Reassessing the Efficiency-Equity Trade-off: Progressivity's Impact on Growth*

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PRELIMINARY

Abstract

In this paper, we revisit the efficiency-equity trade-off of optimal tax theory by emphasizing the consequences of increased progressivity on growth, instead of income levels. We use an endogenous growth framework that takes into account both the decision to become a researcher and the effort that established entrepreneurs make on improving their products. We find that the optimal level or progressivity is lower than the current one but that welfare gains are moderate. However, if one disregards the growth impact one would prescribe a substantially higher level of progressivity at significant welfare cost.

JEL classification: *O30;H21* **Keywords**: *Creative Destruction; Research Spillovers; Progressive Taxation*

I Introduction

Optimal tax theory typically emphasizes the trade-off between equity and efficiency as captured by the *level* of output that is sacrificed for a fairer distribution of income. Yet,

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a potentially much more consequential side-effect of the disincentives created by tax distortions, its impact on growth, remains understudied.

The purpose of this paper is to investigate the equity/growth trade-off in light of an endogenous growth model along the lines of Jones and Kim [2018]. To motivate a desire for redistribution we assume that individuals are heterogeneous with respect to their labor market productivity and with respect to their cost of engaging in research activities. Beyond this, progressive income taxation permits risk to be better shared between individuals, and so is risk a key ingredient in our model.

We concentrate our analysis on constant progressivity tax schedules, akin to those Benabou [2002] and Heathcote et al. [2017].[1] Our central assessment is done by identifying the Utilitarian optimal level of income tax progressivity, a critical aspect in the pursuit of balancing equity and efficiency. As is customary in macro approaches to redistribution, intensifying progressivity in the tax system enhances income redistribution and fosters better risk-sharing among individuals. However, this comes at the cost of reduced incentives for agents to exert effort, potentially impacting economic growth.

Our investigation unveils two novel dimensions in this context. Firstly, entrepreneurial research efforts may be discouraged under increased progressivity. This aspect is of paramount importance as entrepreneurial research has the potential to contribute to technological advancements and stimulate the creative destruction process by increasing the gains from research by potential entrants.

Secondly, in contrast with linear taxes that uniformly reduce income across all economic activities, progressive taxes make activities characterized by convex payoffs, with research being a prominent example, less attractive.

By considering these multifaceted effects, we obtain novel insights into the optimal level of progressivity for the U.S. tax system. Our findings indicate that the ideal level of progressivity is slightly lower than the current one. Yet, transitioning to the optimal tax system is expected to yield only moderate welfare gains.

However, it is imperative to emphasize that failing to account for the growth consequences of progressivity could lead to misguided policy prescriptions. Ignoring the effects of entrepreneurial research and the distinct disincentives introduced by progressive taxes in certain activities might erroneously lead to advocating substantially higher levels of progressivity, resulting in significant welfare losses.

[1] The schedule is automatically adjusted to keep the budget constraint satisfied. This implies that the income-weighted average marginal tax rate is not mechanically changed because of economic growth.

The rest of the paper is organized as follows. After a brief literature review, Section II displays the environment. In Section III the definition of a balanced growth path equilibrium and its characterization are offered. In Section IV we describe the calibration and present our main quantitative findings. Section V concludes the paper. Longer proofs are collected in the appendix.

Lit. Review

The main question we are after in this paper relates to the consequences of taxation on economic growth. This is also what Jaimovich and Rebelo [2017] investigates. They consider a version of Romer's [1990] growth model in which researchers are heterogeneous with regard to their innate talent. As in our case, taxes can affect growth rates. They show how non-linear effects arise from linear taxes due to the researcher's heterogeneity.

Similarly, Li and Sarte [2004] study the growth impact of progressivity in an endogenous growth model using Rebelo's [1991] approach, which is substantially different from Jones and Kim's [2018] on which our approach is based.

Growth arises in our model for the reasons explored by Jones and Kim [2018]. Individuals may opt to engage in research hoping to get a breakthrough that allows them to become entrepreneurs. When successful this creative destruction process generates spillovers that allow the economy as a whole to move up a rung in the technology ladder. Moreover, entrepreneurs are actively involved in innovation, which, while not directly contributing to other firms creates incentives for those who aspire to become entrepreneurs. We contribute to the literature by assessing the consequences of progressivity on all aspects of this process.

Progressivity is considered by Jones [2022] which focuses on the taxation of top incomes in a world of ideas. The special nature of ideas as a non-rival factor leading to increasing returns to scale magnifies the distortionary consequences of progressivity. In contrast with Jaimovich and Rebelo [2017], taxes affect levels but not GDP growth rates, which are still determined by the population growth rate.

In a very early version of this paper, Rodrigues [2021] goes after some of the same questions we explore, but in a world without worker heterogeneity. He finds substantially lower progressivity to be optimal, which emphasizes the importance of taking redistributive models into account.

II The Model

The core of our model is a variation on Jones and Kim's [2018] Schumpeterian economy, which itself builds on an older tradition of modeling and understanding entrepreneurship and innovation.[2] We enrich their framework by assuming agent heterogeneity to add a redistributive motive to the use of progressive income taxes. The economy has a continuous time dimension denoted by t. Whenever convenient, we omit from the variables the time subscript.

II.1 Demographics and preferences

In the economy, there is a continuum of utility-maximizing individuals of size N and no population growth. Each individual is characterized by a pair of parameters (ν, κ), where ν is his or her labor market productivity and κ is the cost of entering research.[3]

We assume that types are assigned through i.i.d. draws from the distributions,

$$\log \nu \sim \mathcal{N}(0, \sigma_\nu^2) \quad \text{c.d.f. } F_\nu(\nu)$$
$$\kappa \sim \text{Exp}(\psi) \quad \text{c.d.f. } H(\kappa)$$

Before entering the labor market, workers face a lottery that, at a probability $b(\nu)$ of success, determines whether they will be able to choose to become researchers,

$$\chi(\nu) \in \{0, 1\}, \quad \text{Pr}(\chi(\nu) = 1) = b(\nu)$$
$$b(\nu) \in [0, 1], \quad b'(\nu) \geq 0.$$

Note that we allow the probability to depend on the agent's labor productivity, ν. Under this assumption, a fraction, $1 - b(\nu)$, of each type, ν, faces no occupational choice. This two-stage determination of each agent's prospect allows us to keep the model tractable by making the (conditional on being potentially a researcher) distribution of researchers independent of ability types while accommodating the empirical ratio of workers and researchers for each type.

Individual preferences are defined over (generally random) streams of consumption and effort discounted at rate ρ. In t, the associated expected utility is, therefore, given

[2] E.g., Aghion and Howitt [1992], Grossman and Helpman [1991], Schumpeter [1950].

[3] Equivalently, households whose members are perfectly altruistic towards the next generations and where types are perfectly inherited by the new generations.

by

$$\mathbb{E}_t \int_t^{\infty} e^{-\rho(s-t)} \left(\log c_s - \frac{\ell_s^{1+\eta}}{1+\eta} \right) \mathrm{d}s, \tag{1}$$

in which $c_t \geq 0$ and $\ell_t \geq 0$ denote consumption and effort, respectively. The parameter η corresponds to the inverse of the Frisch elasticity of effort. This kind of utility function is in alignment with the empirical evidence reviewed in Chetty [2006] and is often adopted in the taxation literature.

Effort ℓ by a (ν, κ)-agent generates $l = \nu\ell$ efficiency units of labor as a worker.

II.2 Production technology

There are two sectors in the economy: one produces final consumption goods from intermediate goods, and the other produces intermediate goods from labor. The final goods sector consists of a price-taking representative firm that combines a unit-measure continuum of varieties of intermediate goods according to the CES technology,

$$Y(\mathbf{q}) = \left(\int_0^1 q_i^{\theta} \, \mathrm{d}i \right)^{\frac{1}{\theta}}, \quad 0 < \theta < 1, \tag{2}$$

where $\mathbf{q} \equiv \{q_i\}$. The index $i \in [0, 1]$ specifies the goods' variety, and θ governs the degree of substitution between them.

