

# Bonds and Stocks Returns Comovements in US and Brazil: Are they Different?

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July, 2023

## Abstract

This paper studies the comovements of bond and stock returns over time. Using the 17 years available data for Brazil, rolling regressions of bonds into stocks excess returns document a positive beta, substantially high between 2015 and 2020, showing that bonds are risky and badly hedge the stock market. Although macro variables alone poorly explain this comovement, the correlation of inflation and output gap significantly does: higher (lower) inflation with lower (higher) output decreases (increases) both bond and stock prices. As previously documented for the USA, with negative bond-stock beta for the last 20 years, these macro factors are relevant. For Brazil, as an emerging economy, risk and international flows arise as also relevant variables behind this positive co-movement: higher credit risk is associated with lower bond and stock prices and vice versa, moreover higher investment by foreigners is associated with higher bond and stock prices and vice versa.

## 1 Introduction

This paper studies the correlation of bonds returns with the stock market. Positive bond-stock betas<sup>1</sup> means that bonds are risky: they co-move together with the stock market. Since they are risky, they input a risk premium for holding it.

In the USA, the bond-stock beta was positive in the 1980's and 1990's, but from the 2000' and on has became negative, making the US treasury bond a safe asset. According to [Campbell, Pflueger and Viceira \(2020\)](#), in the 1980's the bond betas were positive, because the stagflation was driving the bond prices down due to inflation, and the stocks down due to recession. Since the 2000's the bond-stock betas there turned negative because, in growth moments, stock prices increase and inflation decreases bonds prices, and, in recessions, stock prices fall but desinflation increases bond prices. So, since then, bonds are safe and hedge the stock market. Once they became good hedges, their price also increases in recessions. Analogously, when the bonds were risky, their price used to fall even more in bad moments of high marginal utility and high risk aversion.

[Pflueger \(2023\)](#) also argues that a quickly responsive inflation-focused monetary policy in the USA 1980's were crucial for the positive bond-stock beta for driving down stock prices. In the USA 2000's a more inertial output-focused monetary policy contributed for having pro-cyclical inflation and negative bond-stock beta. She builds counter-factuals in a new-keynesian asset-pricing model and shows that, even in the USA 1980's with those supply shocks, if the monetary policy was not quickly responsive and inflation-focused, the bond-stock betas would be negative, as well. So supply shocks and a responsive monetary policy are both necessary for positive beta.

In the last 17 years, we find that the bond-stock beta was very positive in Brazil for a continuous interval of 5 years, and around 0 on the other years. Macro factors as inflation and output do play a significant role here too,

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<sup>1</sup>The beta of regressing bonds returns on stock returns.

but we also investigate the effects of other factors as risk and international flows, given the Brazilian condition of emerging economy. For example, in Brazil, not only the non-hedge property of the asset in a recession may drive the price down, but also other factors may as fiscal sustainability, generating a systematic risk component that makes bonds and stock prices comove.

## 2 Data

For Brazilian bonds we use the IDkA provided by Anbima, starting in Jan/2006. The "IDkA Pré" for nominal bonds is made with LTNs and NTN-Fs with available maturities of 3M, 1Y, 2Y, 3Y, 4Y and 5Y, which are built as: "Os Índices de Duração Constante são gerados a partir de uma aplicação teórica inicial de 1.000,00 unidades monetárias no tempo (t), no ativo sintético (vértice n da ETTJ), que é vendido no dia útil imediatamente posterior (t+1), pela taxa da ETTJ de n-1, gerando um novo valor financeiro a ser reinvestido pela taxa do vértice n nesta mesma data."<sup>2</sup>

Let  $x = \ln(X)$ . All the returns are annual (Ex. holding an asset from July to July). Let  $p_t^{(n)}$  be the price of a bond in time  $t$  with maturity  $n$ .  $i_t^{(n)}$  is the log yield, then

$$i_t^{(n)} = -\frac{1}{n}p_t^{(n)}.$$

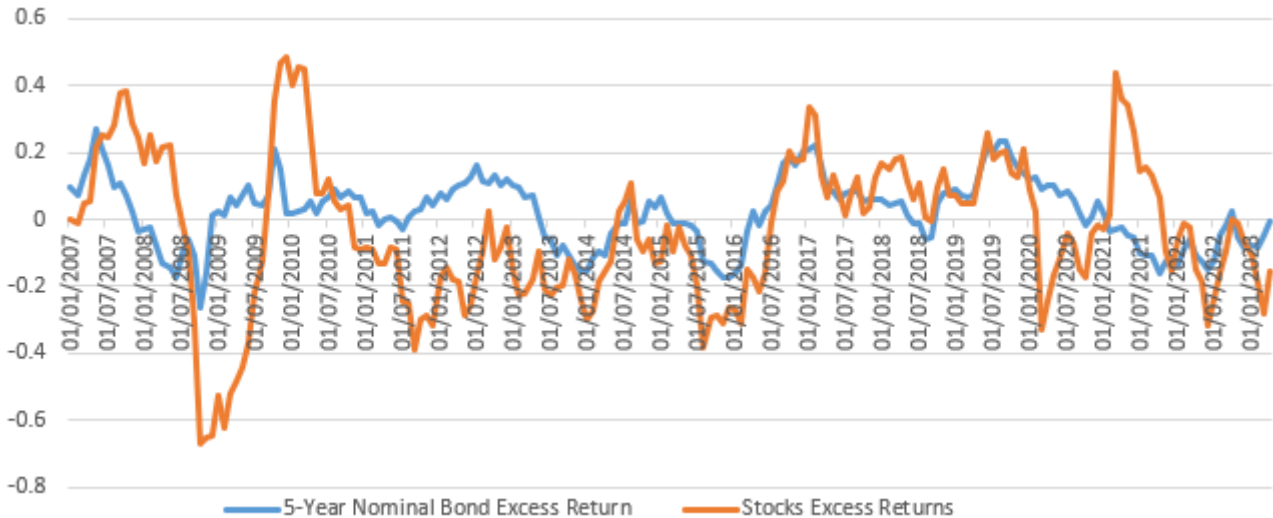
As described by Anbima, the IDkA ( $IDkA_t$ ) would evolve as

$$IDkA_{t+1} = IDkA_t \cdot \frac{P_{t+1}^{n-1}}{P_t^n} \rightarrow \ln\left(\frac{IDkA_{t+1}}{IDkA_t}\right) = p_{t+1}^{n-1} - p_t^n,$$

and  $r_{t+1}^n = p_{t+1}^{n-1} - p_t^n$  is exactly the log of the ex-post return of holding the asset. The excess log return is  $rx_{t+1}^n = r_{t+1}^n - i_t^{(1)}$ , which is the return in excess of the 1-year bond return<sup>3</sup>. For  $i_t^{(1)}$  we compute it from the time series "Taxa de juros prefixada - estrutura a termo - LTN - 12 meses - (% a.a.)" from Ipeadata. [Cochrane and Piazzesi \(2005\)](#): "By focusing on excess returns, we net out inflation and the level of interest rates, so we focus directly on real risk premia in the nominal term structure."

The return of the stock ( $s$ ) market  $r_t^s$  is given by the difference of the log of the Ibovespa. The stock excess return is  $rx_t^s = r_t^s - i_t^{(1)}$ .

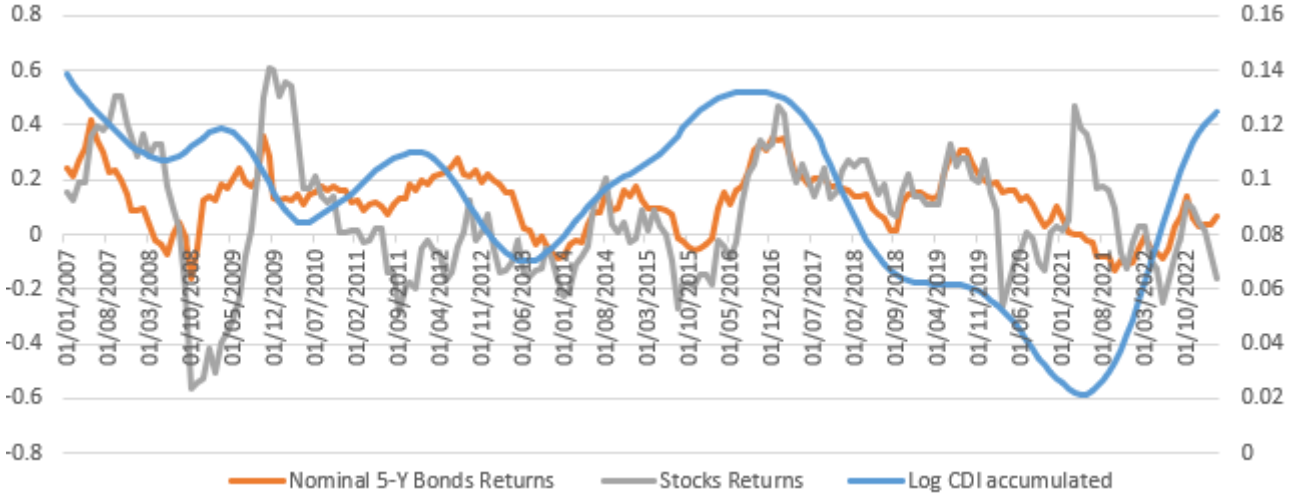
Figure 1: Excess Returns



<sup>2</sup>Further details on the methodology on [https://www.anbima.com.br/pt\\_br/informar/precos-e-indices/indices/idka.htm](https://www.anbima.com.br/pt_br/informar/precos-e-indices/indices/idka.htm)

<sup>3</sup>We use excess over 1-Y bond for having comparability with the Carry Trade. But, it is very similar computing excess returns over the accumulated interbank rate (CDI), see Figure 16.

