ex-ante individual has the same job-finding probability, thus, the same scale parameter $\xi_s$, which is equal to 0.6.[9] Also, in the line of Shimer (2005) as well other papers in the literature, we choose the elasticity of the matching function

[9] One can claim that job-finding rate may differ from individuals along wealth quintiles. We keep the same job-finding rate among the wealth distribution since, following Yum (2018), the relationship between employment and wealth is almost null.

to be equal for ex-ante educational level, and, hence, equal to the bargaining parameter, satisfying the Hosios (1990) condition. Thus, $\eta = \zeta_e$, for each sector

**UI design:** Specifically, about the UI structure, we follow the literature to discipline the parameters. The replacement rate $\varrho$ is going to be equal to 40% of the previous wage since in U.S. and the UI cap $\bar{b}$ will be equal to 48% of the median wage in the model. This calibration is based on data on maximum benefits by state, calculated according to Setty and Yedid-Levi (2021).

**Government:** We set the government consumption, $G_c$, to 17% of the economy's output under the baseline calibration. Following the literature, we assume a consumption tax of 6%. The parameter $\tau$, which governs the progressivity of the capital tax, is calibrated in line with the literature. Marginal tax rates are chosen to raise enough revenue to balance the government budget constraint.

| Parameter | Description | Values | Source |
|---|---|---|---|
| $\alpha$ | Capital share | 0.30 | Literature |
| $\varrho$ | Replacement rate | 0.40 | Literature |
| $\psi$ | Survival probability | 1-1/480 | 40 years in the market |
| $\mu$ | Risk aversion coefficient | 2 | Literature |
| $\chi_1, \chi_2, \chi_3, \chi_4$ | Matching function scale parameter | 0.60 | Setty and Yedid-Levi (2021) |
| $\omega_1, \omega_2, \omega_3, \omega_4$ | Educational population share | 0.11, 0.31, 0.26, 0.32 | CPS |
| $\tau_c$ | Consumption tax | 6% | Literature |
| $\tau$ | Progressivity tax | 10% | Literature |
| $G_c$ | Unproductive government spending | 17% of output | Literature |
| $\zeta_e$ | Matching function elasticity | 0.6 | Hosios Condition |
| $\eta$ | Bargaining power | 0.6 | Literature |
| $\rho$ | Idiosyncratic persistence | 0.98208 | Setty and Yedid-Levi (2021) |

Table 2: **External calibration**: Education group parameters are: 1 = less than high-school; 2 = high-school, 3 = some college, and 4 = college graduate.

# 5 Benchmark economy

In Figure 2 we present the fit of the model (black bars) relative to the data (red bars).

The table shows the share of wealth inequality that accrues to each wealth quintile. The model captures the distribution nicely with, for instance, 0.02% (-0.2%) of wealth being held by the bottom wealth quintile in the model (data), and 69.9% (82.5%) by the top wealth quintile. Hence, the model does a good job of capturing the distribution of wealth,

| Parameter | Description | Values | Target |
|---|---|---|---|
| $\delta_1, \delta_2, \delta_3, \delta_4$ | Job separation rate | 0.0571, 0.0384, 0.0305, 0.0174 | Unemployment rate |
| $\beta_1, \beta_2, \beta_3, \beta_4$ | Discount factor | 0.98, 0.98189, 0.98289, 0.99954 | Wealth distribution |
| $\phi_1, \phi_2, \phi_3, \phi_4$ | Permanent productivity | 0.72, 0.8615, 0.9683, 1.2527 | Wage premium |
| $\kappa_1, \kappa_2, \kappa_3, \kappa_4$ | Costing of posting a vacancy | 1.83, 1.83, 1.83, 1.83 | Normalization of $\theta_e = 1$ |
| $\gamma$ | Depreciation rate | 0.011 | $\frac{I}{Y} = 20\%$ |
| $\bar{b}$ | Cap of UI | 1.081 | 0.48 x $\bar{w}$ |
| $\lambda$ | Tax function adjustment | 0.9573 | Balance government budget constraint |

Table 3: **Internal calibration**: The internally calibrated parameters are estimated using the simulated method of moments (SMM) in which we minimize the sum of the equally weighted squared distance between model and data moments. Education group parameters are: 1 = less than high-school; 2 = high-school, 3 = some college, and 4 = college graduate

which is visibly very unequal, reflecting the U.S. pattern.

The model fits almost perfectly the wage distribution by quintile. For instance, the wages being earned by the bottom wage quintile in the model (data) are 4.9% (4.2%), and 46.7% (47.1%) by the top wage quintile. Also, in the bottom left-graph, we see that the model reproduces the wage premium among educational groups. The median wage premium of someone with college is 128%, the same pattern found in the data (127%).

The model matches the patterns of the unemployment rates across education groups closely. Out of those with less than high school education (LHS), 8.7% (8.7%) are unemployed, while for those with a college degree (C), the unemployment rates in the model are much lower at 2.9% (2.9%). The model also captures well the unemployment rate for those in the middle educational group. The unemployment for those with high-school (HS) is about 6.06% (6.06%) while those with educational level above than high-school (AHS, some college) have an unemployment rate of 4.84% (4.84%).

Also, the model fits well the aggregate outcomes, such as the average job finding rate and investment to output ratio, respectively, 60% (60%) and 19.6 % (20%). Of course, one of the main components of the paper is also perfectly calibrated, the debt-to-output ratio, targeted at 66% (66%), as displayed by Table 2.

| Moments | Data | Model |
|---|---|---|
| Average job finding rate | 60% | 60% |
| Debt/GDP ratio | 66% | 66% |
| Investment/GDP ratio | 20% | 19.6% |

Table 4: **Moments**: The table shows the moments targeted by the parameters in Table 2.