These intermediate goods are in turn produced by entrepreneurs with exclusive rights to produce them. The measure of goods' varieties is fixed and each entrepreneur produces exactly one of them at a time. Their variety q_i is produced proportionally to effective labor hired, L_i,

$$q_i(L_i) = A_{it} L_i. \tag{3}$$

The time subscript made explicit in the factor productivity A_{it} is to suggest in advance the central role of this factor in the model dynamics. As we shall see this is ultimately related not only to inequality among entrepreneurs but also, in the spirit of Romer [1990], to the diffusion of ideas and their non-rival nature which enables growth. The determinants of A_{it} are explained next in the sections on entrepreneurship and innovation.

II.3 Entrepreneurship

Those engaged in entrepreneurship, broadly speaking, are subdivided into two categories representing their current state: *established entrepreneurs*, who own the right to produce a variety and do so under the conditions previously described, hereafter simply called entrepreneurs; and *researchers*, who are trying to come up with new and better marketable ideas and displace the entrepreneurs who occupy limited market space. We first consider the entrepreneurs.

An entrepreneur's productivity A_{it} is ultimately given by

$$A_{it} = \bar{A}_t x_i^{\phi}, \quad \phi > 0, \tag{4}$$

where $\bar{A}_t$ measures the aggregate technological level and x_i measures idiosyncratic productivity. We focus on x_i in this section.

The relative position of the entrepreneur in the market, which they hope to improve through effort, is measured by his or her idiosyncratic productivity denoted by x. For a given i, x is governed by the law of motion

$$dx_{it} = \mu(\ell_{it})x_{it}dt + \sigma x_{it}dB_{it}, \quad dB_{it} \sim \mathcal{N}(0, dt), \tag{5}$$

that is a geometric Brownian motion with mean growth rate $\mu(\ell_{it}) \geq 0$ and percentage volatility $\sigma > 0$. As an increasing function of ℓ, the growth rate of x can thus be influenced (in expected value) by the effort put in by the entrepreneur. It may be convenient to view this improvement effort as a kind of incumbent research.

Every new business starts with a normalized base productivity $x^0 = 1$. However, if left unchecked, x would tend to infinity so that a stabilizing force is needed. As is known in the literature on income dynamics, the distribution of x converges to particular distributions for some assumptions and a given resetting mechanism.[4] Entrepreneurs can retire, die, or simply be competitively displaced by new entrepreneurs. Above all, in a free market economy the heavier the competition, the harder it is to maintain one's business profitable and the more likely it is for old entrepreneurs to go out of business. Here we model exit as the first occurrence between two independent Poisson processes, faced equally by all entrepreneurs. One has an exogenous arrival rate $\bar{\delta}$ while the other has a rate δ^{cd} increasing in "outside pressure", more precisely defined in the next section.

[4] E.g., Gabaix et al. [2016].

Let us call the former process *natural retirement* and the latter *creative destruction* and denote the total resulting exit rate faced by an entrepreneur by[5]

$$\delta_t \equiv \bar{\delta} + \delta_t^{cd}. \tag{6}$$

Once the entrepreneur of a given variety, i, exits, the production of i is taken over by a new entrepreneur. However, if that happens through the natural retirement process, the variety's x_{it} accumulated up to that point is destroyed and set back to its base level x^0.

Thus, we have the necessary resetting mechanism and have fully described the "life-cycle" of an entrepreneur. Later we elaborate on how in equilibrium this whole dynamic results in a distribution of x_{it} which converges to a Pareto distribution and how this relates to the economy's top income distribution.

II.4 Research

Entrepreneurs were researchers at some time in the past. Since the economy is served by a fixed unitary measure of intermediate varieties, those involved in research are actively trying to come up with a better version of an existing variety to take the place of an incumbent entrepreneur. This research is undirected, that is, not tied to a particular variety, and success comes randomly at a Poisson rate $\bar{\lambda}$ for any given researcher.

If successful, the researcher gains exclusive production rights over a better-quality version of a randomly defined existing variety, rendering its former version obsolete. The entrepreneur who produced that old version must now become a researcher again in order to regain the position. This corresponds to the previously mentioned process of creative destruction.

So, equating entrepreneurial entry and exit flows, and letting R_t denote the measure of researchers, we get

$$\delta_t^{cd} = \bar{\lambda} R_t.$$

This means entrepreneurs are subject to the pressure of competition also from the outside: the more outside research and innovation there is, the faster the rate at which they go out of business. Note, however, that it is not within the powers of an entrepreneur to change the probability with which he or she is displaced.

[5] Note that natural retirement refers to the product or service that the entrepreneur is offering. The agent himself returns to the drawing board to try to return to being an entrepreneur.

Random researchers, with equal probability, end up taking over varieties left by entrepreneurs through the natural retirement process which happens at rate $\bar{\delta}/R$ for any given researcher. This generates a second mechanism through which researchers take the positions of entrepreneurs. Although the switch in positions has the same consequence from a private perspective, this form of replacement does not generate innovation like creative destruction does.

Taking these two processes into account, the total rate at which researchers individually become entrepreneurs is given by

$$\lambda_t \equiv \bar{\delta}/R_t + \bar{\lambda}. \tag{7}$$

This is the counterpart to the definition in equation (6) for entrepreneurs.

The cost of researching is that agents must sacrifice their labor market production. In particular, we assume that a (ν, κ)-agent who is engaged in research generates only $\bar{l} = \xi\nu\ell$ efficiency units of labor for an effort ℓ, with $0 < \xi < 1$ common to all. As previously mentioned, it costs, in addition, κ units of utility in order to enter research. Note that in this latter respect, types differ.

II.5 Innovation

We now explain more precisely the role of innovation and its mechanics. Innovation is the sole responsible for long-term growth since it can indefinitely expand the stock of ideas and technology which determine the total factor productivity of the economy. In order to differentiate the economy's technological level from entrepreneurs' idiosyncratic productivity x_i, we use terms such as *quality* and *technology* to refer to them, though they play the same role of a labor-augmenting factor in production.

Innovation occurs and is spread throughout the economy as a side effect, through two processes already described: creative destruction and incumbent research. When creative destruction occurs, i.e. when the entrant entrepreneur comes up with a new idea, the technological level of a given variety i is raised, increasing A_{it} by a factor of $\gamma > 1$, the step size of innovation. We assume for simplicity that technological diffusion, a positive spillover effect of innovation, is instant and universal so that *all* varieties have their technological level raised by this same factor.

Incumbent research done by entrepreneurs, i.e. x-increasing effort, also contributes to innovation and also has a spillover effect to varieties not their own. Likewise, it generates an increase by factor γ to all varieties' quality but weighted in proportion to their own-productivity growth rate $\mu(\ell_{it})$.

Let n_t denote the cumulative stock of innovation steps at time t and assume $n_0 = 0$. We can define $\bar{A}_t$ in equation (4) as $\bar{A}_t \equiv \gamma^{n_t}$ and get the final form of A_{it},

$$A_{it} = \gamma^{n_t} x_i^{\phi} \tag{8}$$

The aggregated contribution of researchers and entrepreneurs towards innovation at any given time is then,

$$\dot{n}_t = \delta^{cd}(R_t) + \iota\mu_t, \tag{9}$$

where $\iota > 0$ and $\mu_t \equiv \int \mu(\ell_{it})\,\mathrm{d}i$.

Since we have a uniform effect of innovation on all varieties, we get to simplify the analysis by having a single n_t that tracks the common progress of technology, eliminating the need for integration across varieties on different levels of quality.

II.6 Market arrangements

The final good is the numeraire of the economy and the firms producing it are perfectly competitive. Entrepreneurs, in contrast, operate in monopolistic competition, setting prices p_i for their products and hiring labor from workers in a competitive market that pays w per unit of effective labor.

There are no traded assets available.

