Semivolatility-managed portfolios

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Abstract

There is ample evidence that volatility management helps improve the risk-adjusted performance of momentum portfolios. However, it is less clear that it works for other factors and anomaly portfolios. We show that controlling by the upside and downside components of volatility yields more robust risk-adjusted performances across a broad set of factors and anomaly portfolios, as well as exchange-traded funds. In particular, we propose semivolatility-managed portfolios that, apart from deleveraging if downside volatility is high, also exploits the higher expected returns in times of good volatility. We find that our semivolatility-managed portfolios that control for both skewness and downside volatility perform better than unmanaged portfolios and the extant (semi)volatility management proposals.

1 Introduction

There is a hot debate on whether volatility management indeed leads to higher risk-adjusted performance and utility gains for the investor (Barroso and Santa-Clara, 2015; Eisdorfer and Misirli, 2017; Moreira and Muir, 2017; Liu et al., 2019; Cederburg et al., 2020; Barroso and Detzel, 2021). The motivation for such a strategy is simple. Volatility is more persistent than expected returns, and hence timing risk exposure makes more sense than expected returns to boost the return-to-variability ratio. This should work as long as past volatility does not positively correlate with future returns (see, for instance, the excellent discussion in Ai et al., 2022).

In view that returns are not Gaussian, investors should perhaps beware more of downside risk given that they worry mostly about underperforming (Roy, 1952; Markowitz, 1959; Sortino and van der Meer, 1991; Ang et al., 2006). Accordingly, we offer a framework that relies on semivolatility management in order to reflect how investors perceive risks. Wang and Yan (2021) take a similar route. They find that scaling by downside volatility yields significantly better performance relative to scaling by volatility because the former is relatively better at predicting future returns. We complement their analysis by showing how to exploit upside risks, as well.

Our approach rests on realized partial second moments, allowing us to control not only for downside volatility, but also for conditional skewness and kurtosis. In particular, we consider three partial second moments: downside volatility (or negative semivolatility), upside volatility (or positive semivolatility), and partial volatility below a certain quantile. Downside volatility has a long pedigree in finance (Roy, 1952; Markowitz, 1959). See, among others, Markowitz et al. (1993), Ballestero (2005), and Estrada (2007) for recent applications in portfolio allocation. There is also a vast literature on decomposing the realized volatility into their downside and upside components (Barndorff-Nielsen et al., 2008; Bollerslev et al., 2020b, 2022b). For instance, Bollerslev et al. (2020a) find that downside and upside variances behave very differently over time, even if both help predict future realized volatility. As for finer decompositions based on partial second moments, Bollerslev et al. (2022a) show how to employ machine-learning methods to choose quantiles that maximize the out-of-sample forecast performance of time series models based on realized partial (co)variances.

The interesting aspect of combining these partial variances is that it permits controlling for higher moments. For instance, the downside-to-upside volatility ratio reflects skewness, whereas the partial volatility below a low quantile (say, first quartile) gauges tail thickness, and hence to some extent kurtosis. Accordingly, apart from scaling portfolios by the total variance as in Moreira and Muir (2017), we propose scaling by the ratio of the downside to upside variance, the ratio of the downside variance to the upside volatility, and by the product between the downside-to-upside variance ratio and the partial variance below the first quartile. Our first proposal not only controls for downside risk, but also exploits upside risk. It does not affect much the investment position in times of conditional symmetry, though. The second scaling strategy handles this issue by using the upside volatility rather than variance. Even if the downside and upside volatilities coincide, risk exposure changes because of the extra downside volatility. The same happens in our third scaling method, with the difference that it focuses more on the tail of the conditional distribution. This means that our second and third proposals actually control for higher-order moments: skewness and downside variance in the second, whereas skewness and (downside) kurtosis in the third.

We empirically assess our semivolatility-managed portfolios using a very broad set of test strategies. It features not only factors and anomaly portfolios as in Barroso and Santa-Clara (2015), Moreira and Muir (2017), Liu et al. (2019), Cederburg et al. (2020), Barroso and Detzel (2021), and Wang and Yan (2021), but also exchange-traded funds

(ETFs). We include the latter for two reasons. First, they are tradeable, in contrast to factors and anomaly portfolios. Second, we can obtain very precise realized estimates of partial variances using ETF returns at the 1-minute frequency. To carry out a robust assessment, we account for every point that Liu et al. (2019) and Cederburg et al. (2020) raise against Moreira and Muir's (2017) empirical analysis. The first point is about restricting attention exclusively to real-time information. Moreira and Muir (2017) estimate the scaling constant using the full sample, thereof inducing a look-ahead bias. This is a minor point because, in principle, the scaling constant does not affect risk-adjusted performance measures. However, the optimal weight between the managed and unmanaged portfolios that expands the mean-variance frontier depends on the alpha estimate (i.e., intercept of the spanning regression). As before, this obviously depend on the full sample, casting doubt on the genuine ability of spanning regressions to distinguish performance.