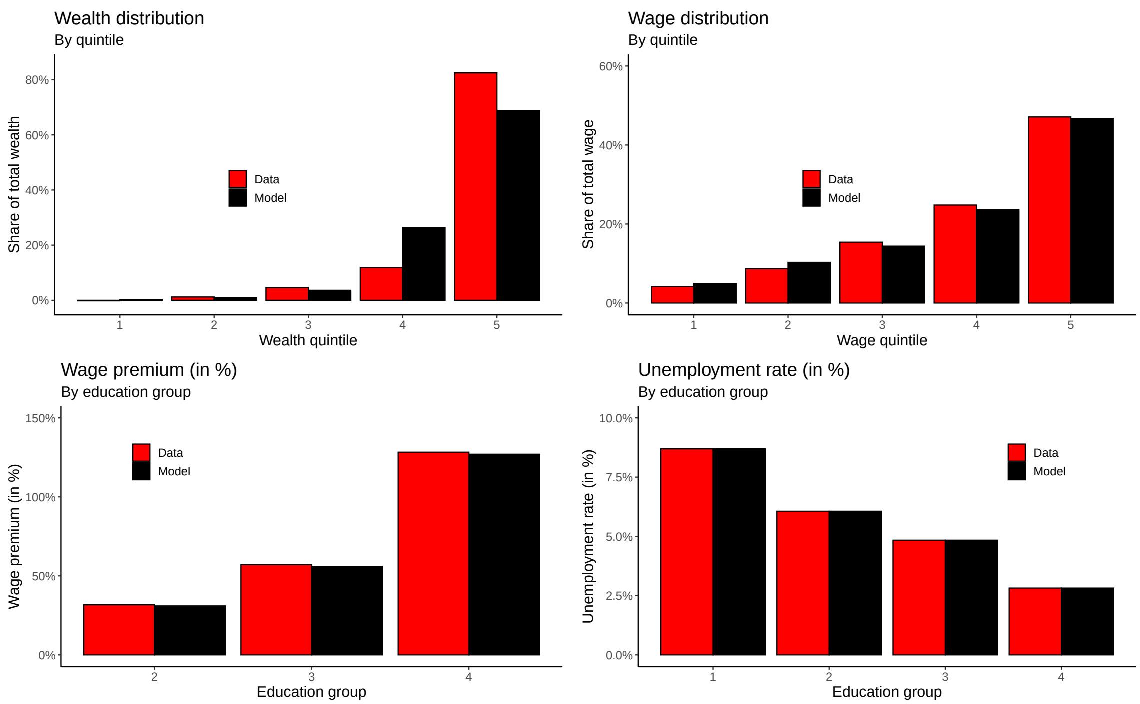


Figure 2: **Model fit**: In the top row, we have the wealth and wage distribution received by each quintile in the model (black) and data (red). In the bottom row, we have wage premium, measured as the percent difference between the median wage of type $i$, where $i$ can be 2 (high-school), 3 (above high-school) and 4 (college), and the median wage of type 1 (lower than high-school). The right-graph show unemployment rates for each educational group.

# 6 Results

## 6.1 Labor-Market Aggregates

In this section, we study how the aggregate variables change when the planner decreases/increases the debt-per-output ratio in this economy. Figure 3 display such findings in the first row to labor-market aggregates and, in the second row, to aggregate outcomes.

We start by describing the effects on labor-market outcomes. As expected, the unemployment rate increases when the public debt rises. When the public debt rises from 0% to 390% of the output, the unemployment rate increases by 1.1 percentage points, reaching at 6.0%. We can see also, in the right column in the first row, that the average wage for some employed individuals with high-school education falls when the public debt rises. Such effect is immediately explained by looking at the bottom left graph, which shows a sharp fall in the aggregate capital in this economy. Hence, the mechanism described by Aiyagari and McGrattan (1998), where the public debt crowds out capital, leading to an increase in the interest rate, drives the unemployment rate up. With this increase in the interest rate, there is a decline in the profit's net present value, reducing the incentives to firms posting vacancies in this economy. Consequently, firms create fewer vacancies, and the unemployment rate goes up. Neither the wage decrease is enough to change firms' behavior. As an aggregate result, the output declines as well.[10]

Now, we present the same labor-market outcomes from the last figure but considering different educational levels and normalized to 100 when the debt over output equals 0. Hence, inspecting Figure 4, we can see that the rise of the public debt is quite harmful to those with lower educational levels than with higher education. The unemployment rate increases by almost 1.5 percentage points for LHS individuals, while only increases by 0.55 percentage points for individuals with college. When we inspect the wage response to the increase in the debt per output ratio, we see that all wages fall in the same proportion, from lower than high-school (LHS) to college (C). We can see that although wages fall in the same proportion, wages for those with more educational levels are higher than those

[10]In general, those models that combine incomplete market with search frictions à la Diamond-Mortensen-Pissarides have two channels that might impact the unemployment rate. The first one, which we call the demand channel, is absent in this paper although present in papers such as Setty and Yedid-Levi (2021) and Santos and Rauh (2022). It refers to changes in the worker's search behavior due to some leisure (or UI) on the utility function that dampens the intensity to search for jobs. The second channel, the only one that acts through the lens of our model, is the supply channel. It refers to changes in the firm's incentive to post vacancies due to changes in the economy, in our case, the aggregate capital evasion due to public debt. Hence, we are mainly concerned about how firms respond to changes in public debt level.