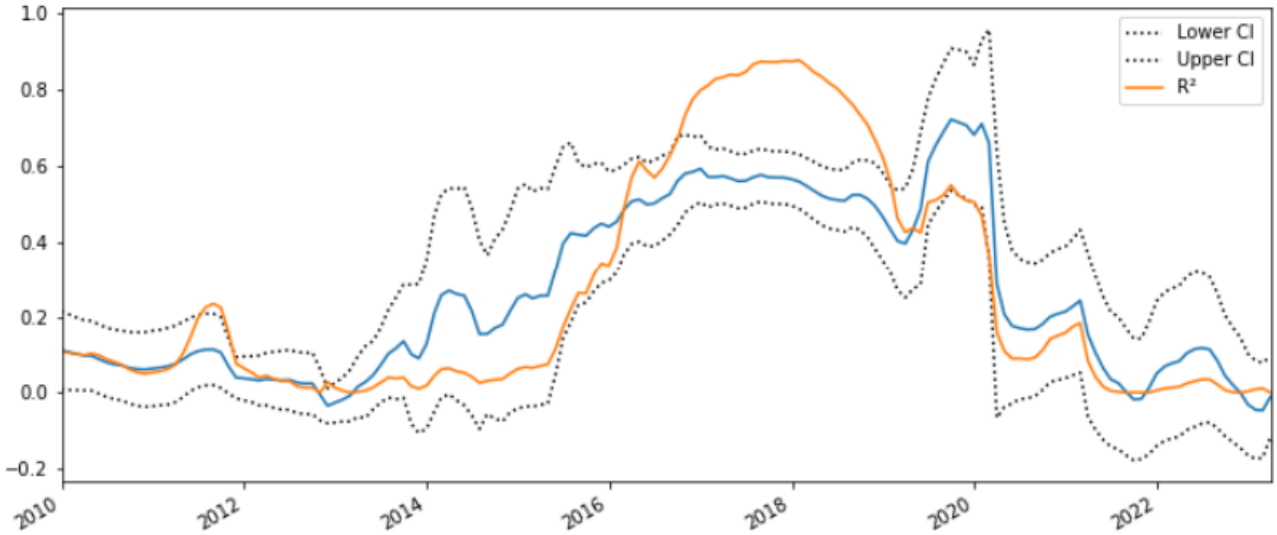
Figure 2: CDI accumulated (left), Ibov and 5Y Nominal Bond Return (right)



### 3 Nominal Bond Betas

Now we run Rolling OLS, that is OLS regressions on moving windows. Here I use a window of 3 years (36 observations for each regression). Estimates covariance matrices calculated using Newey-West correction method with 18 lags for autocorrelation and heteroscedasticity issues. We compute the nominal bond-stock betas, and plot with a 95% confidence interval and the  $R^2$ s (Figure 3). From 2015 to beginning of 2020, the series are highly positively correlated. From 2006 to 2014, and after 2020, they are non-significantly positively correlated.

Figure 3:  $rx_t^{Pre5} = \alpha + \beta.r x_t^s + \epsilon_t$



Below, we plot rolling correlations (2 and 3 years windows) of IPCA 12 months with both the output gap and the output growth.<sup>4</sup> It is visible that the period of higher  $R^2$  and most positive bond-stock beta coincides with the period of sustained negative correlation between the inflation and output (2015 ~ 2019). In 2015-16, the stagflation promoted the positive bond beta as in the 1980's USA. In 2017-2019, the desinflation implemented increased the bond prices, and slightly positive GDP growth lead to increase in stock prices as well. A responsive inflation-focused monetary policy might have contributed as well. In 2015-16 with high interest rates decreasing

<sup>4</sup>I use the IBC-br for proxying the GDP to have monthly data, the gap comes from the HP filter and the growth is the annual log difference.

the bond and stock prices. And, in 2017-19 achieving lower inflation, then with lower interest rates increasing bond and stock prices.

Figure 4: 2 and 3 Year Rolling Window Correlation Between Inflation and Output Gap

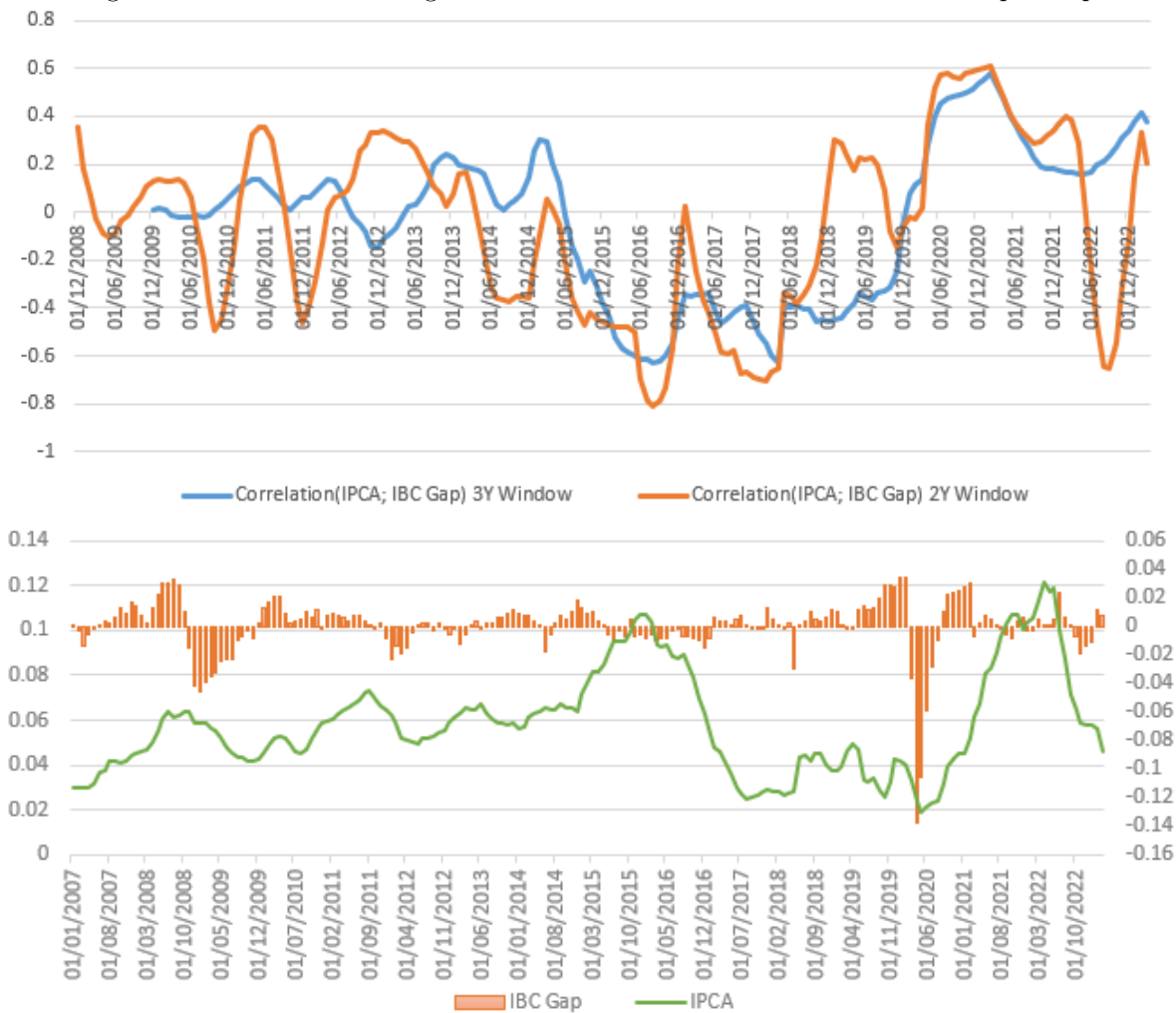
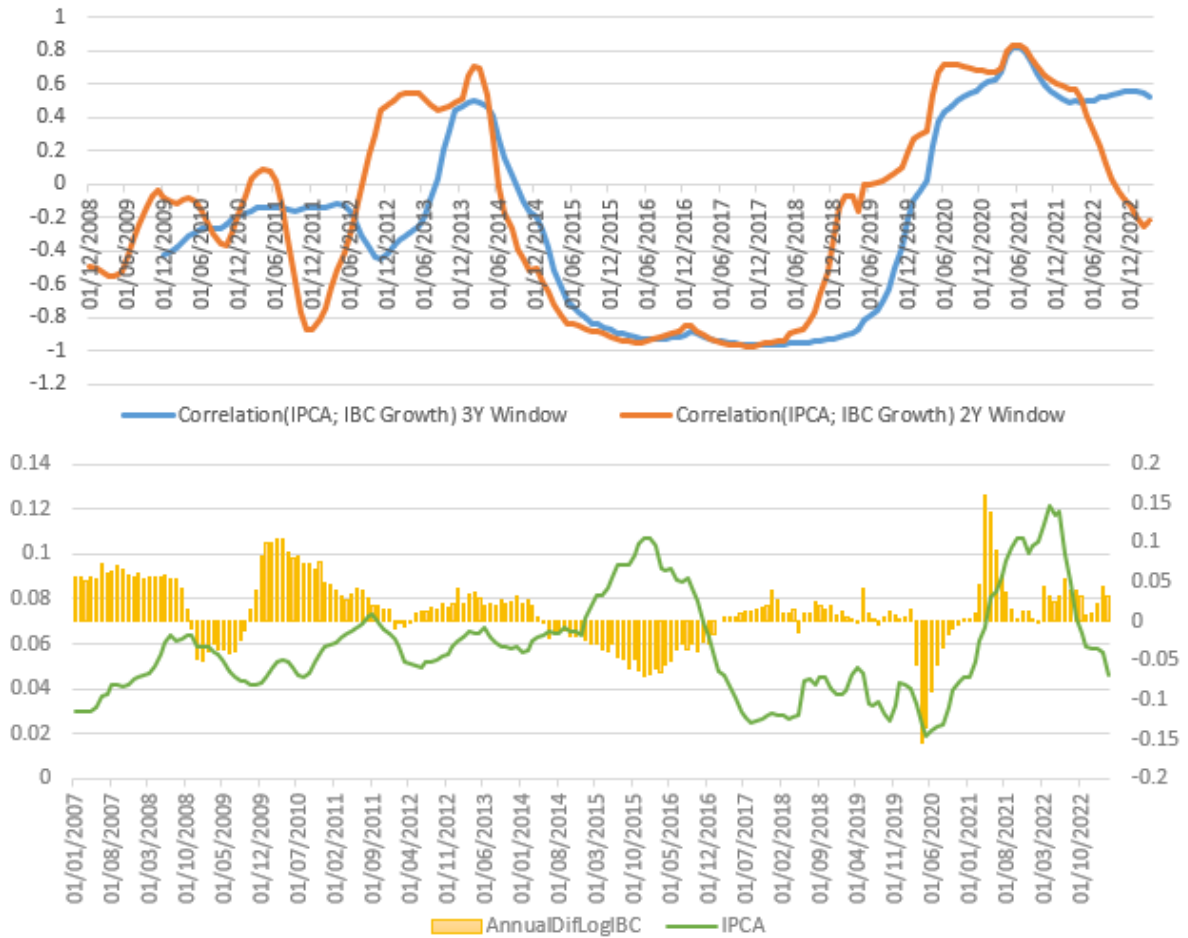
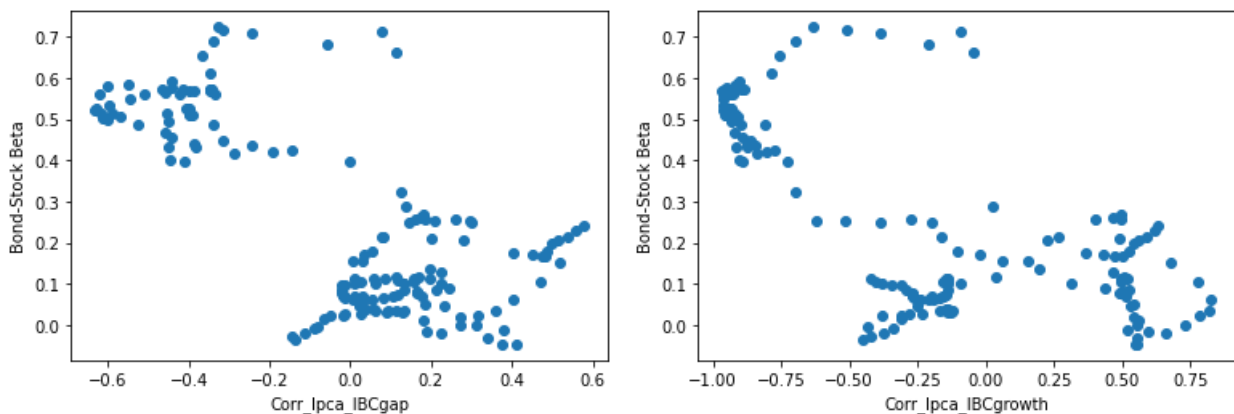