II.7 Government

The government redistributes income through taxes and transfers. We adopt the constant progressivity tax schedule of Benabou [2002] and others, in which the disposable income of household h is defined by

$$\hat{y}_{ht} \equiv y_{ht}^{1-\tau} \tilde{y}_t^{\tau}, \tag{10}$$

where y_{ht} denotes pre-tax income and the break-even income level $\tilde{y}_t$ is determined by the budget constraint

$$\int_h y_{ht}^{1-\tau} \tilde{y}_t^{\tau}\,\mathrm{d}h = \int_h y_{ht}\,\mathrm{d}h, \quad \forall t. \tag{11}$$

The implied tax function for income y_{ht} is, therefore,

$$T(y_{ht}) \equiv y_{ht} - y_{ht}^{1-\tau} \tilde{y}_t^{\tau}. \tag{12}$$

The policy parameter $\tau \leq 1$ governs the progressivity of the schedule as well as the elasticity of disposable to pre-tax income, $(1 - \tau)$. If $\tau > 0$, then the schedule is progressive and both marginal and average tax rates are increasing in pre-tax income. Otherwise, it is either regressive ($\tau < 0$) and the opposite occurs, or $\tau = 0$, so that no taxes are levied at all. Notably, τ is equivalent to the income-weighted average marginal tax rate:

$$\int_h T'(y_{ht}) \left(\frac{y_{ht}}{Y_t} \right) \mathrm{d}h = \tau.$$

Finally, it is worth noting that the term $\tilde{y}_t$ ensures tax rates are always relative to average income so that the schedule automatically adjusts for the general income growth.

II.8 The household's problem

The household initially chooses the occupation of either worker or researcher. As mentioned, there are no financial assets available to the household to smooth consumption, and they are endowed only with their capacity to exert effort. A (ν, κ)-worker can convert a unit of effort into ν units of effective labor depending. These are, then, used in the production of intermediate goods. Each unit of effective labor is paid a wage, w.

Researchers engage in the process of research already described, but must also supply work in the market in order to finance their consumption. They have, however, an opportunity cost of not working full-time, so that effort is converted at a rate $\xi\nu < \nu$ into effective labor.

Denote by $V_t \equiv \max\{V_t^W(\nu), V_t^R(\nu) - \kappa\}$ the value of the (ν, κ)-non-entrepreneur household at t, where $V_t^W(\nu)$ and $V_t^R(\nu)$ are, respectively, the value of being a worker and of being a researcher, net of κ. Furthermore, let $V_t^E(x)$ be the value of an entrepreneur with productivity x; then we can represent the problem recursively through the Bellman equations,

$$\rho V_t^W(\nu) = \max_{\ell_t} \ \log c_t^W(\ell_t, \nu) - \frac{\ell_t^{1+\eta}}{1+\eta} + \frac{dV_t^W(\nu)}{dt}, \tag{13}$$

$$\rho V_t^R(\nu) = \max_{\ell_t} \ \log c_t^R(\ell_t, \nu) - \frac{\ell_t^{1+\eta}}{1+\eta} + \frac{dV_t^R(\nu)}{dt} + \bar{\lambda} \left(\mathbb{E}_x[V_t^E(x,\nu)] - V_t^R(\nu) \right)$$
$$+ \bar{\delta}/R_t \left(V_t^{E_0}(\nu) - V_t^R(\nu) \right). \tag{14}$$

where consumption is equal to disposable income, i.e., $c_t^W(\ell_t, \nu) = (w_t \nu \ell_t)^{1-\tau} \, \tilde{y}_t^\tau$ and $c_t^R(\ell_t, \nu) = (w_t \xi \nu \ell_t)^{1-\tau} \, \tilde{y}_t^\tau$, and we use the shorthand $V_t^{E_0}(\nu) \equiv V_t^E(x^0, \nu)$. As usual, with log-utility, we get, for both occupations, an optimal choice of effort invariant to productivity, ν, time, t, and wage rate, w: $\ell^W = \ell^R = (1-\tau)^{\frac{1}{1+\eta}}$.

The last two terms in equation (14) account for the researchers' expected value change due to their becoming entrepreneurs in both possible ways. The entrepreneur's value function is somewhat more complicated for the fact that it has a diffusion process for its state variable. Nevertheless, because of that the formulation is apt for the use of Ito calculus so that it can be expressed as

$$\rho V_t^E(x_t, \nu) = \max_{\ell_t} \ \log c_t^E(x_t) - \frac{\ell_t^{1+\eta}}{1+\eta} + \frac{\mathbb{E}_t[dV_t^E(x_t, \nu)]}{dt}$$
$$+ \delta_t \left[V_t^R(\nu) - V_t^E(x_t, \nu) \right]. \tag{15}$$

Given the law of movement of x_t in equation (5), the expected rate of change term is established using Ito's lemma:

$$\frac{\mathbb{E}_t[dV_t^E(x_t, \nu)]}{dt} \equiv \mu(\ell_t) x_t \frac{\partial V_t^E(x_t, \nu)}{\partial x} + \frac{\sigma^2}{2} x_t^2 \frac{\partial^2 V_t^E(x_t, \nu)}{\partial x^2} + \frac{\partial V_t^E(x_t, \nu)}{\partial t}. \tag{16}$$

Note that the entrepreneur's consumption c_t^E is not dependent on ℓ_t. Instead, it is a function of productivity x_t which ultimately determines their income and is influenced by the agent's history of efforts. That is, the way effort creates value for the entrepreneur is by influencing their projected value growth through x_t, as made evident by the first term of equation (16). Therefore, ℓ_t plays a fundamentally different role for entrepreneurs than it does for workers and researchers, a role that more closely resembles "investing into one's business" than anything else.

The derivation of the optimal decision rule is dependent on the effort-converting technology $\mu(\ell_t)$ as well as the relationship between x_t and income, which still needs explaining.[6]

III Equilibrium

To define an equilibrium for our model economy, it will be useful to first characterize the aggregate variables for which we will impose market clearing.

We start by noting that the net value of entering research for type (κ, ν) is

$$\mathcal{D}(\kappa, \nu) \equiv -\kappa + \rho V^R(\nu) - \rho V^W(\nu).$$

This defines thresholds, $\kappa^*(\nu)$, through $\mathcal{D}(\kappa^*(\nu), \nu) = 0$, that can be shown to be, in equilibrium, given by

$$\kappa^*(\nu) = -\underbrace{\frac{(1-\tau)\lambda}{\rho+\lambda+\delta}}_{a} \log \nu + \underbrace{[\rho V^R(1) - \rho V^W(1)]}_{C} = -a \log \nu + C.$$

We can use this expression to define the share of each type, ν, that have the opportunity to become researchers and do become, $r^*(\nu) \equiv \Pr\left(\mathcal{D}(\kappa, \nu) > 0 \mid \nu,\ \chi(\nu) = 1\right)$. In this case, $r(\nu) = b(\nu) r^*(\nu)$ is the share of ν-types that become researchers and $1 - r(\nu)$, the share that are workers. Following from definitions and the distribution of κ,

$$\begin{aligned} r^*(\nu) = H(\kappa^*(\nu)) &= \begin{cases} 1 - e^{-\psi \kappa^*(\nu)} & k^*(\nu) \geq 0 \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} 1 - e^{-\psi C} \nu^{\psi a} & 0 \leq \nu \leq e^{C/a} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Those who opt to engage in research do so hoping to become entrepreneurs at which moment they cease to contribute their effort directly to the production process. Hence, to characterize the aggregate supply of efficiency units of labor we return to the population

[6] While the utility of an entrepreneur with a higher ν is higher than that of an agent with a lower ν through V_t^R this has no bearing on the entrepreneur's choice of effort ℓ, hence on $\mathbb{E}_t[dV_t^E(x_t)/dt]$. The probability δ_t of being replaced is independent of x_t, hence, of ℓ. As a consequence, the additional expected utility that an entrepreneur gets from having a higher ν cannot be altered by his or her choices.

constraint,

$$N = W + R + 1.$$

Among workers, there are those who have no option but to remain workers, comprising a share of $1 - b(\nu)$ for each ν, and those who can become researchers but choose not to, the share $b(\nu)(1 - r^*(\nu))$ for each ν. Adding the two groups we obtain the number of workers in the population,

$$\begin{aligned} W &= N \int \left[1 - b(\nu) + b(\nu)(1 - r^*(\nu))\right] f_\nu(\nu) \mathrm{d}\nu \\ &= \int \underbrace{\left[N(1 - r(\nu)) f_\nu(\nu)\right]}_{\equiv W(\nu)} \mathrm{d}\nu = \int W(\nu) \mathrm{d}\nu. \end{aligned} \tag{17}$$

Even more important for our purposes will be to distinguish, among those who are engaged with entrepreneurship at any given time, those who are established entrepreneurs from those who are in the phase of research:

$$\begin{aligned} R + 1 &= N \int \left[b(\nu) r^*(\nu) f_\nu(\nu)\right] \mathrm{d}\nu \\ \therefore \quad R &= \int \underbrace{\left[N r(\nu) f_\nu(\nu) \frac{R}{R+1}\right]}_{\equiv R(\nu)} \mathrm{d}\nu = \int R(\nu) \mathrm{d}\nu. \end{aligned} \tag{18}$$

It is appropriate to define $R(\nu)$ as the measure of researchers at each ν this way because in equilibrium the ratio of researchers to entrepreneurs must be the same across ν, since all face the same rate of entry and exit.