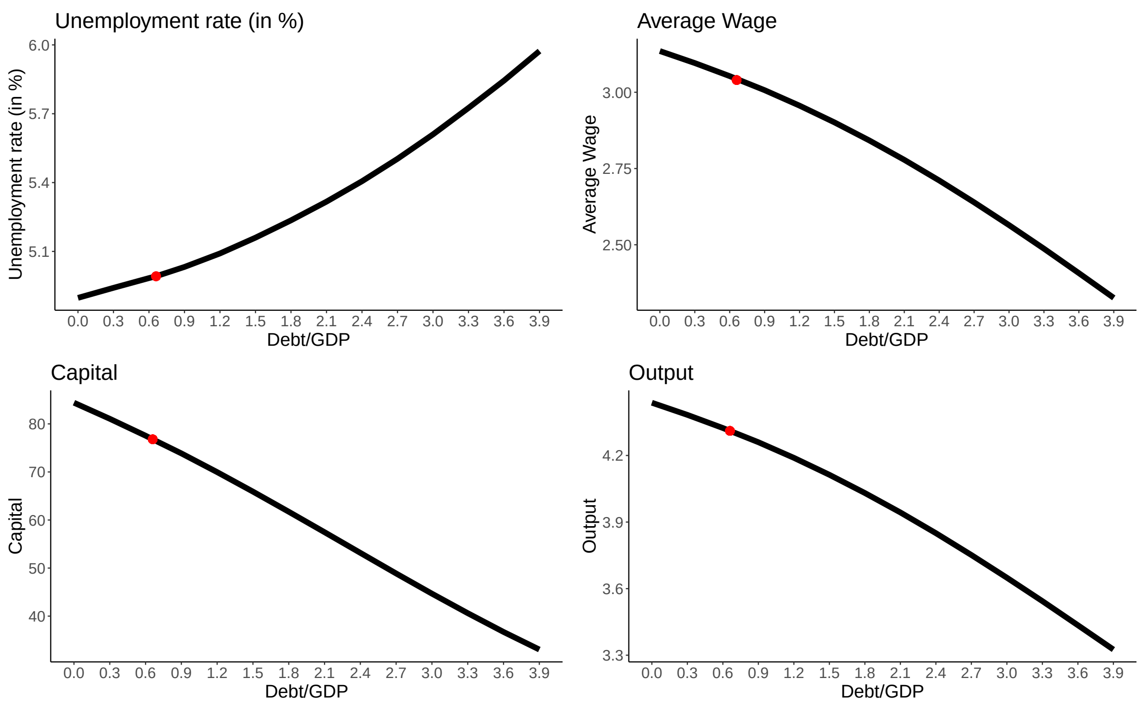


Figure 3: **Aggregate outcomes**: In the top row, we have the unemployment rate and average wage, respectively. In the bottom row, we have capital and output, respectively. The red dot represent the benchmark situation. All graphs show the change of outcomes with respect to a change in the debt-to-GDP ratio. For instance, the number 2.1 in the horizontal axis represents the level of public debt equal to 210% of GDP.

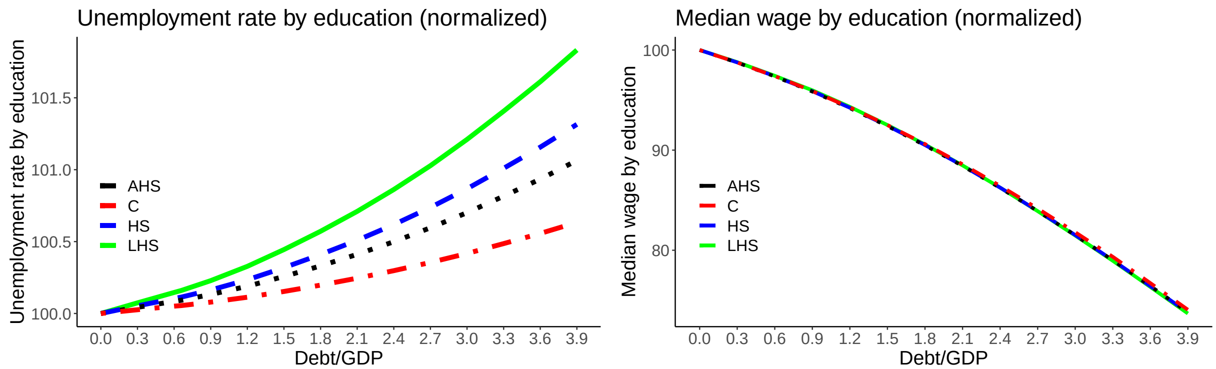


Figure 4: **Labor-market outcomes for educational level**: In the top row, we have the unemployment rate and median wage (both normalized to base 100 when Debt/GDP is equal to 0) for all educational levels. In both graphs, the solid green line represents those with lower than high-school (LHS). The dashed blue line show for those with high-school (HS) while the black dotted line display for those with educational level higher than high-school, but no completed college (AHS) and the red dot-dashed line show the results for those with college degree (C).

who do not.[11]

Inspecting a little bit more the fall in unemployment rate, we analyze how much the expected value of creating a vacancy changes when public debt rises. The results can be seen in Table 5. In this specific case, we see for each educational group, how much fall the vacancy creation benefit when the public debt is 240% of output compared to benchmark situation. We realize that the fall is large for those that have education lower than high-school (9.06%), followed by high-school (-8.60%), above high-school (-8.33%) and the vacancies for college degrees (-5.85%). This is the reason that unemployed rate rises much more for lower than high-school compared to college.

| Education group | Expected benefit |
|---|---|
| Lower than high-school (LHS) | -9.06% |
| High-school (HS) | -8.60% |
| Above high-school (AHS) | -8.33% |
| College (C) | -5.85% |

Table 5: **Expected benefit of creating a vacancy**: The table shows the the fall in the expected value of creating a vacancy ( $\mathbb{E}\left[\max\{J(e,a',z'),0\}\middle|z\right]\frac{S_U(e,a,z)}{u}$) when debt reaches 240% of output compared to the benchmark situation.

[11] In the Figure 9, on the appendix, we present the same labor-market outcomes for educational level without the normalization.

## 6.2 Optimal public debt

In this section, we estimate the optimal public debt level in terms of CEV, based on the welfare measure presented on Section 3. We conduct our experiment by changing the debt level in the economy, keeping other aggregate variables constant. The government budget constraint adjustment comes from the parameter $\lambda$, which adjusts each time the public debt changes. The result is displayed in Figure 5. The left graph in the top row shows the welfare result (vertical axis) for each public debt level (horizontal axis), while the right graph displays such welfare measured but decomposed by uncertainty and level. The left graph in the bottom row shows how the welfare measure is given by labor-market status, i.e., employed and unemployed, and the right graph shows how the change in public debt affects each educational group. In all graphs, the red dot represents the benchmark situation, i.e., where the CEV equals 0.

The optimal level of public debt is about 240% of GDP. The welfare in this economy approximates from a laffer-curve type, where it only increases from the benchmark situation (66% of GDP) until the level that maximizes the CEV (240% of the GDP). After that, the welfare diminishes to the point that it becomes negative when the public debt level is around 390% of GDP. When it reaches the top, it gets to 3%, i.e., the economy's agents would give up roughly 3% of lifetime consumption to go to this economy with a higher public debt level. This result goes in the opposite direction that the literature points out.[12] To understand it, we decompose such welfare measures to see what components drive this higher level of optimal public debt.

The right graph in the top row decomposes the welfare result in components as level and uncertainty. Those two components go in different directions because of the twofold effect of evasion of capital in this economy. The striking effect of lower capital accumulation is exemplified by the decrease in the welfare level component (the dashed blue curve), which shows that individuals in this economy are reducing their participation in this economy. On the other hand, capital evasion pushes interest rates up, reducing the cost of saving and uncertainty in this economy, which is explained by the increase in the uncertainty component (the red dot-dashed line). In the net overall effect, the uncertainty component overcomes level one and drives the welfare increase until it becomes flatter.