We tackle both these issues. First, we employ only real-time information to estimate the scaling constant. Perhaps surprisingly, this actually improves the performance gains of managed portfolios. Second, we do not rely exclusively on spanning regression tests to check whether semivolatility management indeed entail performance gains relative to unmanaged and volatility-managed portfolios. We provide direct comparisons of Sharpe ratios and, due to our interest in downside risk, of Sortino ratios. To test whether the difference in Sharpe and Sortino ratios are significant, we employ Ledoit and Wolf's (2008) bootstrap-based test rather than Jobson and Korkie's (1981) test as in Cederburg et al. (2020). The former avoids the assumption of iid Gaussian returns in the latter, which makes no sense in the context of volatility timing. We nonetheless report the results of both tests, revealing that the Jobson-Korkie test apparently rejects too often the null hypothsis of equal risk-adjusted performances. Finally, Cederburg et al. (2020) argue that Moreira and Muir's (2017) volatility-management strategy could well reach impracticable leverage levels in the absence of a more realistic restriction. To make sure our (semi)volatility-managed portfolios are indeed feasible, we truncate scaling to a reasonable value of at most two.

We find that our semivolatility-managed proposal that controls for both downside variance and skewness offers more robust risk-adjusted performance gains than the extant methods in the literature. In particular, it yields significant gains in both Sharpe and Sortino ratios, especially for the worst-performing factors and anomaly portfolios. Scaling by the downside-to-upside volatility ratio has little effect for the bulk of the factors and anomaly portfolios, though. Controlling either for downside volatility or for both skewness and kurtosis pay off across the board, as opposed to volatility management that helps mostly the best-performing factors and anomaly portfolios.

Finally, we also discuss how the pricing errors of the Fama-French three- and five-factor models change as we move from unmanaged to managed factors. To do so, we employ Barillas et al.'s (2020) procedure for nonnested factor model comparisons. We find that, using the three-factor model, semivolatility management pays off at the 5% significance level if we time either downside volatility or both volatility and skewness. For the five-factor model, scaling by downside volatility yields Sharpe ratio improvements at the 5% level, whereas managing both volatility and skewness significantly ameliorates the Sharpe ratio at the 1% level. In particular, the maximum attainable Sharpe ratio of the Fama-French factor model increases from 1.08 to 1.38 once we replace unmanaged factors by our semivolatility-managed factor that controls for both volatility and skewness.

Our study on (semi)volatility management is part of a larger literature on factor timing. Moreira and Muir (2017) claim that investors can improve Sharpe ratios by decreasing exposure to risk factors when their volatility is high. Barroso and Detzel (2021) show however that these gains disappear in times of low sentiment once we account for transaction costs. Bianchi et al. (2022) show how to improve the volatility-managed momentum portfolio by controlling for conditional skewness. DeMiguel et al. (2021) entertain weights on multifactor portfolios that depend on the market volatility, instead of focusing on individual portfolios. They show that volatility-management of the multifactor portfolio weights outperforms the unconditional multifactor portfolio, net of transaction costs, regardless of whether sentiment is high or low. Volatility is not the unique timing variable in this literature, though. For instance, Haddad et al. (2020) time their risk factors using the book-to-market spread of the principal components of a large panel of equity factors. There are also many papers that condition risk premia on macroeconomic indicators: e.g., Bass et al. (2017), Amenc et al. (2019), Bender et al. (2019), and Gómez-Cram (2022).

The remainder of the paper proceeds as follows. Section 2 describes the data and methodology, whereas Section 3 discusses whether (semi)volatility management indeed yields risk-adjusted performance gains across different factors, anomaly portfolios, and exchange-traded funds. Section 4 concludes.

2 Data and methodology

In this section, after describing the data, we discuss the extant volatility and semivolatility management strategies in the literature, and then propose our version based on partial variances. Next, we go through our assessment methodology. Apart from spanning regressions as in Moreira and Muir (2017) and the Jobson-Korkie test as in Cederburg et al. (2020), we entertain Ledoit and Wolf's (2008) test for Sharpe ratio differentials as well as Barillas et al.'s (2020) cross-section relative test for nonnested factor models. Finally, given our interest in downside risk, we also extend the Ledoit-Wolf test to compare Sortino ratios.