We can then characterize the aggregate supply of efficient labor as,

$$\begin{aligned} L &= \ell^W \left(\int \nu W(\nu) \mathrm{d}\nu + \xi \int \nu R(\nu) \mathrm{d}\nu \right) \\ &\equiv \ell^W \left(\bar{\nu}^W + \xi \bar{\nu}^R \right), \end{aligned}$$

noting that $\bar{\nu}$ denotes aggregate ν.

III.1 The balanced growth path

Now we lay out an appropriate definition of equilibrium compatible with a balanced growth path and then proceed to characterize it.

Given a government policy τ, a balanced growth path equilibrium consists of a measure of workers and researchers $\{W_t(\nu), R_t(\nu)\}$ of each type; choices of effort $\{\ell_t^W, \ell_t^R, \ell_t^E(x)\}$; intermediate input lists $\{\mathbf{q}_t\}$; tax levels $\{\tilde{y}_t\}$; a distribution $f(x)$; growth rate g; and prices $\{w_t, \{p_{it}\}\}$ which satisfy the following conditions for every t:

1. Each (ν, κ)-agent solves their recursive problems: $V_t(\nu, \kappa)$ for non-entrepreneurs, and; $V_t^E(x, \nu)$ for entrepreneurs. This defines the associated $\ell_t^W, \ell_t^R, \ell_t^E(x)$ decision rules.[7]

2. The final goods firm maximizes profit choosing inputs $\mathbf{q}_t = \{q_{it}\}$ given prices $\{p_{it}\}$; the intermediate goods firms maximize profits given wage rate w_t by setting prices $\{p_{it}\}$ under monopolistic competition.

3. Prices $\{p_{it}\}$ clear the intermediate goods market; wage rate w_t clears the labor market:
$$\int_0^1 L_{it}\,\mathrm{d}i = \ell^W\left(\bar{\nu}^W + \xi\bar{\nu}^R\right). \tag{19}$$

4. W_t, R_t satisfy the population constraint,
$$W_t + R_t + 1 = N, \tag{20}$$
where W and R defined according to (17) and (18), respectively.

5. The government chooses $\tilde{y}_t$ balancing its budget as in equation (11).

6. $f(x)$ is the stationary distribution of x, which satisfies the proper Kolmogorov forward equation.

7. Output grows at constant rate g.

[7] Using the convention that an agent who did not get the chance of becoming a researcher, drew a value $\kappa = \infty$, the definition also applies to these agents.

III.2 Output, income, and productivity

We first characterize output, wage, and profits in equilibrium. All proofs are provided in the appendix.

Proposition 1. *Let $X_t \equiv \int x_{it}\,\mathrm{d}i$ and $L_t \equiv \int L_{it}\,\mathrm{d}i$. Given the market arrangements described and assuming technology parameters are such that $\phi = (1-\theta)/\theta$, output at t is given by*

$$Y_t = \gamma^{n_t} X_t^{\phi} L_t, \tag{21}$$

wage rate by

$$w_t = \theta\gamma^{n_t} X_t^{\phi}, \tag{22}$$

and profits for entrepreneur i by

$$\pi_{it} = (1-\theta)\gamma^{n_t} X_t^{\phi-1} L_t x_{it}. \tag{23}$$

Thus, profits are linear in x_{it} and we can write the entrepreneur's pre-tax income as $y_t^E(x_{it}) = m_t x_{it}$ with m_t defined to conform to (23):

$$m_t \equiv (1-\theta)\gamma^{n_t} X_t^{\phi-1} L_t. \tag{24}$$

The assumption about ϕ and θ is necessary for linearity, so that x_{it} and income share the same distribution. This is not essential to our results but makes the algebra cleaner.

Next, we consider the entrepreneurs' productivity distribution. Let $f(x,t)$ be the probability density function of x at time t and take $f(x,0)$ as given. Assume a fixed $\mu(\ell_{it}) = \mu^*$ common to all entrepreneurs, as will be in our equilibrium. Furthermore, we assume for simplicity that base productivity $x^0 = 1$ is also the minimum productivity possible, i.e. there is a "reflecting barrier" at x^0 which impedes x_t from getting lower than it.[8]

On these conditions, the distribution $f(x,t)$ satisfies, outside the point of reinjection x^0, the following Kolmogorov forward equation:

$$\frac{\partial f(x,t)}{\partial t} = -\bar{\delta} f(x,t) - \frac{\partial}{\partial x}[\mu^* x f(x,t)] + \frac{1}{2}\cdot\frac{\partial^2}{\partial x^2}[\sigma^2 x^2 f(x,t)]. \tag{25}$$

[8] More precisely, it means $x_{t+dt} = \max\{x^0, x_t + dx_t\}$ for small dt. It is possible to relax this assumption — we get a stationary double Pareto distribution [Reed, 2001] —, but the algebra gets more unwieldy for little to no added insight.

If a stationary distribution $f(x) \equiv \lim_{t\to\infty} f(x,t)$ exists, then it must satisfy

$$0 = -\bar{\delta} f(x) - \frac{\partial}{\partial x}[\mu^* x f(x)] + \frac{1}{2} \cdot \frac{\partial^2}{\partial x^2}[\sigma^2 x^2 f(x)], \tag{26}$$

whence we derive the following proposition.

Proposition 2. *Given the assumptions above, the stationary distribution of x which satisfies* (26) *is the Pareto distribution,*

$$f(x) = \begin{cases} z^* x^{-z^*-1} & \text{for } x > 1 \\ 0 & \text{for } x < 1 \end{cases} \tag{27}$$

where

$$z^* = \frac{-\tilde{\mu}^* + \sqrt{(\tilde{\mu}^*)^2 + 2\sigma^2\bar{\delta}}}{\sigma^2} \tag{28}$$

and $\tilde{\mu}^* \equiv \mu^* - \sigma^2/2$.

With this result we settle both the stationary distribution of entrepreneurs' income and the aggregate (or mean) productivity X_t, provided (i) the shape parameter z^* is larger than one, so that $f(x)$ has a finite mean, and (ii) the assumption about constant $\mu(\ell_{it})$ is in fact valid in equilibrium. The latter we show in the next section.

III.3 Research allocation

The important remaining piece of the equilibrium is the allocation of work and research among households. We are interested in equilibria with interior solutions for (W, R) and show later that the allocation must be constant to be consistent with a balanced growth path.

Assume hereafter that

$$\mu(\ell) = \beta \ell^{1-\alpha}, \quad \beta > 0,\ \alpha < 1. \tag{29}$$

Proposition 3. *Given equilibrium allocation R, the optimal entrepreneurial effort on the balanced growth path is*

$$\ell^E = \left(\frac{\beta(1-\alpha)(1-\tau)}{\rho + \delta(R)} \right)^{\frac{1}{\eta+\alpha}}. \tag{30}$$

Two observations about proposition 3 are in order. The first is that the only endogenous variable that affects entrepreneurial effort is the number of researchers, through the creative destruction rate, and nothing else. The second is that the elasticity to tax progressivity τ is higher in absolute value than that of workers and researchers, which equals $-1/(\eta+1)$.

[TO BE DONE]

IV Quantitative Findings

In this section, we compare balanced growth path equilibria for different levels of progressivity. Ours is a dynastic economy, so we consider a social welfare function that aggregates the value functions in period 0. More specifically we consider a Utilitarian metric and convert utility differences by dividing it by the marginal value of resources as measured by the sum of the inverse of marginal utilities of consumption.