Figure 5: 2 and 3 Year Rolling Window Correlation Inflation and Annual Log Difference of IBC-Br



The scatter plots below of the bond-stock betas against the inflation-output correlation shows clearly this. But one question that arises is why in Brazil the positive inflation-output correlation did not generate negative bond betas? Before 2015 there was not a persistent period of positive correlation, except by around 2013. But from 2020 to 2022 this happened and the betas did not flip sign. Why? Maybe there is a systematic variable that drives both bond and stock prices together.



Then, we regress the Bond-Stock Beta on these variables (results on Table 2). For this analysis all the variables are standardized. The IBC-br gap alone or the IPCA alone explain very few of the bond-stock beta variation. In spite of this, regressing on the moving 3-year window correlation between IPCA and IBC-br presents an  $R^2$  of 0.527 (gap) and 0.495 (growth), with a coefficient statistically and economically significant: a more negative correlation (Ex: Stagflation) is associated with a more positive bond-stock beta. The inflation-

output relation is priced on this variable.

Table 1:  $BondStockBeta = \alpha + \beta.x_t + \epsilon_t$ ; for different  $x_t$ .

$x_t$	$\hat{\beta}$	P-Value	$R^2$
Corr(IPCA;IBC gap) 3Y Window	-0.7259	0.000	0.527
Corr(IPCA;IBC growth) 3Y Window	-0.7038	0.000	0.495
Annual Log Diff IBC-Br	-0.3708	0.006	0.137
IPCA	-0.2814	0.223	0.079
IBC Gap	0.1259	0.231	0.016

Then, we analyze the associations of different variables with the bonds excess returns and the stocks excess returns. Below I run regressions of the excess returns separately onto different variables.

Table 2: Returns Regressions (Standardized)

<i>Regression</i>	$\hat{\beta}$	P-Value	$R^2$
$ExcssPre5_t = \alpha + \beta.IPCA_t + \epsilon_t$	-0.5873	0.000	0.345
$ExcssPre5_t = \alpha + \beta.AnnualLogDiffCDS_t + \epsilon_t$	-0.5789	0.001	0.335
$ExcssPre5_t = \alpha + \beta.AnnualLogDiffEMBI_t + \epsilon_t$	-0.4477	0.007	0.200
$ExcssPre5_t = \alpha + \beta.LogCDS_t + \epsilon_t$	-0.3734	0.008	0.139
$ExcssPre5_t = \alpha + \beta.AnnualLogDiffDebtGDP_t + \epsilon_t$	0.2501	0.127	0.063
$ExcssPre5_t = \alpha + \beta.PortfInvest_t + \epsilon_t$	0.2418	0.131	0.058
$ExcssPre5_t = \alpha + \beta.AnnualLogDiffFX_t + \epsilon_t$	-0.1967	0.277	0.039
$ExcssPre5_t = \alpha + \beta.FinancialAccount_t + \epsilon_t$	0.1535	0.363	0.024
$ExcssPre5_t = \alpha + \beta.CurrentAccount_t + \epsilon_t$	0.1358	0.421	0.018
$ExcssPre5_t = \alpha + \beta.FXOrders_t + \epsilon_t$	-0.0608	0.711	0.004
$ExcssPre5_t = \alpha + \beta.ForeignFlowsCVM_t + \epsilon_t$	-0.0128	0.910	0.000
$ExcssIbov_t = \alpha + \beta.AnnualLogDiffCDS_t + \epsilon_t$	-0.8443	0.000	0.713
$ExcssIbov_t = \alpha + \beta.AnnualLogDiffEMBI_t + \epsilon_t$	-0.7504	0.000	0.563
$ExcssIbov_t = \alpha + \beta.AnnualLogDiffFX_t + \epsilon_t$	-0.6389	0.000	0.408
$ExcssIbov_t = \alpha + \beta.AnnualLogDiffDebtGDP_t + \epsilon_t$	0.5344	0.001	0.286
$ExcssIbov_t = \alpha + \beta.IBCGrowth_t + \epsilon_t$	0.4542	0.002	0.206
$ExcssIbov_t = \alpha + \beta.CurrentAccount_t + \epsilon_t$	0.3515	0.005	0.124
$ExcssIbov_t = \alpha + \beta.IBCGap_t + \epsilon_t$	0.3399	0.050	0.116
$ExcssIbov_t = \alpha + \beta.FinancialAccount_t + \epsilon_t$	0.3387	0.023	0.115
$ExcssIbov_t = \alpha + \beta.FXOrders_t + \epsilon_t$	0.1556	0.439	0.024
$ExcssIbov_t = \alpha + \beta.ForeignFlowsCVM_t + \epsilon_t$	0.0844	0.267	0.007
$ExcssIbov_t = \alpha + \beta.PortfInvest_t + \epsilon_t$	-0.0002	0.999	0.000

Matching the idea of the inflation-output correlation as main driver of the bond-stock beta, we have that inflation was the variable with highest  $R^2$  for explaining nominal bonds returns. The output was also relevant for explaining stocks returns, but was not the most relevant. Risk variables as the CDS and EMBI arised with higher explanatory power. They also showed up relevant for bond returns in the same direction (with negative coefficient) forcing positive bond-stock beta. In the USA, on [Campbell, Pflueger and Viceira \(2020\)](#)

the risk aversion comes from recessions (high marginal utility periods) and booms (low marginal utility). In high marginal utility moments, people run for risky assets, as 1980's nominal bonds, driving down its prices. When the bonds turned safe due to the inflation-output correlation, in recession moments a flight to safety movement happens forcing bond prices up. In Brazil, there was positive inflation-output correlation in 2013 and 2020-22, but the bond-stock beta did not flip sign, and on this period the correlation with CDS keeps very negative (see Figures 6). So the risk dynamics is different.