2.1 Data description

We employ a wide array of test assets that we classify in three groups. The first set comprises ten equity factors: market (MKT), size (SMB), and value (HML) from Fama and French (1993); momentum (MOM) from Carhart (1997); betting against beta (BAB) from Frazzini and Pedersen (2014); investment (CMA and IA) and profitability (RMW and ROE) from Fama and French (2015) and Hou et al. (2015); and expected growth (EG) from Hou et al. (2021). Daily data on MKT, SMB, HML, MOM, RMW, and CMA returns are from Kenneth French's website at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french, whereas BAB returns come from Andrea Frazzini's website at http://people.stern.nyu.edu/afrazzin. Data on IA, ROE and EG are available at http://global-q.org/factors.html. This is exactly the group of factors that Moreira and Muir (2017) consider, except for the addition of the EG factor.

Table 1 displays the (annualized) descriptive statistics for the monthly factor returns. EG, MKT and BAB exhibit the highest average returns respectively at 9.65%, 8.26% and 8.21%, whereas SMB, RMW and CMA display the smallest mean returns respectively at 2.45%, 3.10% and 3.17%. Although it features not-so-high average returns, momentum is definitely the riskiest factor, with a volatility of 16.30%, downside semivolatility of 12.48%, and a shocking minimum return of -52.27%. This is actually consistent with the stylized fact that momentum typically goes up by escalator, but goes down by elevator (Barroso and Santa-Clara, 2015). For the sake of comparison, the second riskiest is MKT, with a slightly higher volatility of 18.52%, downside semivolatility of 12.27%, and a minimum return of -29.13%. As expected, all factor returns are far from Gaussian, displaying some skewness and very thick tails judging by their excess kurtosis. In particular, the distributions of MOM, SMB and HML returns are heavily skewed and leptokurtic.

The second refers to a large panel of anomaly portfolios that helps characterize the cross-section of expected returns (Kozak et al., 2020). We extract daily returns on 207 anomaly portfolios from Chen and Zimmermann's (2022) database, available at https://www.openassetpricing.com/. They construct each portfolio following closely the guidelines of the corresponding original paper.[1] As such, their returns strongly correlate with the returns on the anomaly portfolios that appear in Serhiy Kozak's website at https://www.serhiykozak.com/data, for instance. The sample period varies across anomaly portfolios. The longest time spans are for price from Blume and Husic (1973) and spinoff from Cusatis et al. (1993), with 25,297 daily observations from January 2, 1926 to

[1] See full documentation at https://drive.google.com/file/d/1PDFl3pKwbY8DH5S9PWH_0p16HPo2wZL1/view.

December 31, 2021. The shortest sample lengths are for deferred revenues from Prakash and Sinha (2013) and for takeover vulnerability and active shareholders from Cremers and Nair (2005), with 4,137 daily observations from October 1st, 1990 to February 28, 2007.

Figure 1 exhibits box plots for the main descriptive statistics of the 207 anomaly portfolios. Average returns are typically between 3% and 8% per annum, with very few anomaly portfolios displaying negative returns. The exceptions are governance index from Gompers et al. (2003); firm age from Barry and Brown (1984); pension funding status from Franzoni and Marin (2006); sales growth over overhead growth from Abarbanell and Bushee (1998); and cash-flow-to-price variance from Haugen and Baker (1996). Most anomaly portfolios have annualized volatility between 8% and 16% and downside semivolatility between 5% and 11%. There is a lot of dispersion as what concerns skewness, whereas anomaly portfolios typically display a large amount of excess kurtosis.

The third group of test assets consists of 71 exchange-traded funds, whose 1-minute returns we collect from the Pi Trading database, available at https://pitrading.com/.[2] Access to intraday data is paramount to alleviate concerns about the precision of the monthly realized partial moments. Indeed, there is not much room for precision if we have to estimate monthly realized semivariances using daily data. Accordingly, we compute their monthly realized measures using 1-minute returns to ensure small-sample bias does not drive the results. Although the ETF data covers the period running from July 1998 to February 2017, the starting date varies across funds. Table 2 describes the sample period of each ETF we consider. In particular, Dow Jones Industrial Average ETF (DIA), iShares MSCI Japan Index Fund (EWJ), and SPDR S&P 500 (SPY) are the only funds for which we have data for the entire sample period of 224 months. In turn, the shortest time series is of 70 months for PowerShares S&P 500 Low Volatility Portfolio (SPLV), ranging from May 2011 to February 2017.