Before we provide the details of our exercises it is important to deal with some issues that are particular to our setting. First, economies grow which means that if we compare welfare in different moments in time we will find different levels of welfare. Hence, we must choose a common departure point for the economies we study. Our choice is to compare economies that start at the same total consumption level. That is, we normalize the technological level at period 0 in such a way that the economy under the different policies produces the same total consumption.

Note that this choice, combined with our use of a $\ln$ specification for consumption utility has the advantage of freeing us from choosing a specific policy under which the utility gain is measured: the inverse of the marginal value of resources is the same across all policy specifications in period 0.

A second issue regards the fixed κ. In describing our environment we have assumed that this is a price paid once and for all. The economy starts and in the long run, we reach (approximate arbitrarily well) the balanced growth path. Since the economies have different growth rates, comparing the costs after a large number of periods is problematic.

To make sense of our simply taking the difference between the average value of κ, we interpret κ as a flow that is proportional to the aggregate consumption level of the economy. Under this assumption, we simply compare the aggregate cost by adding all the costs paid and use the same conversion to represent the utility difference in consumption units.

In the following exercises we assume the functional form $b(\nu) = 1 - e^{-\bar{b}\nu}$.

The social welfare criterion we use is Utilitarian, given by[9]

$$\mathcal{U} \equiv \int V_{h,0}\, \mathrm{d}h. \tag{31}$$

Let us define D as the average household expected lifetime disutility from effort and κ at time zero. Furthermore, define $\bar{c}_{ht}$ and $\bar{y}_{ht}$ as, respectively, the certainty equivalent stream of consumption of household h and the corresponding pre-tax income necessary to attain it, i.e. $\bar{c}_{ht} = \tilde{y}_t^{\tau} \bar{y}_{ht}^{1-\tau}$, when holding D unchanged. It can be shown that in a balanced growth path, the criterion is given by

$$\rho\mathcal{U} = g/\rho - \rho D + \log y_0 + \int \log \left(\frac{\bar{y}_{h0}}{y_0}\right)^{1-\tau} \mathrm{d}h - \log \int \left(\frac{y_{h0}}{y_0}\right)^{1-\tau} \mathrm{d}h, \tag{32}$$

where y with no household subscript denotes per capita income.

IV.1 Parametrization

The parameters of the model are either calibrated using empirical targets or taken from estimates and conventional values in the literature and we assume that a unit of time t corresponds to a year. The full baseline parametrization is summarised in table 1.

Entrepreneurial sector. We target a δ corresponding to the yearly rate of establishment exit recorded by the Business Dynamics Statistics survey from the U.S. Census Bureau, which averages 9.80% since the year 2000. Setting the exogenous part $\bar{\delta}$ to 40% of the total rate, we determine $\bar{\lambda}$ using the remaining endogenous part $\delta^{CD} = \bar{\lambda}R$. Both parameters concerning access to entrepreneurship are set to reasonable values which allow mid-range ability workers to access and choose research: $\psi = 1$ and $\bar{b} = 3$. Figure 1 plots both the share of households that have the opportunity and choose entrepreneurship

[9] Consider all integration over households normalized by population size: $\int \mathrm{d}h = 1$.

	Parameter	Value	Source
Population size	N	17.74	SCF
Discount rate	ρ	0.015	—
Labor supply elasticity	η	2	Chetty [2012]
Workers' ability dispersion	σ_ν	0.806	SCF
Exog. researcher entry rate	$\bar{\lambda}$	0.033	U.S. Census Bureau
Exog. entrepr. exit rate	$\bar{\delta}$	0.040	$\bar{\delta}/\delta = 40\%$
Barriers to entrepreneurship.			
Cost parameter	ψ	1	—
Opportunity parameter	$\bar{b}$	3	—
Productivity growth technology			
Curvature parameter	α	0	—
Level parameter	β	0.039	SCF
Productivity/income volatility	σ	0.197	Guvenen et al. [2021]
Incumbent innovation parameter	ι	9.363	Garcia-Macia et al. (2019)
Researcher's effective labor	ξ	0.634	SCF
Final production parameter	θ	0.675	SCF
Intermediate production parameter	ϕ	0.482	$\phi = (1-\theta)/\theta$
Tax progressivity	τ	0.181	Heathcote et al. (2017)
Growth step size	γ	1.083	Growth $= 2\%$

Table 1: Baseline parameter values

given their type ν, i.e. $r(\nu)$, as well as the (scaled) distribution of ν in each occupation which results. For income volatility we use an estimate of Guvenen et al. [2021] who study income dynamics in the U.S. We set the volatility $\sigma = 0.197$ of persistent innovations. σ was estimated for their intermediate specification with Gaussian innovations and unemployment shocks, which best resembles our model. Their study is based solely on labor income data but does include self-employment income attributed to labor and uses a very large dataset, yielding a precise estimation and a good fit.

Income inequality. Using data from the Survey of Consumer Finances (SCF) we calculate five moments, averaged over the seven last surveys (2001 through 2019), which are targeted for calibration. Those moments are: the Gini coefficient of income (0.56); the ratio of the 90th to the 10th percentile of income, P90/P10 (11.17); the share of income due to the top 1% of earners (19.8%); the share of income due to entrepreneurs (35.3%); and the share of entrepreneurs in the population (16.36%). To match the

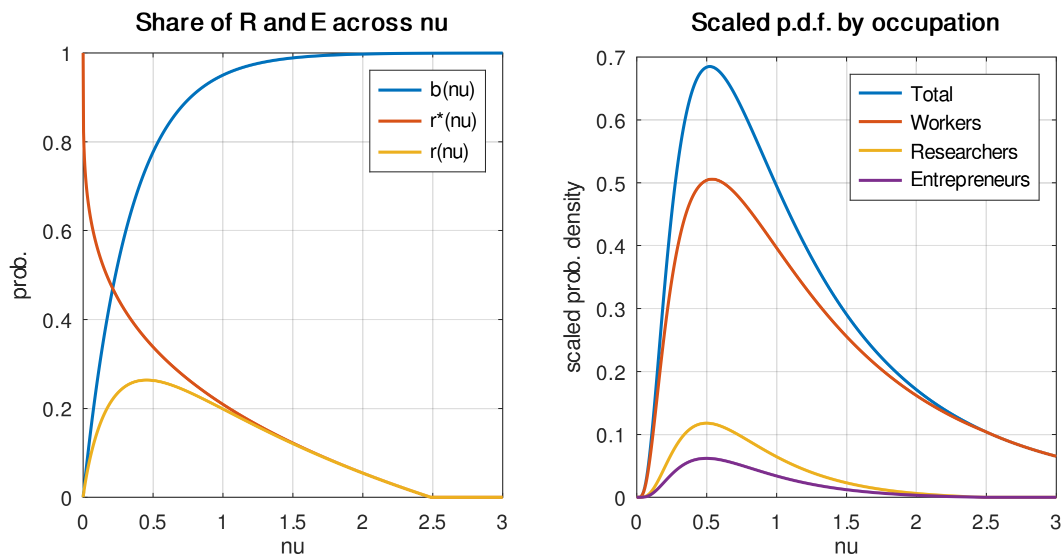

Figure 1: The entrepreneurial sector across ν: The entrepreneurial sector across ν': The left panel displays the share of the population for each type, ν in the three occupations: researcher and entrepreneur. The panel on the right displays the scaled (by the fraction of workers in each occupation) densities for each of the three occupations.

model's concept of entrepreneurship with the data we define it in the broadest sense: the last two moments, and any others concerning entrepreneurship, are computed for the model taking into account both researchers and established entrepreneurs. In the data, we consider entrepreneurs as those who reported being either self-employed or private business owners. Using these targets we manage to replicate the data well along some relevant dimensions using the values for $N, \sigma_\nu, \beta, \xi$, and θ reported in table 1. Beyond the targeted moments, the model closely replicates some other income distribution features reported in table 2 and in the Lorenz curve plotted in figure 2. One feature the model does not succeed in replicating is the inequality among entrepreneurs, which it overshoots. Figure 3 plots the distribution of income for all occupations, making it clear that entrepreneurship is characterized by either low or high income depending on the state one occupies, giving rise to a high disparity in the cross-sectional distribution.