The Debt to GDP has statistically significant coefficients, but with a positive sign that makes no economic sense at first.

For international flows variables I used the Current Account, Financial Account and Portfolio Investment from the Balance of Payments, the Foreign Exchange Orders Movement from the BCB, and the Foreign Investment Flows from CVM. None of them presented significant explanatory power for bonds excess returns, with higher  $R^2$ s for Portfolio Investment (0.06) and Financial Account (0.02). For explaining stocks returns the Current Account is significant ( $R^2 = 0.124$ ) and after it comes the Financial Account ( $R^2 = 0.115$ ). Other measures are not apparently correlated. Intriguingly, risk variables presented high correlations with the returns, the exchange rate presented high correlation ( $R^2 = 0.04$  for bonds and  $R^2 = 0.41$  for stocks), but this movement on prices did not appeared as much on quantities.

Figure 6: Excess Returns and CDS

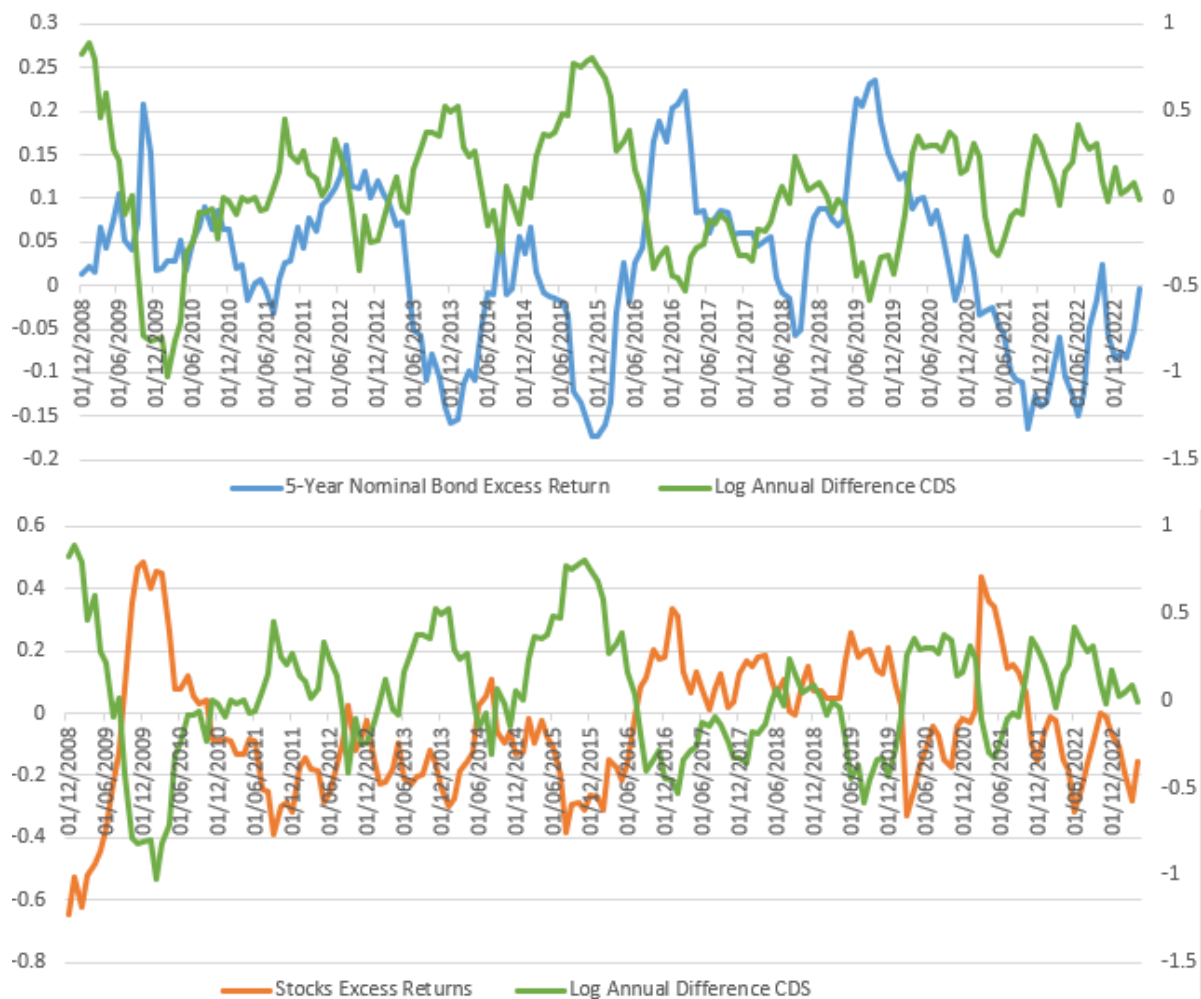