[12] Aiyagari and McGrattan (1998) show that the optimal public debt level is around their benchmark, which is the same as ours (66% of GDP). Flodén (2001) show that such result persists if transfers are low, but when high, the optimal public debt varies roughly. Peterman and Sager (2022) show that the optimal public debt level is around -160 of output (i.e., the government must have savings) when life-cycle components are introduced.

In the bottom row, we decompose such results to see how heterogeneous are the effects of the public debt increase in the economy. The right graph shows that the public debt hike harms unemployed individuals more than employed. The crowding out effect of capital leads to a rise in the interest rate, which decreases the net present value of firms' profits, leading to less vacancy creation, and pushing the unemployment rate up. The job-finding rate decreases, which leads to an increase in the unemployment duration. However, since most individuals are employed, they accumulate more assets and benefit from such hike in the asset return. Actually, this pattern is more visible in the right graph, which shows which individuals by educational level benefit more from the increase in public debt. We can see that the individuals with college (red dashed line), i.e., those who have higher wages and accumulate more savings, accrue rents to benefit from the interest rate hike. In contrast, those individuals with lower educational levels are harmed by such policy since, in their income composition, the wage has a higher share than the income from accumulated assets.[13]

### 6.3 CEV decomposition

In this section, we decompose the effects of this optimal public debt. To this end, we construct a decomposition exercise to quantify the importance of: *i*) labor-market tightness $\theta_e$, *ii*) wages, *iii*) $\theta_e$ and wages, *iv*) tax adjustment $\lambda$ and, *v*) the interest rate $r$ in shaping the public debt effects. Given that the welfare gains peak for a public debt over GDP around 240%, we narrow the analysis under this scenario. Table 6 display the findings.[14]

Assuming that labor-market tightness is kept unchanged, a public debt of 240% of output does not display large changes compared to the optimal situation. The aggregate output and capital dropped 9.78% and 29%, respectively. Of course, as the labor-market tightness remains constant, the unemployment rate does not change, neither the job finding probability. [15]

When the wages are kept constant, the effects are much more substantial.[16] Since the labor price cannot adjust to the new condition of the economy, all the adjustment occurs through the firm's vacancy posting and, consequently, affect the job-finding probability,

[13]See Vogel (2014) for a similar reasoning.

[14]When the $\lambda$ remains constant, we set the government expenditure to adjust to keep the government budget constraint constant.

[15]As explained before, we do not have any component of search intensity in our model. Hence the demand channel does not play a role, only the supply channel. Keeping the tightness constant is the same as keeping the vacancy posting constant, thus, keeping the supply channel constant.

[16]Remark that wages here are calculated as an outcome of bilateral bargaining.

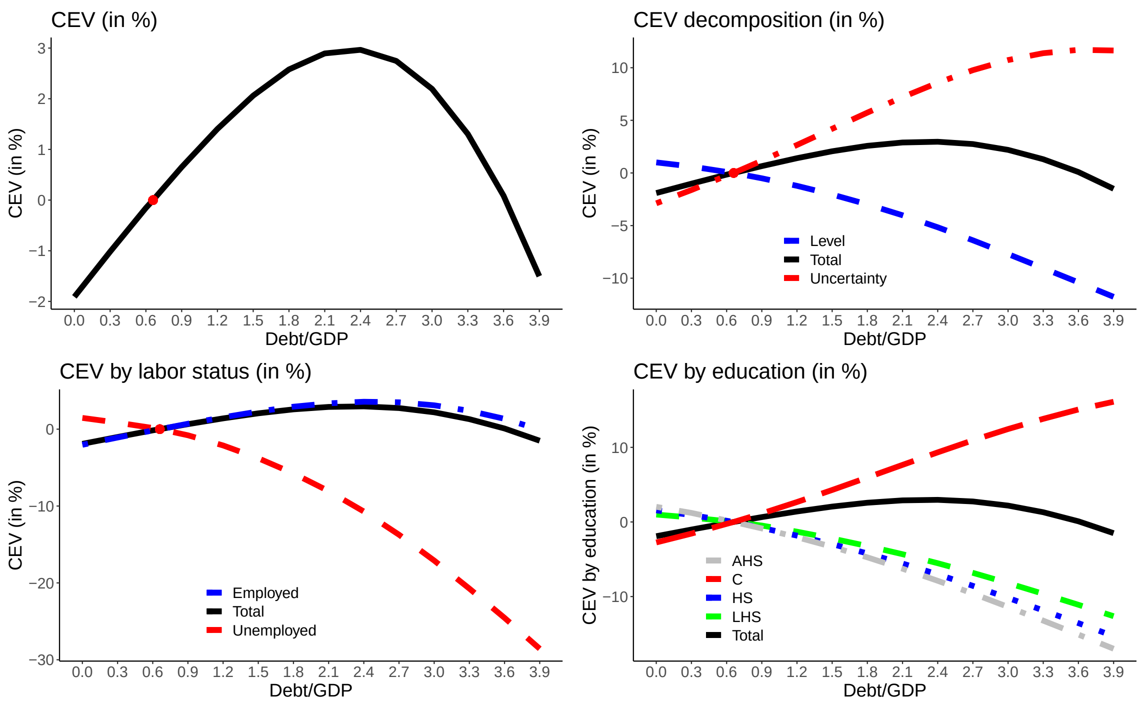


Figure 5: **Consumption Equivalent Variation (CEV)**: In the top row, we have the consumption equivalent variation (CEV) in the left graph (black solid line) and CEV decomposition in level (blue dashed line) and uncertainty (red dotted-dashed line) in the right hand side. In the bottom row, we have the CEV for employed (blue dashed-dot) and unemployed (red dashed) in the left graph while in the right graph, the CEV is displayed by educational level, where the green dashed line is the CEV for people with lower than high-school education (LHS), the blue dotted for people with high-school education (HS) , the gray dot-dashed line for individuals with education above than high-school (AHS) and the big red dashed line to workers with college (C).

which fell to 8.80%. The unemployment rate shot up to 25.83% from 5.41% compared to the optimal situation but with flexible wages. Such results dropped output, capital, and consumption by 21.54%, 24.46%, and 27.59%, respectively. The massive unemployment hike drove the economy's production to the bottom. Even the interest rate felt, which explains the mean income decline of 9.75%. This result shed some light on the wage role in this economy. Firms cannot pass their losses through wages to adjust their profit in this situation, which induces a large drop in the decision to post vacancies. The huge increase in public debt crowds out a large portion of capital in the optimal situation. This can affect the labor-market outcomes in two ways: *i*) the congestion effect in the labor-market frictions and *ii*) the price of labor, i.e., wage. The results in Table 5 show that the latter is much more important than the former since the adjustment in the economy occurs through wage bilateral bargaining problem instead of the congestion effect in the labor-market frictions. This wage channel dampens the effect of changing labor-market conditions, which makes capital evasion increase unemployment rate only by 0.5 percentage points when the tightness is constant. Hence, wages are much more critical as an adjustment variable than labor-market tightness.