Demography and preferences. The time discount rate is set at a standard value, $\rho = 0.015$. The disutility of effort parameter is set at $\eta = 2$, generally consistent with estimates of the Frisch elasticity [Chetty, 2012]. As already mentioned, population size N is calibrated so that entrepreneurs comprise the population share of 16.4% one

	Data	Model
Targeted moments		
Gini coefficient	0.566	0.566
P90/P10	11.17	11.17
Top 1% share of income	19.79	19.79
Entrpr.'s share of income	35.27	35.27
Entrpr.'s share of population	16.36	16.36
Non-targeted moments		
Bottom 5% share of income	0.39	0.47
Top 5% share of income	35.72	34.60
Top 10% share of income	46.15	45.74
Entrpr.'s Gini	0.646	0.767

Table 2: Moments of the income distribution: data vs. model

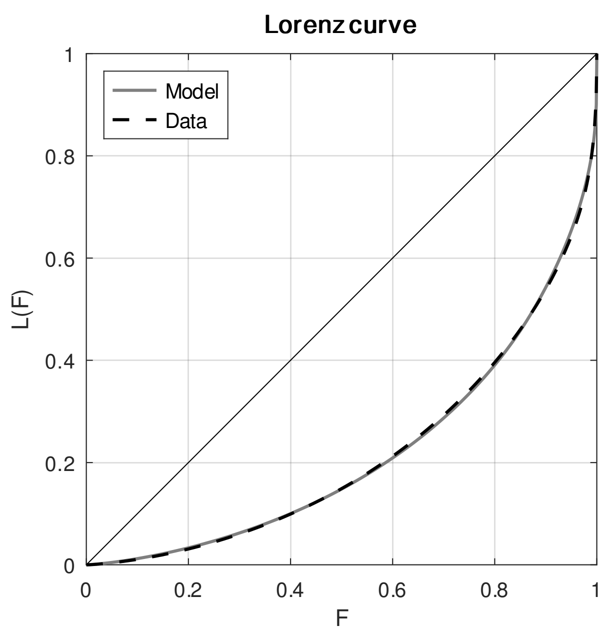

Figure 2: Lorenz curve for income: data vs. model

observes in the data.

Taxation and growth. The baseline tax progressivity parameter $\tau = 0.181$ is taken from the estimation of Heathcote et al. [2017] for the statutory tax rates in the U.S.

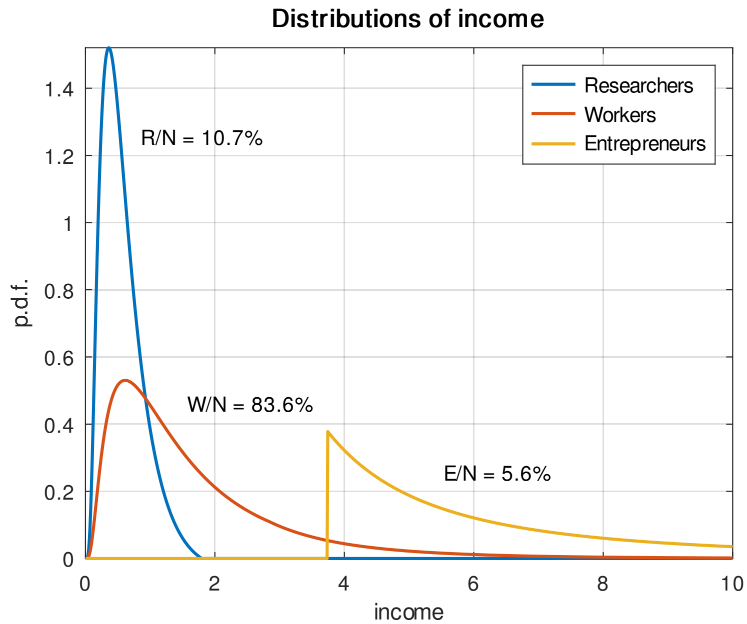

Figure 3: Distributions of income (before taxes) for each occupation.

using the same functional form for the tax schedule we use here.[10] Finally, in relation to innovation and growth we need to set ι and γ. The former we calibrate so that the innovation due to creative destruction is one-fourth of the total [Garcia-Macia et al., 2019] and the latter is set so that the economy grows at a rate $g = 0.02$.

IV.2 Optimal taxation

Now we turn to some exercises of optimal taxation. We adjust the progressivity parameter, τ, keeping the government budget constraint fixed on a period-by-period basis and compute the Utilitarian value attained. Our main findings are displayed in Figure 4. The optimal value for τ is 0.129 which is lower than the current value of 0.181. The concavity on τ of the Utilitarian criterion optimum is explained by the action of some countervailing forces. By reducing incentives to work and to become a researcher increased progressivity lowers the aggregate costs of effort and economizes on the fixed value κ. On the other hand, it hurts growth and general output.

Figure 5 describes the following experiment. We consider the impact of progres-

[10]Their estimation shows that this simple tax schedule fits very well the actual distribution of pre- and post-government income ($R^2 = 0.91$).

sivity on the Utilitarian objective when growth is not responsive to taxes. We find that a substantially larger progressivity level is optimal, τ. If we now apply this level of progressivity in our economy, we find a 9% welfare loss, thus pointing to relevant consequences of abstracting from the impact of progressivity on growth.

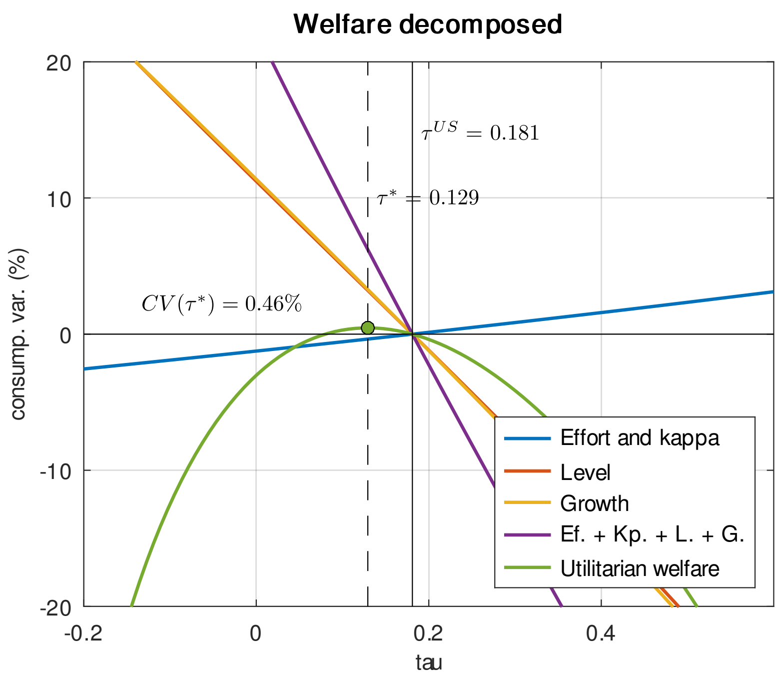

Figure 4: Welfare variations across τ decomposed into some of its components.

V Conclusion

In this paper, we have explored the equity-efficiency trade-off due to progressivity in labor income tax in a world of endogenous growth. We find that the optimal level of progressivity for the U.S. tax system is slightly lower than the current one. However, transitioning to the optimal system results in only moderate welfare gains. On the contrary, if the growth consequences of progressivity are disregarded, the prescription would advocate substantially more progressivity, leading to significant welfare losses.