Figure 7: Excess Returns and Portfolio Investment

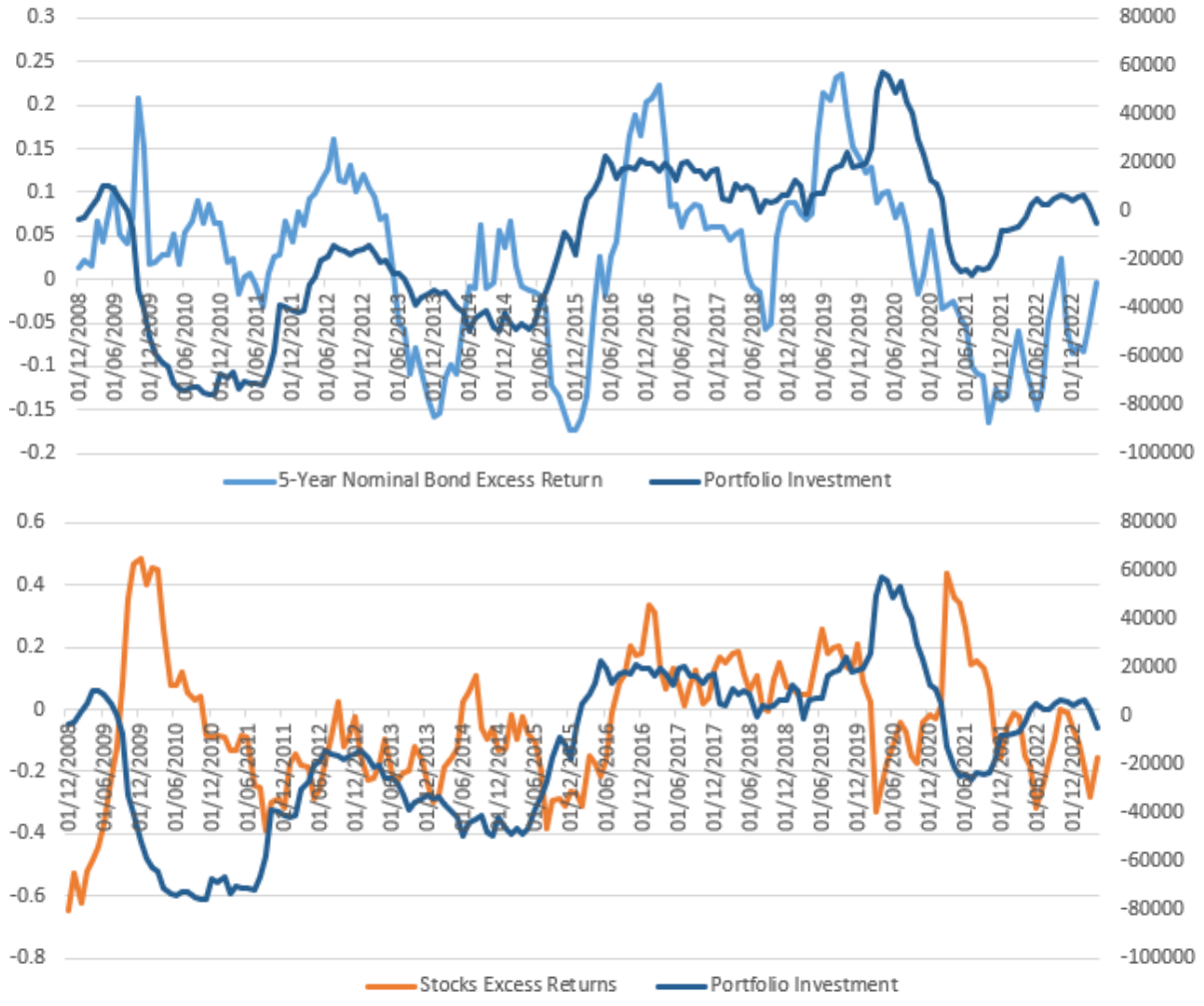
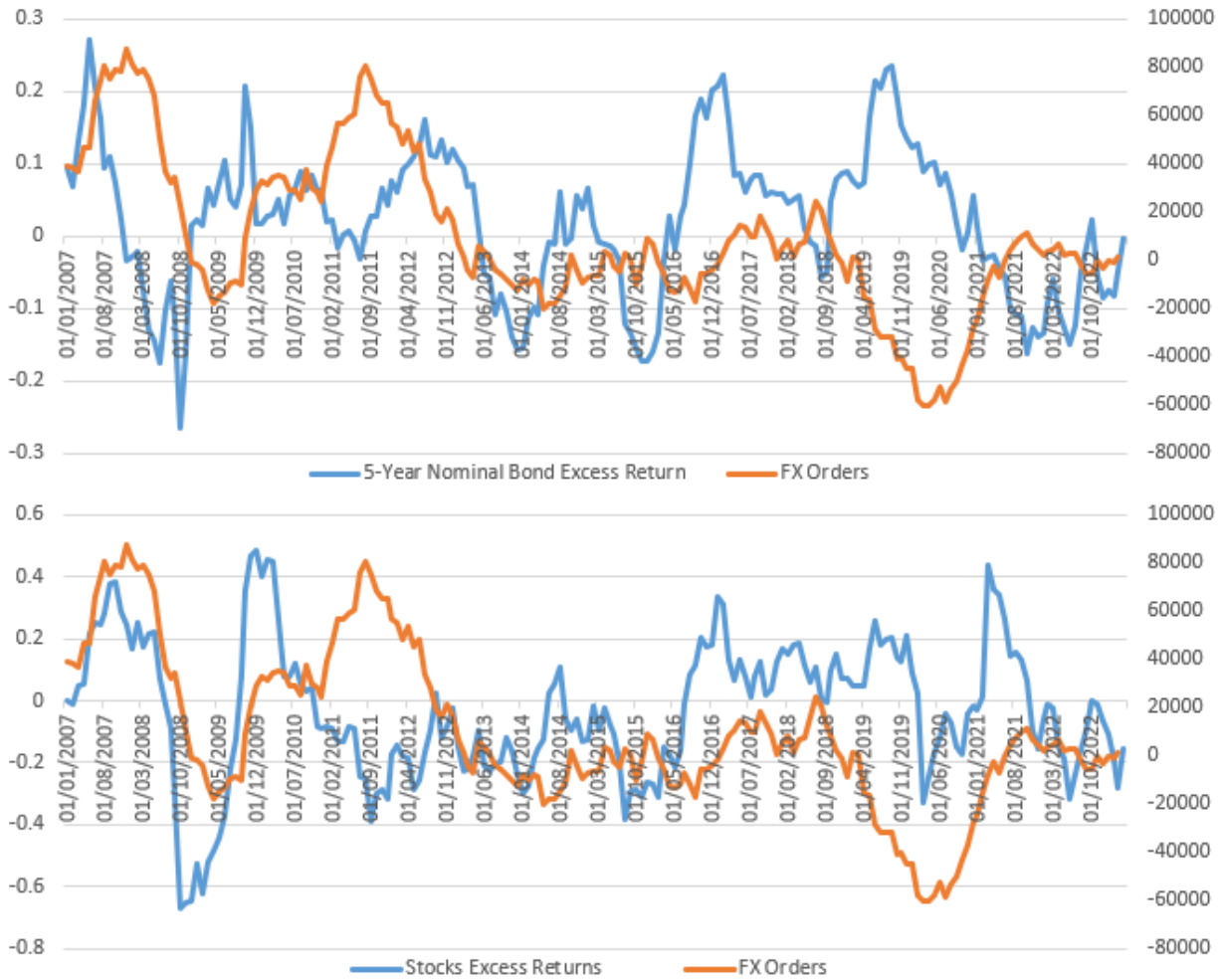




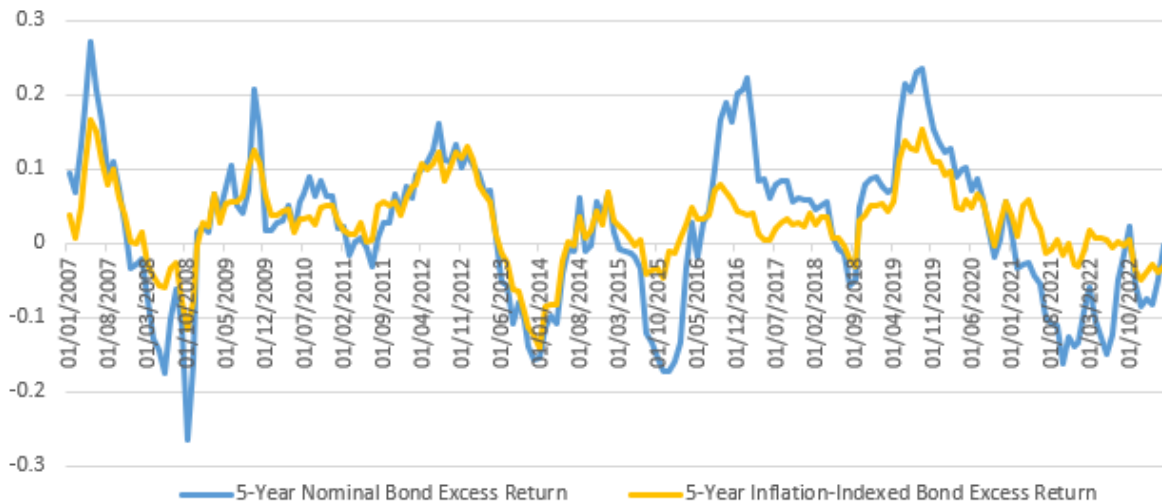
Figure 8: Excess Returns and FX Orders



## 4 Inflation-Indexed Bond Betas

Anbima also provides the "IDkA IPCA" for inflation-indexed bonds, build with NTN-Bs for maturities of 2, 3, 5, 10, 15, 20 and 30 years. It works similarly as the "IDkA Pré" but the variation of the inflation (given by the IPCA) is always incorporated to the index/price. For comparison purposes I use here the 5-year bond.

Figure 9: Nominal and Inflation-Indexed Bond Excess Returns



I run the rolling regressions  $rx_t^{IPCA} = \alpha + \beta.r x_t^s + \epsilon_t$  for computing the inflation-indexed bond-stock betas. The patterns of the coefficients, P-values and  $R^2$ s are very similar from the nominal bond beta, but here the  $R^2$ s are slightly smaller showing that nominal bonds are more correlated with stocks than inflation bonds.