We perform an exercise where both tightness and wages are kept constant. The aggregate output and capital fell less than in others exercises. The first one dropped 2.54% while capital declined 8.23%. Surprisingly, mean income increased by 3.79% as a consequence of wage constancy.

In the last two columns, we display the result where the $\lambda$ and $r$ are in the same value as the benchmark. The constancy of $\lambda$ brings no adjustment in the tax code, being such tax level the same as in the benchmark situation, which indicates that the tax function appropriates less income than in the optimal situation. This is why consumption increases by 10.87% even though the unemployment rate increases, capital and output decline. The individuals that are benefited from the rise in public debt (which can be seen from the increase in mean income) do not have their income taxed, which leads to this consumption hike compared to the optimal debt situation.

When $r$ is kept constant, the aggregate capital (thus, aggregate savings in the economy) falls by almost 86.24%. As the cost of precautionary savings becomes too high, individuals postpone savings, which drives capital down. However, since firms use capital per labor in the production function, and that is determined by the interest rate, capital per labor remains constant, which explains why the labor-market aggregates remain almost constant, as job finding probability, unemployment rate and wages. One interesting aspect is that even though wages and mean income remain practically constant, consumption drops 13.39% as a result of the adjustment of $\lambda$, which balances the government budget constraint and reduces consumption in this economy.

| Variable | Benchmark | Debt 240% | Keeping at Benchmark $\theta_e$ | Wages | Both | $\lambda$ | $r$ |
|---|---|---|---|---|---|---|---|
| Aggregate output $Y$ | 100 | -10.69% | -9.78% | -21.54% | -2.54% | -9.36% | + 0.13% |
| Aggregate capital $\bar{K}$ | 100 | -30.77% | -29.0% | -24.46% | -8.23% | -27.33% | -86.24% |
| Consumption | 100 | -5.16% | -5.34% | -27.59% | -8.80% | +10.87% | -13.39% |
| Unemployment rate | 4.99% | 5.41% | 4.99% | 25.83% | 4.99% | 5.36% | 4.97% |
| Job finding probability | 60.00% | 55.00% | 60.00% | 8.80% | 60.00% | 55.60% | 60.20% |
| Wage | 100 | -10.77% | -10.01% | 0.00% | 0.00% | -9.40% | +0.40% |
| Mean income | 100 | +11.26% | +10.80% | -9.75% | +3.79% | +11.00% | -0.98% |
| Interest rate $r$ | 0.58% | 1.07% | 1.04% | 0.07% | 0.07% | 1.07% | 0.58% |
| Lambda $\lambda$ | 100 | -14.75% | -14.54% | -20.17% | -11.99% | 0.00% | -12.18% |
| Debt/Y | 66% | 240% | 240% | 240% | 240% | 240% | 240% |
| **Gini** | | | | | | | |
| Gross income | 0.598 | 0.584 | 0.584 | 0.579 | 0.592 | 0.594 | 0.592 |
| Net income | 0.439 | 0.453 | 0.451 | 0.405 | 0.425 | 0.489 | 0.409 |
| Consumption | 0.581 | 0.563 | 0.564 | 0.546 | 0.572 | 0.579 | 0.568 |
| Assets | 0.712 | 0.699 | 0.701 | 0.707 | 0.711 | 0.691 | 0.719 |
| **Welfare (in %)** | | | | | | | |
| CEV | - | 2.97 | 2.47 | -24.77 | -6.41 | 18.83 | -12.44 |
| CEV Level | - | -5.16 | -5.34 | -27.59 | -8.80 | 10.87 | -13.39 |
| CEV Uncertainty | - | 8.57 | 8.25 | 3.89 | 2.61 | 7.18 | 1.10 |

Table 6: **Decomposition of Optimal Public Debt**: The table presents changes to the economy when we keep constant the labor-market tightness, i.e, $\theta_e$, wages, both $\theta_e$ and wages, the tax adjustment $\lambda$ and the interest rate $r$. Marked with +/- are relative changes to the benchmark economy set for 100 for outcomes for which levels that do not have an obvious interpretation. For outcomes for which levels have a clear interpretation, we present actual numbers in the benchmark as well others columns.

## 6.4 An alternative utilitarian welfare function

Our main welfare result is based on a utilitarian welfare function that faces an equality-versus-efficiency trade-off. Following Bénabou (2002) and, more recently, Dyrda and Pedroni (2022), we consider a welfare function where the planner cares only about *efficiency*. Take the following welfare function:

$$\mathbb{U}^{\hat{\mu}} = \left\{ \int \mathbb{E}\left[ \sum_{t=0}^{\infty} \beta_e^t \left( \prod_{s=0}^{t} \psi \right) u(c_t) \right]^{\frac{1-\hat{\mu}}{1-\mu}} d\mathbb{S}(e,a,z) \right\}^{\frac{1-\mu}{1-\hat{\mu}}} \tag{24}$$

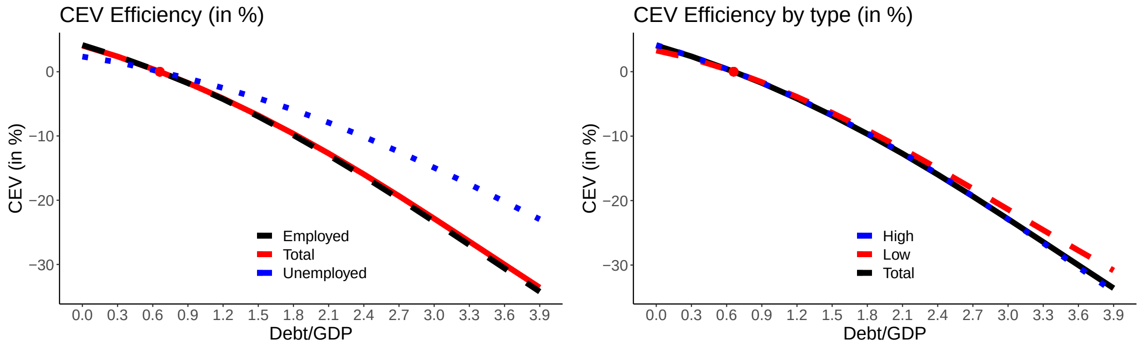


Figure 6: **Consumption Equivalent Variation (CEV) in Efficiency**: In the left-hand side, we have the new welfare measure that displays the CEV of Efficiency for all individuals (red solid line), employed (black dashed line), unemployed ones (blue dotted line), while in the right-hand side we have the same measure but for high (blue dotted line) and low (red dashed) individuals.