This study sheds light on the multifaceted implications of tax policies on growth and distribution, informing policymakers and academics alike. However, for the model to remain tractable we have not taken into account the potential heterogeneity in research ability. As Jaimovich and Rebelo [2017] have shown this may cause (even linear) taxes

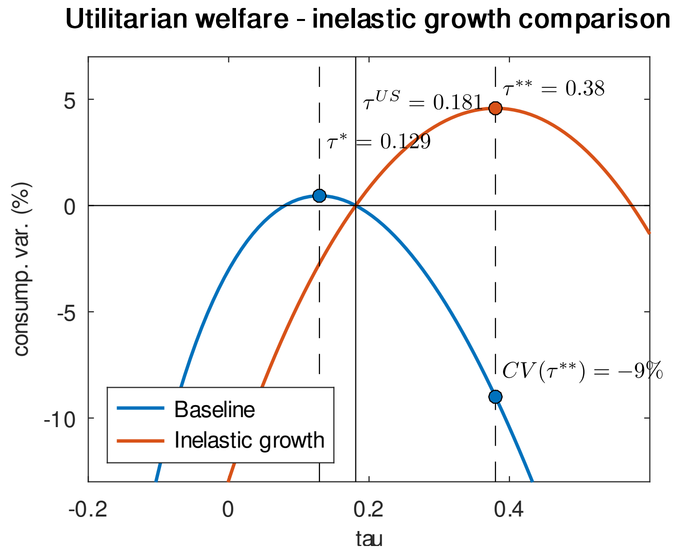

Figure 5: Welfare variations when comparing economies with elastic and inelastic growth.

to have a non-linear impact on growth. We believe that this type of consideration would reinforce our concerns with increased progressivity, but leave this potentially important dimension of the problem for future work.

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A Derivations

Proof of Proposition 1. The final goods firm has to solve at a given time

$$\max_{\mathbf{q}} \left(\int_0^1 q_i^\theta \, \mathrm{d}i \right)^{\frac{1}{\theta}} - \int_0^1 p_i q_i \, \mathrm{d}i, \tag{33}$$

so that the first-order conditions for q_i yield the inverse demand curve

$$p_i(q_i) = \left(\frac{Y}{q_i} \right)^{1-\theta}. \tag{34}$$

The entrepreneurs solve, through their intermediate firms,

$$\max_{q_i} \; p_i(q_i) q_i - w L_i(q_i), \tag{35}$$

so that optimal pricing consists of the usual $1/\theta$ markup over marginal cost. When combined with (34), we can write

$$q_i = \left(\frac{1}{\theta} \frac{w}{\gamma^n x_i^\phi} \right)^{\frac{1}{\theta-1}} Y. \tag{36}$$

Now if we plug (36) into the final goods production function, using our assumption of $\phi = (1-\theta)/\theta$ and definition $X \equiv \int x_i \, \mathrm{d}i$, we get our final equation for wages:

$$w = \theta \gamma^n \left(\int_0^1 x_i^{\phi \frac{\theta}{1-\theta}} \, \mathrm{d}i \right)^{\frac{1-\theta}{\theta}} = \theta \gamma^n X^\phi. \tag{37}$$

In order to obtain the expression for Y, simply substitute (37) into (36), and then again into $L_i(q_i)$ and the aggregate labor definition $L \equiv \int L_i \, \mathrm{d}i$ to get

$$L = \frac{Y}{\gamma^n X^\phi} \tag{38}$$

which is the final form of Y.

At last, the expression for profits follows from substituting the expression for prices and wages into the profit equation,

$$\pi_i = \max_{q_i} \left(\frac{Y}{q_i}\right)^{1-\theta} q_i^\theta - \theta\gamma^n X^\phi \frac{q_i}{\gamma^n x_i^\phi}, \tag{39}$$

and then plugging the optimal q_i in order to get

$$\pi_i = (1-\theta)\gamma^n X^{\phi-1} L x_i \tag{40}$$

□

Proof of proposition 2. Let us guess that the stationary distribution takes the form $f(x) = Cx^{-z-1}$ and substitute into (27) to get

$$\begin{aligned} 0 &= -\delta^* C x^{-z-1} + z\mu^* C x^{-z-1} + z(z-1)\frac{\sigma^2}{2} C x^{-z-1} \\ &= C x^{-z-1}\left[-\delta^* + z\mu^* + z(z-1)\frac{\sigma^2}{2}\right]. \end{aligned}$$

For the equation to hold for all x the term in brackets must be zero. Solving for the positive root we get z^* in (28) and $C = z^*$ follows from $f(x)$ integrating to one.

□

Constant growth. Observation: constant R_t is sufficient for constant growth. Remember that

$$\dot{n}_t = g_t/\log\gamma = \delta_t^{cd} + \iota\mu(\ell_t^E) = \bar{\lambda} R_t + \iota\beta(\ell_t^E)^{1-\alpha},$$

and therefore constant R_t, which is also sufficient for constant ℓ_t^E as stated in proposition 3, is all that is required for constant growth.

Workers' value function. On a BPG, workers' value is given by

$$\begin{aligned} \rho V_t^W &= gt + \frac{g}{\rho} + u_0^W \\ &= gt + \frac{g}{\rho} + \tau\log\tilde{y}_0 + (1-\tau)\left(\log w_0 + \frac{\log(1-\tau)-1}{1+\eta}\right) \end{aligned}$$

where u_t^W denotes utility flow at t from optimal consumption and effort. To show this, simply note that $\tilde{y}_t$ and wages w_t grow at constant rate, i.e. substitute $w_t = w_0 e^{gt}$ etc., substitute optimal effort $\ell^W = (1-\tau)^{\frac{1}{1+\eta}}$, and solve the integral in (1).

Entrepreneur and researcher's value function. From (15) we can establish the first-order condition for optimal entrepreneurial effort,

$$\ell_t^E(x_t) = \left(\beta(1-\alpha)x_t \frac{\partial V_t^E(x_t)}{\partial x}\right)^{\frac{1}{\eta+\alpha}}. \tag{41}$$

Judging the value function of the entrepreneur, we begin by guessing it is of the form $V_t^E(x) = V_t^{E_0} + C\log x$, where, again, $V_t^{E_0} \equiv V_t^E(x^0)$. Inserting the guess in equation (41) we get a constant optimal choice

$$\ell^E = (\beta(1-\alpha)C)^{\frac{1}{\eta+\alpha}}, \tag{42}$$

and the Ito calculus term defined in equation (16) reduces to

$$\frac{\mathbb{E}_t[dV_t^E(x_t)]}{dt} = C\underbrace{\left(\mu(\ell^E) - \frac{\sigma^2}{2}\right)}_{\tilde{\mu}} + \frac{dV_t^{E_0}}{dt}. \tag{43}$$

Then, inserting our guess into the left-hand side of (15) we can compare the coefficients of $\log x$ to get

$$C = \frac{1-\tau}{\rho+\delta}, \tag{44}$$

which is indeed constant in the BGP where R, and therefore δ, are constant.

Next, we form an ordinary differential equations system between V_t^R and $V_t^{E_0}$ in the BGP which, denoting by u_t^R and $u_t^{E_0}$ their respective utility streams, can be written as

$$\begin{pmatrix} \dot{V}_t^R \\ \dot{V}_t^{E_0} \end{pmatrix} = \begin{bmatrix} \rho+\lambda & -\lambda \\ -\delta & \rho+\delta \end{bmatrix} \begin{pmatrix} V_t^R \\ V_t^{E_0} \end{pmatrix} - \begin{pmatrix} u_t^R + \frac{1-\tau}{\rho+\delta}\frac{\bar{\lambda}}{z} \\ u_t^{E_0} + \frac{1-\tau}{\rho+\delta}\tilde{\mu} \end{pmatrix}, \tag{45}$$

using the fact that, if z is the equilibrium $f(x)$ shape parameter, $\log x$ is exponentially distributed with $\mathbb{E}[\log x] = 1/z$. Note that due to constant income growth, both utility streams are linear in time, specifically, $u_t = u_0 + gt$. So we use that functional form with undetermined coefficients to look for a particular solution for the system. Solving for the coefficients, we get

$$\rho V_t^R = gt + \frac{g}{\rho} + \tau \log \tilde{y} + \frac{(\rho+\delta)v^R + \lambda v^E}{\rho+\lambda+\delta} \tag{46}$$

$$\rho V_t^{E_0} = gt + \frac{g}{\rho} + \tau \log \tilde{y} + \frac{(\rho+\lambda)v^E + \delta v^R}{\rho+\lambda+\delta} \tag{47}$$

where v^R, v^E are related to the value streams that are particular to the time spent in each occupation:

$$v^R = (1-\tau)\left[\log(\xi w_0 \ell^R) + \frac{\bar{\lambda}/z}{\rho+\delta}\right] - \frac{(\ell^R)^{1+\eta}}{1+\eta} \tag{48}$$

$$v^E = (1-\tau)\left[\log(m_0) + \frac{\tilde{\mu}}{\rho+\delta}\right] - \frac{(\ell^E)^{1+\eta}}{1+\eta} \tag{49}$$

The value difference between work and research is, then, constant in the BGP and given by

$$\begin{aligned}\rho(V_t^R - V_t^W) &= \frac{\rho+\delta}{\rho+\lambda+\delta}(1-\tau)\left(\log\xi + \frac{\bar{\lambda}/z}{\rho+\delta}\right) \\ &+ \frac{\lambda}{\rho+\lambda+\delta}\left[(1-\tau)\left(\log\left(\frac{m_0}{w_0\ell^W}\right) + \frac{\tilde{\mu}}{\rho+\delta}\right) - \frac{(\ell^E)^{1+\eta} - (\ell^W)^{1+\eta}}{1+\eta}\right]. \end{aligned} \tag{50}$$

The indifference condition implied by utility maximization then sets the equality to zero and pins down the equilibrium number of researchers R.