Figure 10:  $rx_t^{IPCA} = \alpha + \beta.r x_t^s + \epsilon_t$

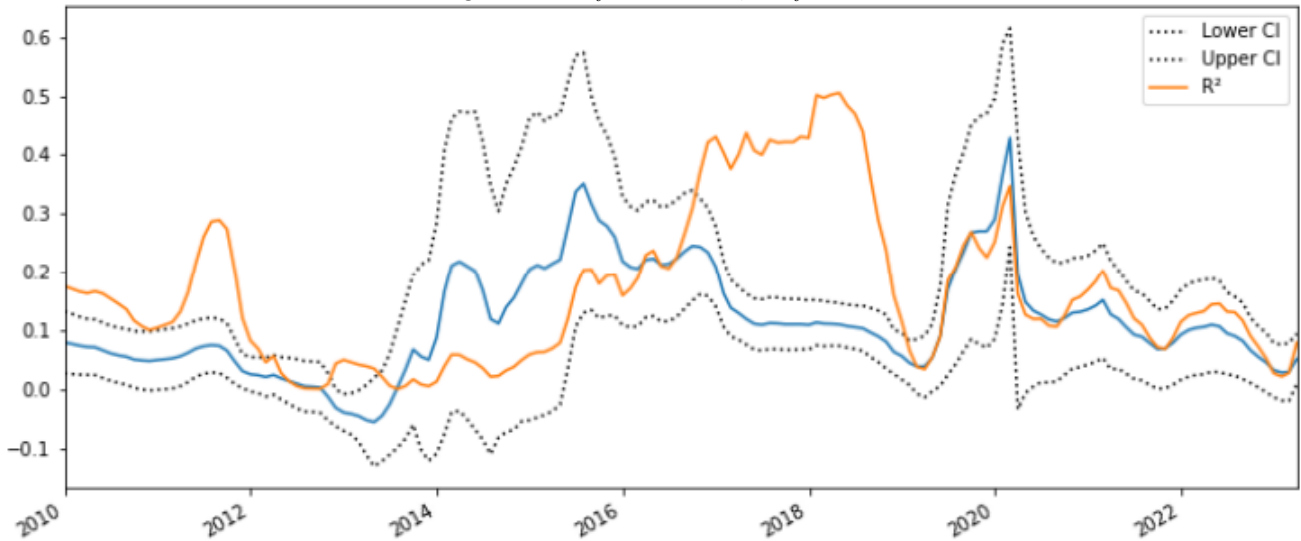
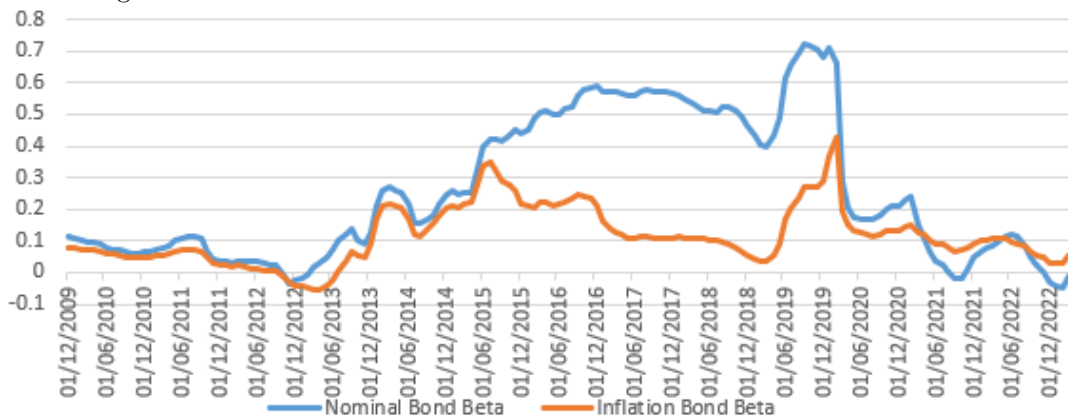
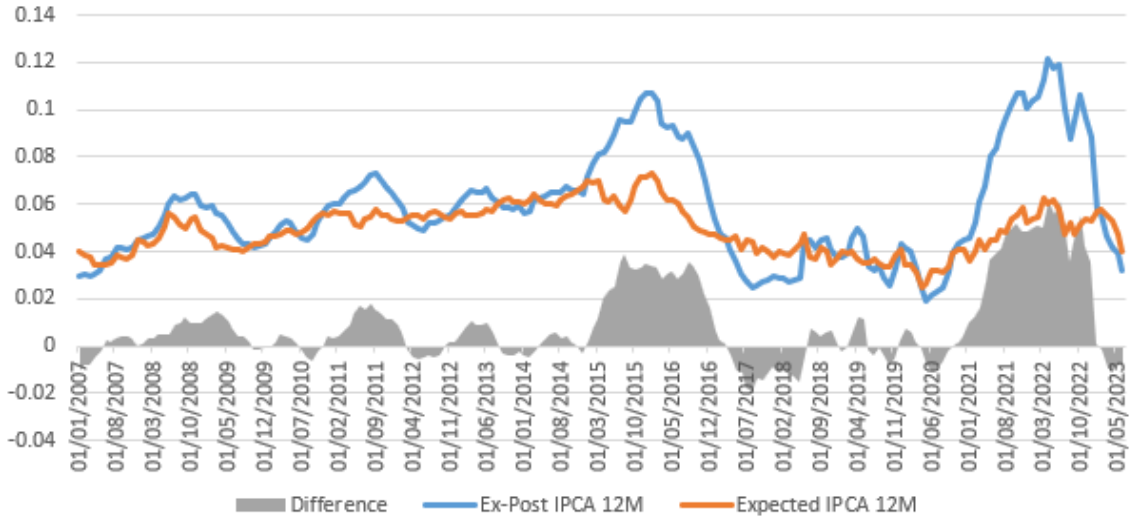


Figure 11: Nominal Bond-Stock beta and Inflation-Indexed Bond-Stock Beta



Mainly on 2015 to 2020 appears a gap between both betas (Figure 11), that was already visible on the difference between their excess returns (Figure 9). Nominal bonds excess returns are more volatile and has larger bond-stock betas. This comes from deviations of realized inflation (adjusted ex-post for the inflation bonds) and expected inflation. In 2015 the inflation was higher than expected, providing higher returns for the inflation bond, and from 2016 to 2020 the opposite happened (see Figure 12). Furthermore, the risk premium, which arises from holding a fixed rate bond when the future inflation is unknown, also may be relevant.

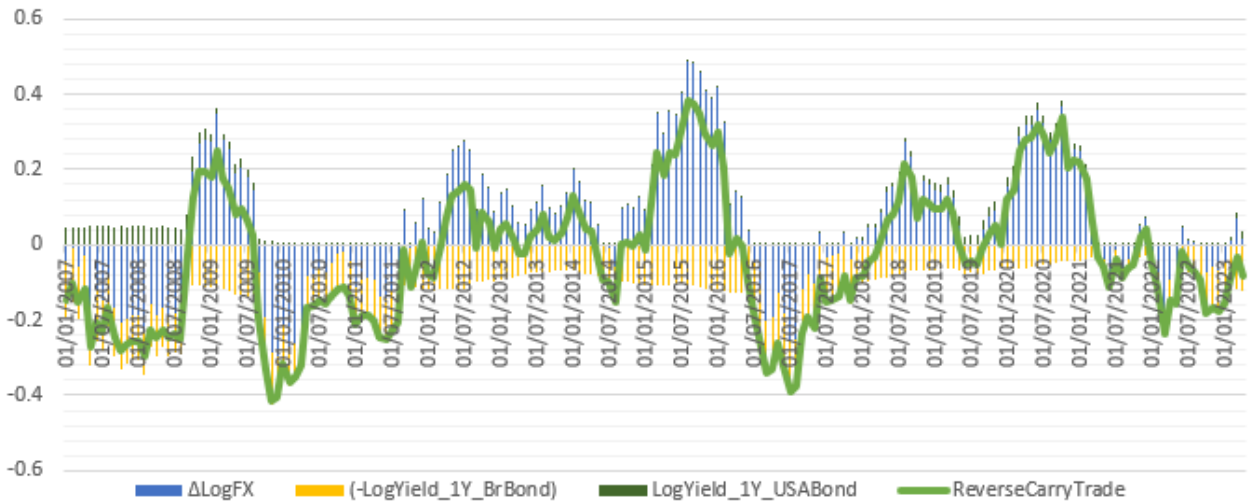
Figure 12: Inflation Ex-Ante and Ex-Post



## 5 Carry Trade

Let  $e_t$  be the log of the nominal exchange rate given by the price of one US Dollar in Brazilian Reals. Then the return on "reverse" carry trade of a Brazilian investing in the USA is given by  $rx_{t+1}^{rc} = i_t^{(1)*} - i_t^{(1)} + \Delta e_{t+1}$ . For the 1-year US bond  $i_t^{(1)*}$  I use the "Market Yield on U.S. Treasury Securities at 1-Year Constant Maturity, Quoted on an Investment Basis" by the FRED.

Figure 13: Reverse Carry Trade Components



As explained, bonds returns co-moves with stocks and are risky. Due to risk being a central determinant of this pattern, and driving both stocks and bonds returns in the same direction, the reverse carry trade arises as an excellent hedge. The reverse carry trade is mainly moved by FX (Figure 13). When risk increases leading to decrease of bonds and stocks returns, the exchange rate depreciates increasing reverse carry trade returns.