Figure 2 displays box plots for the main descriptive statistics of the 71 exchange-traded funds. There is a lot of dispersion in average returns, with the bulk of the cross-section distribution around 5% per annum. There are many ETFs with very negative mean returns, though. Consistently, we also observe many ETFs with very negative skewness. Both volatility and semivolatility are typically higher for ETFs than for factors and anomaly portfolios, ranging mostly from 20% to 40% and from 15% to 30%, respectively. ETF returns are also very far from Gaussian, exhibiting both negative skewness and excess kurtosis.

[2] We do not use all of the 573 exchange-traded funds in the database, because most of them do not have enough liquidity.

2.2 Timing (downside) risk

Although the literature refers to volatility management, the practice is to actually time realized (semi)variances. Moreira and Muir (2017) indeed claim that targeting variance yields slightly better performance than timing volatility. As such, they entertain the following volatility-management strategy:

$$f_{(\sigma),t} = \frac{c_{(\sigma)}}{\hat{\sigma}^2_{t-1}}\, f_t, \tag{1}$$

where $\hat{\sigma}^2_{t-1}$ is the conditional variance estimate at month $t-1$ and f_t is the buy-and-hold portfolio (excess) return. The scaling constant $c_{(\sigma)}$ standardize the managed portfolio so that it has the same unconditional variance of the unmanaged portfolio.

Wang and Yan (2021) argue that (2) should perform better than (1) because downside semivariance negatively correlates with future returns more strongly than variance. Accordingly, they time downside risk by restricting attention to the negative semivariance:

$$f_{(-),t} = \frac{c_{(-)}}{\hat{\sigma}^2_{(-),t-1}}\, f_t, \tag{2}$$

where $\hat{\sigma}^2_{(-),t} = \frac{1}{T}\sum_{d=1}^{T} \min(0, r_{d,t})^2$ and $r_{d,t}$ are returns at day d (or minute in the ETF case) within month t. The scaling constant $c_{(-)}$ now standardizes the managed portfolio so that it has the same unconditional negative semivariance of the unmanaged portfolio.

Next, we introduce our risk timing strategies based on partial second moments that account to some extent for higher-order moments. The first attempts to control for both downside and upside risks, deleveraging if the former is high, while scaling up if the latter is high. In particular, we scale by the downside-to-upside variance ratio:

$$f_{(+/-),t} = c_{(+/-)}\, \frac{\hat{\sigma}^2_{(+),t-1}}{\hat{\sigma}^2_{(-),t-1}}\, f_t, \tag{3}$$

where $\hat{\sigma}^2_{(+),t} = \frac{1}{T}\sum_{d=1}^{T} \max(0, r_{d,t})^2$ and $c_{(+/-)}$ ensures the unconditional downside variance remains the same, as before. A problem with (3) is that, under distributional symmetry, the positive and negative semivariances coincide, keeping the portfolio unchanged irrespective of the volatility levels. As such, it essentially times skewness, scaling up or down according to whether symmetry is positive or negative.

Our second proposal deals with the symmetry issue by replacing the upside variance by the upside volatility in the numerator:

$$f_{(\sqrt{+}/-),t} = c_{(\sqrt{+}/-)}\, \frac{\hat{\sigma}_{(+),t-1}}{\hat{\sigma}^2_{(-),t-1}}\, f_t \tag{4}$$

with $c_{(\sqrt{+}/-)}$ denoting the scalar that keeps the unconditional negative semivariance constant. To understand exactly which risks we control with (4), we decompose the reciprocal of the investment position into the product of the semivolatility ratio $\hat{\sigma}_{(-)}/\hat{\sigma}_{(+)}$ and the downside volatility $\hat{\sigma}_{(-)}$. The downside-to-upside volatility ratio accounts for skewness, exploiting upside risk when symmetry is positive and deleveraging when skewness is negative. At the same time, the second term in the decomposition controls for (downside) volatility, even in times of symmetric returns.

Our last strategy aims to control for tail risk, by contemplating the partial variance below the first quantile: namely, $\hat{\sigma}^2_{(q),t} = \frac{1}{T}\sum_{d=1}^{T} \min(q, r_{d,t})^2$, where q denotes the first quartile of the empirical distribution of the returns at the highest frequency (Bollerslev et al., 2022a). It is obviously much harder to estimate partial moments at the tails. This is why we restrict attention to the first quantile and exclusively to exchange-traded funds, for which we can estimate monthly partial variances very precisely using 1-minute returns.[3] This results in managed portfolios of the form

$$f_{(q),t} = \frac{c_{(q)}}{\hat{\sigma}^2_{(q),t-1}} \frac{\hat{\sigma}^2_{(+),t-1}}{\hat{\sigma}^2_{(-),t-1}} f_t \tag{5}$$

with $c_{(\sqrt{+}/-)}$ denoting the scaling constant that ensures that the managed and unmanaged portfolios have the same unconditional negative semivariance. As before, while the downside-to-upside volatility ratio scales leverage up and down according to whether skewness is positive or negative, the first fraction ensures that we deleverage if there is too much mass on the negative tail. This implies that (5) controls to some extent for both skewness and (downside) kurtosis.