where $\mathbb{U}^{\hat{\mu}}$ is the new welfare function, $\hat{\mu}$ is the planner's degree of inequality aversion, and $\mathbb{S}(e, a, z)$ is the aggregate distribution over individuals states $(e, a, z)$. If $\hat{\mu} = 0$, the planner is maximizing efficiency, or, in our case, the level component of CEV.[17] Figure 6 show the result when considering such measures for employed and unemployed individuals and productivity levels. We can see that, differently from the general CEV results displayed in subsection 6.3, unemployed and employed ones suffer from the public debt increases, with the employed welfare displaying a higher decline than the unemployed ones. The same argument holds for individuals with low and high productivity types. Hence, the crowding-out effect of capital has, as a consequence, the interest rate hike, which leads to a decline in firms' decisions to post vacancies, driving unemployment up and the economy down. Since matching is not being formed, workers are appropriating less of consumption size, reducing our new welfare measure. Since our new measure only concerns efficiency and not inequality, even high types of individuals (those that might accumulate more assets) suffer from the economic downturn.

## 6.5 The role of ex-ante heterogeneity

This subsection presents the results of a series of experiments where we shut down ex-ante heterogeneity. We construct the infinity lived agent model by removing all education-dependent features (e.g., fixed effects, separation rate, discount factor.) Here, our calibration is heavily based on Krusell et al. (2010). We find that the two models generate quantitatively different optimal policies.

[17] As exposed by Dyrda and Pedroni (2022), when $\hat{\mu} \rightarrow \infty$, we get a Rawlsian function welfare function where the planner only concern's with inequality.

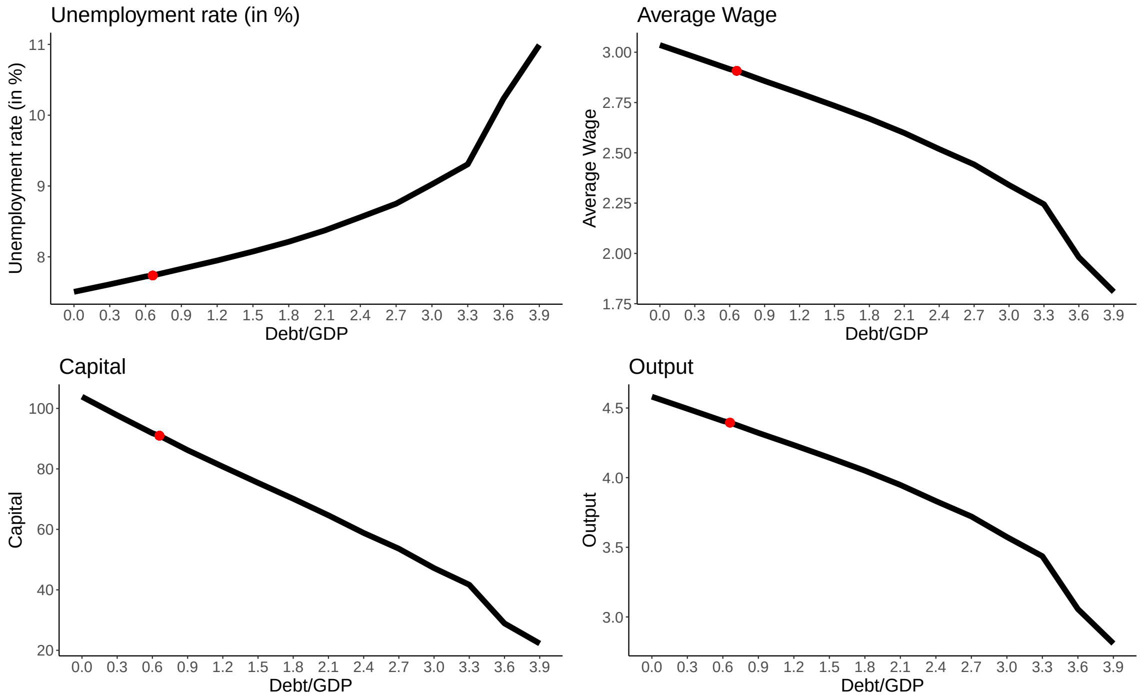


Figure 7: **Aggregate outcomes in an economy without ex-ante heterogeneity**: In the top row, we have the unemployment rate and average wage for workers, respectively. In the bottom row, we have capital and output, respectively. The red dot represent the benchmark situation. All graphs show the change of outcomes with respect to change debt to GDP ratio. For instance, the number 1.8 in the horizontal axis represents the level of public debt equal to 180% of GDP.

First, we see that all aggregate outcomes respond more to a public debt increase than when agents have an innate ability, as shown in Figure 7. The unemployment rate roughly increased 3.5 percentage points, from 7.51% to 10.99%, when public debt hiked from 0% to 390% of output. The average wage has declined 40.41%, capital decreased 78.58% and output 38.68%. Firms are more responsive to an interest rate hike due to capital evasion than in the model with ex-ante heterogeneity. Since we eliminate the most productive individuals (due to their innate ability), they accumulate less assets and are more exposed to changes in the firm's decisions to post vacancies. Such result can be seen in the welfare decomposition in Figure 8.