A note on the "weights" of v^E and v^R. In a continuous-time Markov chain with two states $\{1,2\}$ and rate matrix

$$Q = \begin{bmatrix} -\lambda & \lambda \\ \delta & -\delta \end{bmatrix} \tag{51}$$

the probability matrix $P(t)$ with entries $p_{ij} = Prob(\chi_t = j|\chi_0 = i)$, where χ_t denotes the state at t, is given by

$$P(t) = \begin{bmatrix} \frac{\delta}{\delta+\lambda} + \frac{\lambda}{\delta+\lambda}e^{-(\lambda+\delta)t} & \frac{\lambda}{\delta+\lambda} - \frac{\lambda}{\delta+\lambda}e^{-(\lambda+\delta)t} \\ \frac{\delta}{\delta+\lambda} - \frac{\delta}{\delta+\lambda}e^{-(\lambda+\delta)t} & \frac{\lambda}{\delta+\lambda} + \frac{\delta}{\delta+\lambda}e^{-(\lambda+\delta)t} \end{bmatrix}. \tag{52}$$

The coefficients of v^R and v^E, fundamentally acting as weights, in equations (46) and (47) correspond to the present value (times ρ), discounted at ρ, of the probabilities in $P(t)$, that is,

$$\rho \int_0^\infty e^{-\rho t} P(t) \mathrm{d}t = \begin{bmatrix} \frac{\rho+\delta}{\rho+\lambda+\delta} & \frac{\lambda}{\rho+\lambda+\delta} \\ \frac{\delta}{\rho+\lambda+\delta} & \frac{\rho+\lambda}{\rho+\lambda+\delta} \end{bmatrix}. \tag{53}$$

Thus we have, for example, a researcher weighting the value v^R according to the present value of the probability he remains in research across time.

Social welfare criteria. Let us consider two social welfare criteria: a utilitarian aggregation and an aggregate efficiency criterion Benabou [2002]. First, define D as the average household expected lifetime disutility from effort at time zero; $\bar{c}_{ht}$ and $\bar{y}_{ht}$ as, respectively, the certainty equivalent stream of consumption of household h and the corresponding pre-tax income necessary to attain it, i.e. $\bar{c}_{ht} = \tilde{y}_t^\tau \bar{y}_{ht}^{1-\tau}$, when holding expected disutility from effort unchanged. The criteria are defined as [11]

$$\mathcal{U} \equiv \int V_{h,0} \, \mathrm{d}h \tag{54}$$

$$\mathcal{E} \equiv \int_0^\infty e^{-\rho t} \log \bar{C}_t \, \mathrm{d}t - D, \tag{55}$$

where $\bar{C}_t \equiv \int \bar{c}_{ht} \, \mathrm{d}h$. Then, it can be shown that in a BGP the criteria reduce to

$$\rho\mathcal{U} = g/\rho - \rho D + \log y_0 + \int \log \left(\frac{\bar{y}_{h0}}{y_0}\right)^{1-\tau} \mathrm{d}h - \log \int \left(\frac{y_{h0}}{y_0}\right)^{1-\tau} \mathrm{d}h \tag{56}$$

$$\rho\mathcal{E} = \underbrace{g/\rho - \rho D + \log y_0}_{\equiv \rho\mathcal{W}} + \underbrace{\log \int \left(\frac{\bar{y}_{h0}}{y_0}\right)^{1-\tau} \mathrm{d}h - \log \int \left(\frac{y_{h0}}{y_0}\right)^{1-\tau} \mathrm{d}h}_{\log\left(\frac{\int \bar{y}_{h0}^{1-\tau} \mathrm{d}h}{\int y_{h0}^{1-\tau} \mathrm{d}h}\right)} \tag{57}$$

where y with no household subscript is per capita income.[12] Due to Jensen's inequality, we can see that $\mathcal{U}$ is no greater than $\mathcal{E}$, and due to risk aversion, that $\mathcal{E}$ is no greater than $\mathcal{W}$. The presence of either risk or income inequality determines equality or inequality:

[11] Consider all integration over households normalized by population size: $\int \mathrm{d}h = 1$.

[12] When we present the welfare decomposition we use $\rho\mathcal{U} - \log y_0$ and $\rho\mathcal{E} - \log y_0$, instead of $\rho\mathcal{U}$ and $\rho\mathcal{E}$ to preclude the initial level of income, which is somewhat arbitrary, to affect the wrong attribution of different aspects of allocation changes.

inequality	risk	
yes	yes	$\mathcal{U} < \mathcal{E} < \mathcal{W}$
no	yes	$\mathcal{U} = \mathcal{E} < \mathcal{W}$
yes	no	$\mathcal{U} < \mathcal{E} = \mathcal{W}$
no	no	$\mathcal{U} = \mathcal{E} = \mathcal{W}$

Certainty equivalent. Workers are not subject to risk. Therefore, $c_t^W = \bar{c}_t^W$. As for researchers and entrepreneurs, if we define

$$r^R = \frac{\rho + \delta}{\rho + \lambda + \delta} \quad \text{and} \quad r^E = \frac{\rho + \lambda}{\rho + \lambda + \delta},$$

and s^χ, as the population share of occupation χ, then we may use the value functions to derive the deterministic consumption streams which attain the same utility,

$$\frac{\bar{c}_t^R}{y_t} = \frac{\bar{c}_0^R}{y_0} = \left(\frac{\tilde{y}_0}{y_0}\right)^\tau \left(\frac{\bar{y}_0^R}{y_0}\right)^{1-\tau}, \tag{58}$$

where

$$\left(\frac{\bar{y}_0^R}{y_0}\right)^{1-\tau} = \left[\left(\frac{\theta\xi}{s^W + \xi s^R} \exp\left\{\frac{\bar{\lambda}/z}{\rho+\delta}\right\}\right)^{r^R} \left(\frac{(1-\theta)}{s^E z/(z-1)} \exp\left\{\frac{\tilde{\mu}}{\rho+\delta}\right\}\right)^{1-r^R}\right]^{1-\tau},$$

and

$$\frac{\bar{c}_t^E}{y_t} = \frac{\bar{c}_0^E}{y_0} = \left(\frac{\tilde{y}_0}{y_0}\right)^\tau \left(\frac{\bar{y}_0^E}{y_0}\right)^{1-\tau}, \tag{59}$$

where

$$\left(\frac{\bar{y}_0^E}{y_0}\right)^{1-\tau} = \left[\left(\frac{\theta\xi}{s^W + \xi s^R} \exp\left\{\frac{\bar{\lambda}/z}{\rho+\delta}\right\}\right)^{1-r^E} \left(\frac{(1-\theta)}{s^E z/(z-1)} \exp\left\{\frac{\tilde{\mu}}{\rho+\delta}\right\}\right)^{r^E}\right]^{1-\tau}.$$

Here, we have also used the following facts:

$$w = \theta Y/L, \qquad L = \ell^W (W + \xi R), \tag{60}$$

$$m = (1-\theta)Y/X, \qquad X = z/(z-1). \tag{61}$$