Figure 14: 3Y Rolling OLS Reverse Carry Trade into Nominal Bonds  
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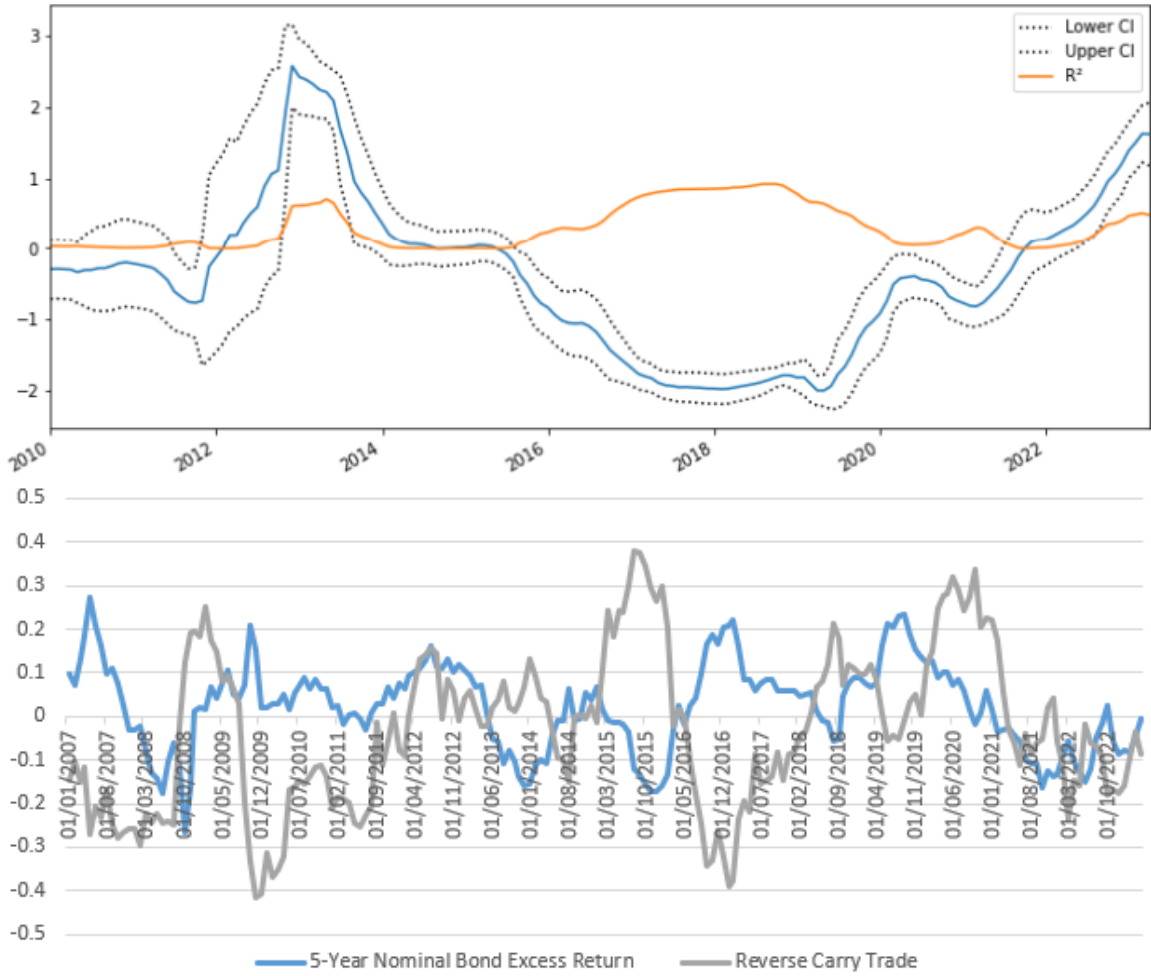
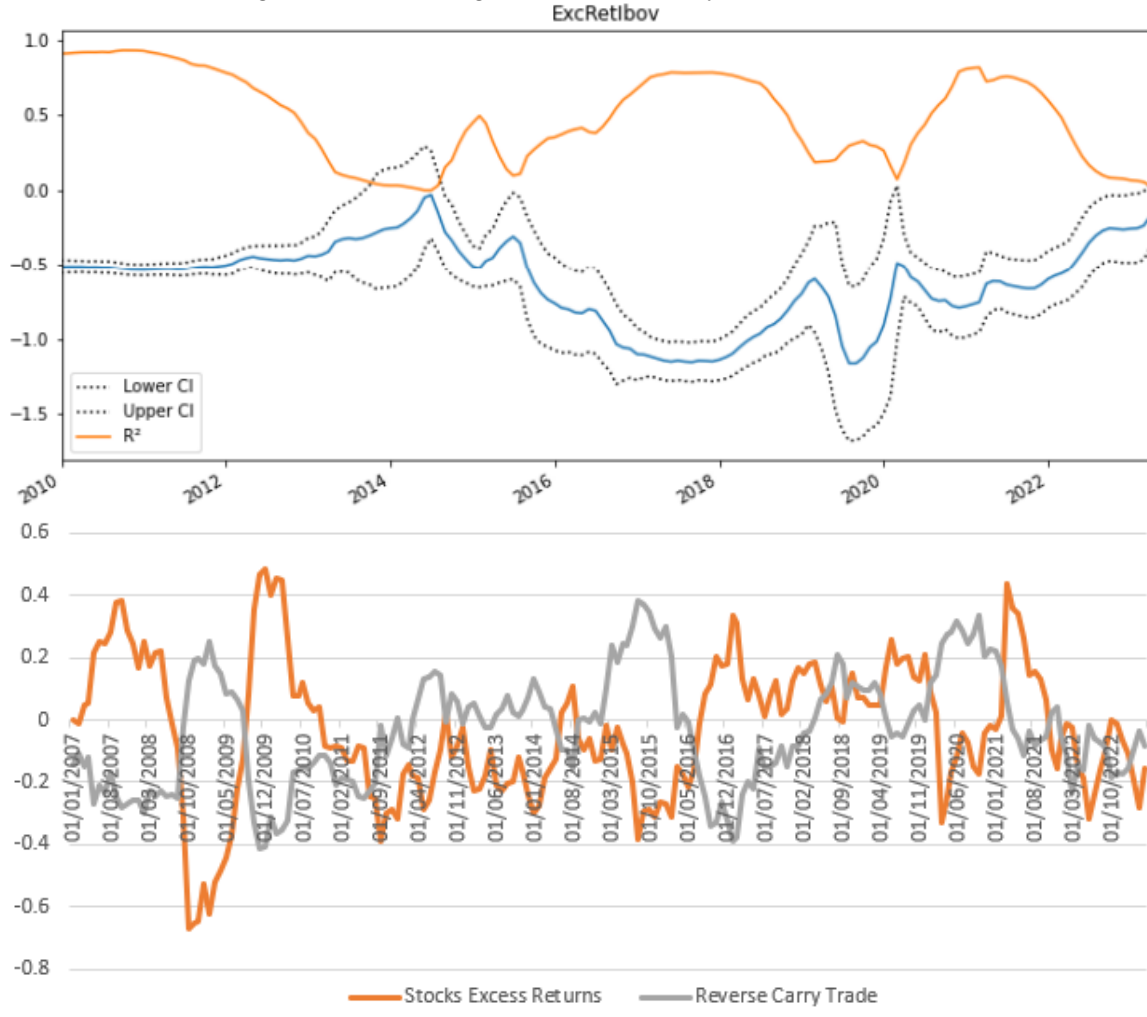


Figure 15: 3Y Rolling OLS Reverse Carry Trade into Stocks

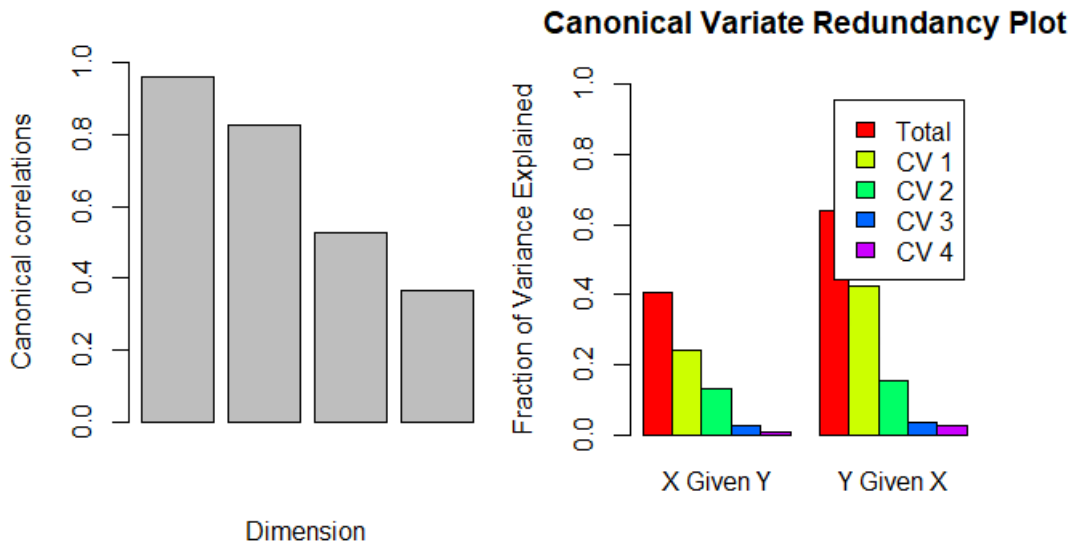


## 6 Canonical Correlation Analysis (CCA)

I create 2 groups of standardized variables. "Returns" group (Y): Nominal Bond Excess Returns, Inflation Bonds Excess Returns, Stocks Excess Returns and Reverse Carry Trade. "Factors" group (X): Output Gap, Inflation, US Term Spread<sup>5</sup>, Financial Account, Portfolio Investment, FX Orders, and the annual log difference of the EMBI and CDS.

The CCA is an interesting method for analyzing correlations with and within 2 groups of variables. The procedure is to compute the correlation matrix  $\sum_{YY}^{-1} \sum_{YX} \sum_{XX}^{-1} \sum_{XY}$ , of the variables of both groups  $Y$  and  $X$ , the eigenvectors and the eigenvalues. Similarly to a Principal Components Analysis, the eigenvalues are ordered and its respective eigenvector is used for building a new matrix of linear combinations of the original variables for each group. So, the algorithm gets the two groups of variables (2 matrices with the variables and its observations), and create a new matrix for each group. Each column of the new matrix is build with linear combinations of the variables of its group. The columns (called canonical variables) in the new matrix are built orthogonally (eigenvectors). Each column of the first group matrix is constructed to match the correspondent column of the second group matrix, with weights that maximize the correlation between the pair of corresponding columns of each matrix (this comes by ordering the eigenvalues).

<sup>5</sup>The Term Spread I use is the difference between the yields of the five and one-year T-bills from the FRED database, as a proxy for international risk premia.



The first 2 pairs of canonical variables have a (canonical) correlation of 0.96 and 0.83 each, and for the last 2 pairs 0.53 and 0.37. The "factors" group (X) explains 0.6 of the variance of the "returns" group (Y). The first 2 Canonical Variables (CVs) embrace most of this variability. So I focus on the first two pairs that presented linear combinations of the variables with highest correlation. Roughly speaking, I look here for synthetic vectors that contains the information of the returns that are most related to the information of the macro factors, cleaning from unrelated variation.