We decomposed the CEV measure in level and uncertainty, as in previous subsections. We see that, differently from the results displayed in Figure 5, here, the level component drives CEV down. From the benchmark situation where the public debt of 66% of GDP, the level CEV component decreases by 14.49% when the debt/GDP reaches 240% while the uncertainty component increases by only 8.01%. In that case, the optimal public debt level is 0%. We can see that the public debt increase harm employed and unemployed and,

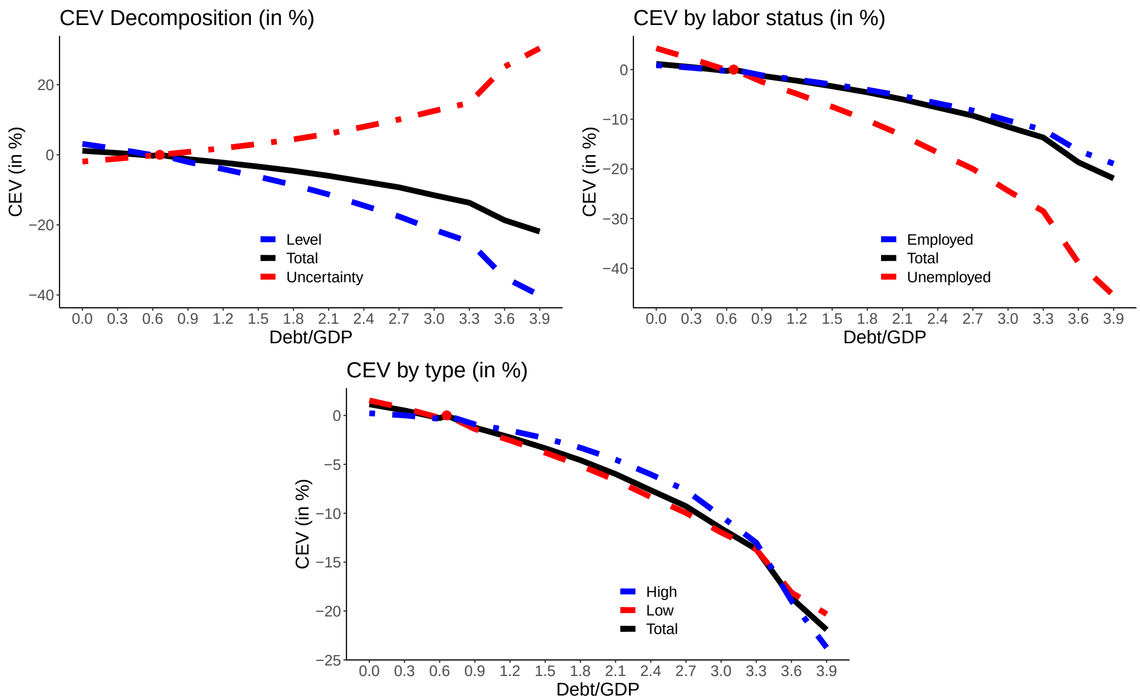


Figure 8: **Consumption Equivalent Variation (CEV) in an economy without ex-ante heterogeneity**: In the top row, we have the consumption equivalent variation (CEV) in the left graph (black solid line) and it decomposition in level (blue dashed line) and uncertainty (red dotted-dashed line) in the right hand side. In the bottom row, we have the CEV for idiosyncratic productivity shocks type, where high (blue dot-dashed line) indicates the most productive individuals in the economy and low (red dashed line) indicates the lowest productive ones.

in the bottom row of the Figure 8, individuals with high and low uninsurable productivity shocks.

Our results demonstrate that ex-ante heterogeneous education is an essential determinant of an optimal policy when infinitely agents models with incomplete markets display labor-market frictions. Without wealth inequality, the main mechanism of vacancy creation gains much traction compared to the model where individuals can accumulate much more asset and dampens such effect.

# 7 Concluding remarks

Public debt has been increasing over the years to accommodate social expenditures such as medicare and job-displacement insurance programs. Also, it's been an important tool when government faces business cycle shocks. This paper analyzes the long-term effects of public debt on labor-market outcomes. To do so, we calibrate a rich heterogeneous agent search-and-matching model with incomplete markets to the U.S. economy while incorporating the effects of public debt.

The main trade-off of the model is that, on the one hand, public debt expansion crowds-out capital and leads to a hike in the return rate, which reduces vacancy creation in the economy. On the other hand, the interest rate hike reduces consumption smoothing costs, which induces agents to engage in precautionary savings. We find that the unemployment rate increases by 1.1 percentage points when public debt varies from 0 to 390% of output. Such benign adverse effects of unemployment occur because part of the adjustment comes via bilateral wage bargaining process, with average wages decreasing almost 20%, damping public debt effect on vacancy creation. When we decompose by innate ability or educational type, we see that individuals with lower than high-school (LHS) education are the most harmed by the increase in public debt. The unemployment rate increases by 1.5 percentage points for them while only increasing by 0.5 percentage points for individuals with college (C) degrees.

We also find that the optimal level of debt-to-GDP is 240%. Individuals employed and with a college degree benefit most from such a hike in public debt compared to the benchmark, mainly because of the higher share of their income that comes from asset accumulation. When we decompose the consumption equivalent variation (CEV) in equity (uncertainty) and level (efficiency), we show that the former overcomes the latter in a matter of importance. Increasing interest rate as a result of capital crowding-out due to public debt leads to a more equal distribution of resources among agents, which raises the uncertainty component of CEV to the top at 240%. After that, the level component overcomes the equity one, and CEV starts to decrease, giving the CEV a laffer-curve shape.

We shut down wealth and skill-wage premiums to see what changes in the optimal public debt result. We find that the unemployment rate becomes much more elastic to changes in public debt than the benchmark model. The unemployment rate increases by 3.5 percentage points, reaching almost 11% when debt varies from 0% to 390%. The optimal level of public debt is equal to 0% due to the effect of public debt on vacancy creation.

This paper sheds some light on the long-term effects of public debt on different individuals' labor markets. Nevertheless, the results of the paper there is a lack of analysis about the transitional dynamics of public debt when labor market frictions are included.

# References

AIYAGARI, R. (1994): "Uninsured idiosyncratic risk and aggregate saving," *The Quarterly Journal of Economics*, 109, 659–684. 3

——— (1995): "Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting," *Journal of Political Economy*, 103, 1158 – 1175. 7

AIYAGARI, R. AND E. MCGRATTAN (1998): "The optimum quantity of debt," *Journal of Monetary Economics*, 42, 447–469. 2, 6, 21, 24

AÇIKGOZ, O., M. HAGEDORN, H. HOLTER, AND Y. WANG (2018): "The Optimum Quantity of Cpaital and Debt," *Working Paper*. 7

BALL, L. AND G. MANKIW (1995): "What do Budget Deficits do? in: Budget deficits and debt: issues and options," *Symposium Proceedings, Federal Reserve Bank of Kansas City*. 2

BEWLEY, T. (1986): "Stationary Monetary Equilibrium with a Continuum of Independently Fluctuation Consumers," *Contributions to Mathematical Economics in Honor of Gérard Debreu*. 3

BÉNABOU, R. (2002): "Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?" *Econometrica*, 70, 481–517. 11, 28

CHAMLEY, C. (1986): "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 54, 607–622. 7

DESBONNET, A. AND T. WEITZENBLUM (2012): "Why do Governments End-up with Debt? Short-run Effects Matter," *Economic Inquiry*, 50, 905–919. 7

DIAMOND, P. (1982): "Wage Determination and Efficiency in Search Equilibrium," *The Review of Economic Studies*, 49, 217–227. 2

DYRDA, S. AND M. PEDRONI (2022): "Optimal Fiscal Policy in a Model with Unisurable Idiosyncratic Income Risk," *Forthcoming The Review of Economic Studies*, 1–49. 7, 28, 29

EECKHOUT, J. AND A. SEPAHSALARI (2021): "The Effect of Wealth on Worker Productivity," *Working Paper*. 7

FLODÉN, M. (2001): "The effectiveness of government debt and transfers as insurance," *Journal of Monetary Economics*, 48, 81–108. 2, 6, 7, 24

GANONG, P., F. GREIG, P. NOEL, D. SULLIVAN, AND J. VAVRA (2022): "Spending and Job-Finding Impacts of Expanded Unemployment Benefits: Evidence from Administrative Micro Data," *Working Paper*, 1–97. 2

HEATHCOTE, J., K. STORESLETTEN, AND G. VIOLANTE (2008): "Insurance and opportunities: A welfare analysis of labor market risk," *Journal of Monetary Economics*, 501–525. 15

——— (2017): "Optimal Tax Progressivity: An Analytical Framework," *Quarterly Journal of Economics*, 132, 1693–1754. 11

HOSIOS, A. J. (1990): "On the efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57, 279– 298. 18

HUGGETT, M. (1993): "The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control*, 17, 953–969. 3

KEHOE, P., V. MIDRIGAN, AND E. PASTORINO (2019): "Debt Constraints and Employment," *Journal of Political Economy*, 127, 1926 – 1991. 7

KRUEGER, D., K. MITMAN, AND F. PERRI (2017): "Macroeconomics and Heterogeneity, Including Inequality," *Handbook of Macro Economics*, 2B, Chap.11. 10

KRUSELL, P., T. MUKOYAMA, AND A. SAHIN (2010): "Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations," *Review of Economic Studies*, 77, 1477–1507. 2, 7, 13, 17, 29

KRUSELL, P. AND A. SMITH (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867– 896. 17

LAMA, R. AND J. P. MEDINA (2019): "Fiscal Austerity and Unemployment," *Review of Economic Dynamics*, 34, 121–140. 7

MORTENSEN, D. (1982): "Property Rights and Efficiency in Mating, Racing, and Related Games," *American Economic Review*, 72, 968–979. 2

PETERMAN, W. AND E. SAGER (2022): "Optimal Public Debt with Life-Cycle Motives," *American Economic Journal: Macroeconomics*, 14, 404– 437. 7, 24

PISSARIDES, C. (1988): "Short-run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages," *American Economic Review*, 75, 676–690. 2

ROHRS, S. AND C. WINTER (2017): "Reducing Government Debt in the Presence of Inequality," *Journal of Economic Dynamics and Control*, 82, 1–20. 7

ROUWENHORST, K. G. (1995): "Asset pricing implications of equilibrium business cycle models," *Cooley, T.F. (Ed.), Frontiers of Business Cycle Research*, 294 – 330. 11, 17

SANTOS, M. AND C. RAUH (2022): "How do transfers and universal basic income impact the labor market and inequality?" *Working Paper*, 1–46. 7, 17, 21

SETTY, O. AND Y. YEDID-LEVI (2021): "On the Provision of Unemployment Insurance when Workers are Ex-Ante Heterogeneous," *Journal of the European Economic Association*, 19, 664–706. 3, 4, 7, 10, 13, 17, 18, 21

SHIMER, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95, 25– 49. 17

VOGEL, E. (2014): "Optimal Level of Government Debt - Matching Wealth Inequality and Fiscal Sector," *European Central Bank - Working Paper*, 1–55. 6, 25

YARED, P. (2019): "Rising Government Debt: Causes and Solutions for a Decades-Old Trend," *Journal of Economic Perspectives*, 33, 115– 140. 2

YUM, M. (2018): "On the Distribution of Wealth and Employment," *Review of Economic Dynamics*, 30, 86–105. 7, 17

# A Additional figures

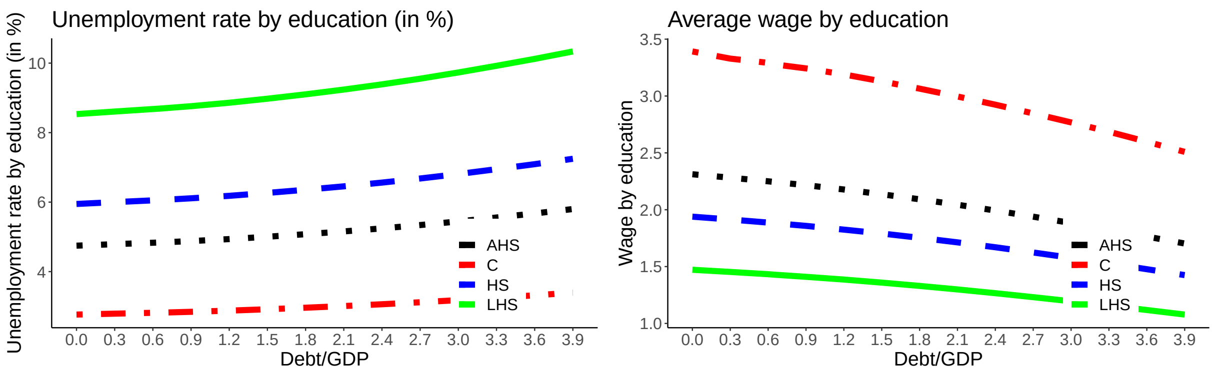


Figure 9: **Labor-market outcomes for educational level**: In the left graph, we have unemployment rate (in %) and average wage by educational levels, respectively. The green solid line show the outcome for those with education level lower than high-school (LHS). The blue dashed line show for those with high-school (HS) while the blue dashed line display for those with educational level higher than high-school, but no completed college (AHS) and the red dot-dashed show the results for those with college degree (C).