Above on Table 3, I compute the correlations between the variables and the canonical variables. Analyzing the first pair of canonical variables, we have that on the returns group, the bonds (0.44, 0.43) and stocks (0.85) returns have positive loadings, with higher weight for stocks, and highly negative loading for the reverse carry trade (-0.84). On the first canonical variable from the "factors" group, the heaviest loadings are negative from the risk variables EMBI (-0.93) and CDS (-0.95). Inflation (-0.14) has also a negative but much smaller loading on this dimension. The output (0.31) and the financial account (0.32) have positive considerable loadings. So on this dimension, the stocks and the reverse carry trade are the most affected variables, and the risk variables are their strongest drivers. Higher risk decreases bonds and stock prices. But also the movements in the output and financial account are considerable. Increase in the financial account or output increases bonds and stocks prices.

On the second dimension of canonical variables, by the returns group, all variables have the same loadings negative sign, with more weights for the nominal and inflation bonds (-0.74 and -0.45). On the factors group, the most relevant variable is now the inflation (0.88), arising as the most relevant variable for explaining the movements on bonds. So on this dimension inflation is highly negative correlated with bonds. Furthermore, portfolio investment (-0.52) and FX orders (0.51) now arise with relevance on this dimension. The signs also have economic interpretation. Portfolio investment in Brazil and the bonds returns have negative sign, so they co-move together. FX orders and bonds returns have opposite signs, so lower demand for foreign exchange is associated with higher bond prices; analogously, higher demand for Brazilian *reais* is associated with higher bond prices. So on this second dimension that gave more weight for bonds and inflation changes, foreign flows received also considerable weights.

I conclude that inflation is the main driver of bond returns. Risk variables channel is the most important for stocks returns, but also is a relevant channel for bonds returns. The literature points the output as the relevant variable for stocks representing the real economy, but in Brazil this variable is relevant but not as much as risk. Furthermore, international flows variables as financial account, FX orders and portfolio investment appear to co-move with stocks and bonds by this Canonical Correlation Analysis. Then, in Brazil, the correlation of

inflation and output is a determinant of bond and stock co-movements. But new variables, as country risk present strong effect on the stocks and bonds in the same direction.

Table 3: CCA

Weighting of each factor for the factors CVs				
	Factors CV1	Factors CV2	Factors CV3	Factors CV4
IBC Gap	0.31214390	0.10254341	-0.441999404	-0.22950084
IPCA	-0.13928602	0.88298986	-0.199325588	0.25349740
Annual Diff Log CDS	-0.94824425	0.29806821	-0.005895068	0.02625274
Term Spread US	0.08735134	0.32262243	0.219583256	-0.42589740
Financial Account	0.32172022	-0.12220263	-0.172593989	-0.17391583
Portfolio Investment	-0.17894485	-0.52243798	-0.153753869	0.27394293
Annual Diff Log EMBI	-0.93039504	0.07251957	-0.159553014	0.20365923
FX Orders	0.26079171	0.51088507	0.570931728	0.03356137

Correlation of each return with the factors CVs				
	Factors CV1	Factors CV2	Factors CV3	Factors CV4
Excss Return Nominal Bond	0.4186291	-0.6154336	0.2201736	0.10596516
Excss Return Inflation Bond	0.4137633	-0.3734304	0.1154623	0.27572566
Excss Return Stocks	0.8151638	-0.2394028	-0.2229738	-0.04981448
Reverse Carry Trade	-0.8259935	-0.2066253	-0.1853274	0.10134690

Correlation of each factor with the returns CVs				
	Returns CV1	Returns CV2	Returns CV3	Returns CV4
IBC Gap	0.30026507	0.08495043	-0.232303393	-0.084295017
IPCA	-0.1339854	0.73149867	-0.104760346	0.093108886
Annual Diff Log CDS	-0.9121579	0.24692979	-0.003098294	0.009642558
Term Spread US	0.0840271	0.26727133	0.115407249	-0.156430922
Financial Account	0.3094769	-0.10123679	-0.090710913	-0.063878798
Portfolio Investment	-0.1721349	-0.43280530	-0.080809035	0.100618472
Annual Diff Log EMBI	-0.8949880	0.06007766	-0.083856915	0.074803466
FX Orders	0.2508670	0.42323447	0.300066870	0.012326997

Weighting of each return for the returns CVs				
	Returns CV1	Returns CV2	Returns CV3	Returns CV4
Excss Return Nominal Bond	0.4351907	-0.7428881	0.4189203	0.2884998
Excss Return Inflation Bond	0.4301324	-0.4507667	0.2196880	0.7506882
Excss Return Stocks	0.8474129	-0.2889824	-0.4242482	-0.1356244
Reverse Carry Trade	-0.8586710	-0.2494167	-0.3526191	0.2759261

## 7 Final Remarks

This paper aims to analyse the determinants of comovements between bonds and stocks returns in Brazil, and understand the similarities and differences from the USA case. We infer that inflation and output are crucial and have an effect similarly to the USA case, but new factors arise making bonds and stocks returns comove

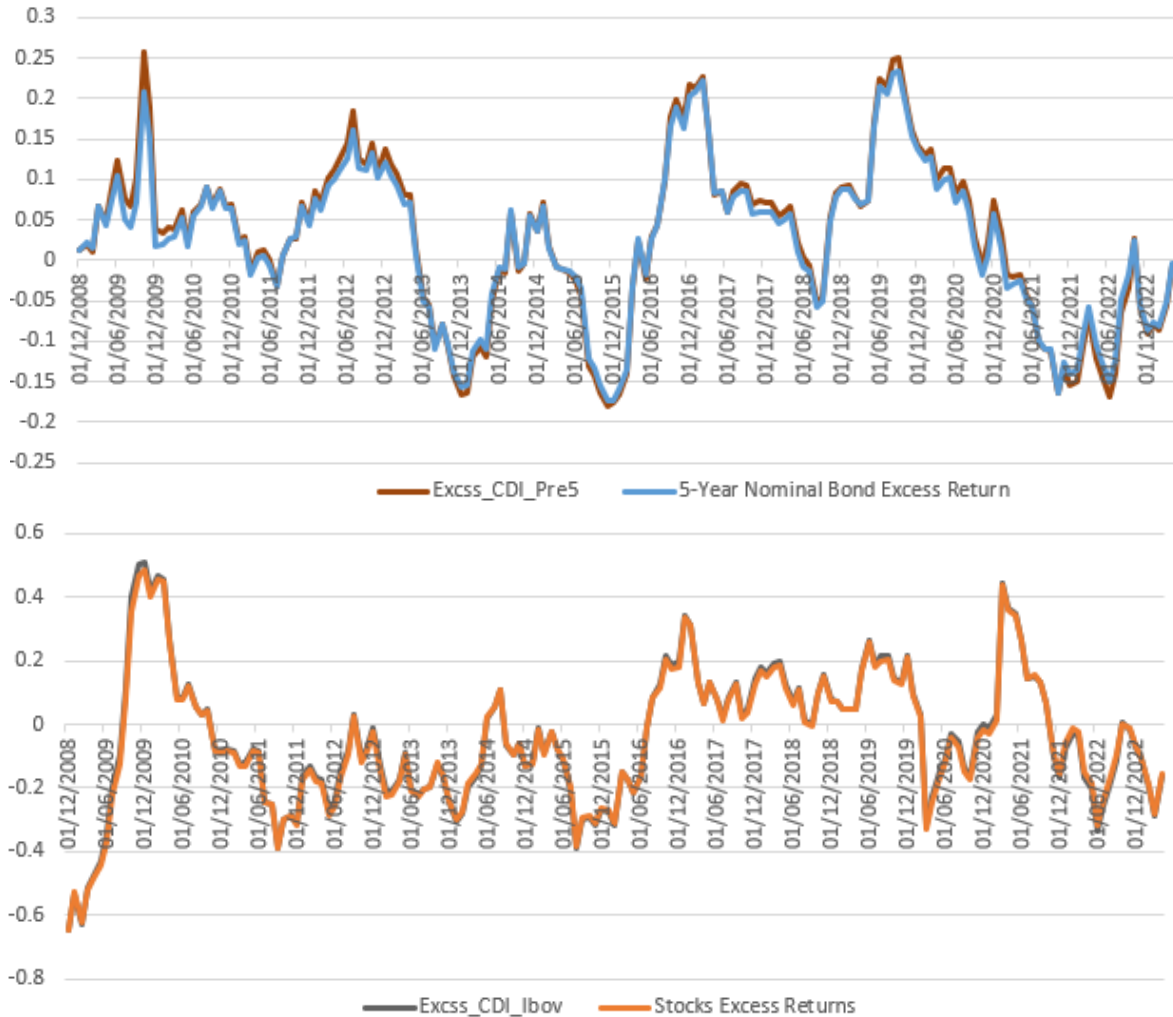


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The next steps of this paper are to incorporate these stylized facts into an asset pricing model and analyse the implications for portfolio formation.

## 8 Appendix

Figure 16: Excess returns over CDI and 1-year Bond



## References

- [1] Campbell, John Y, Carolin Pflueger, and Luis M Viceira, (2020). “Macroeconomic drivers of bond and equity risks,” *Journal of Political Economy*, 128 (8), 3148–3185.
- [2] Cochrane, John H. and Monika Piazzesi, (2005). Bond risk premia. *American economic review*, 95(1), 138-160.
- [3] Pflueger, Carolin, (2023). Back to the 1980s or not? The drivers of inflation and real risks in Treasury bonds. No. w30921. National Bureau of Economic Research.