In the empirical application, we consider several configurations for the above risk timing strategies, in order to take Liu et al.'s (2019) and Cederburg et al.'s (2020) critiques into account. First, we fix the scaling constant as in Moreira and Muir (2017), using the entire sample to compute the unconditional variance of the unmanaged portfolio, as well as using only real-time information. Second, we do not allow leverage to run amok by constraining their magnitude to at most two. The aim is to mitigate concerns about the (in)feasibility, in practice, of the investment positions. To understand how leverage constraints affect the (semi)volatility-management performance, we report results with and without imposing leverage restrictions.

One last remark is that the scaling constant should not affect *per se* Sharpe and Sortino ratios given that they do not depend on the unconditional variance of the portfolio returns. However, once we constrain leverage to prevent unrealistic investment positions, the value of the scaling factor becomes more relevant. By using exclusively real-time information, our estimate of the unconditional (semi)variance of the unmanaged portfolio becomes more local. This also ends up alleviating leverage constraints.

[3] Our findings are nonetheless similar for values of q between the 5th and 25th percentiles. The results are available from the authors upon request.

2.3 Assessing performance gains

Our interest lies on understanding whether timing risk leads to investors' utility gains. As such, we check whether (semi)volatility-managed portfolios indeed outperform their unmanaged counterparts. To do so, we employ not only the testing approaches previously used in the literature, but also some additional analyses.

The first test rests on spanning regressions, as in Moreira and Muir (2017). We regress returns to the managed portfolio on returns to the unmanaged (buy-and-hold) portfolio:

$$f_{(\cdot),t} = \alpha + \beta\, f_t + u_t, \tag{6}$$

where u_t is a white noise. If α differs from zero in (6), the managed strategy helps expand the mean-variance frontier. This means there is a linear combination of the managed and unmanaged portfolios that achieves a higher Sharpe ratio than before. The spanning regression test then checks whether there is enough statistical evidence to reject the null hypothesis $\mathbb{H}_0 : \alpha = 0$ in favor of the alternative hypothesis $\mathbb{H}_1 : \alpha \neq 0$. This does not boil down to a direct comparison of Sharpe ratios, though. Cederburg et al. (2020) show that a positive alpha in the spanning regression does not necessarily lead to Sharpe Ratio improvements. In addition, the optimal weight on $f_{(\cdot)}$ in the factor combination depends on α, thereof requiring ex-post information.

The second set of tests explicitly compares the risk-adjusted performance of managed portfolios relative to their original buy-and-hold versions. For instance, Cederburg et al. (2020) employ Jobson and Korkie's (1981) test to assess whether managed portfolios indeed improve on the Sharpe ratio of unmanaged portfolios. Let the returns on strategies i and j be stationary with mean $\boldsymbol{\mu} = (\mu_i, \mu_j)'$ and covariance matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_i^2 & \sigma_{ij} \\ \sigma_{ij} & \sigma_j^2 \end{pmatrix}.$$

Jobson and Korkie's interest is in the difference between their Sharpe ratios, namely, $\Delta = \mathrm{SR}_i - \mathrm{SR}_j$, with $\mathrm{SR} = \mu/\sigma$. We estimate both means and variances by their realized counterparts.

Let $\boldsymbol{\theta} = (\mu_i, \mu_j, \sigma_i^2, \sigma_j^2)'$. Under the assumption of iid Gaussian returns, Memmel (2003) first establishes that $\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{d} N(0, \boldsymbol{\Omega})$, with

$$\boldsymbol{\Omega} = \begin{pmatrix} \sigma_i^2 & \sigma_{ij} & 0 & 0 \\ \sigma_{ij} & \sigma_j^2 & 0 & 0 \\ 0 & 0 & 2\sigma_i^4 & 2\sigma_{ij}^2 \\ 0 & 0 & 2\sigma_{ij}^2 & 2\sigma_j^4 \end{pmatrix},$$

and then obtains the asymptotic variance of $\widehat{\Delta}$ by the delta method. The main problem with the Jobson-Korkie test is that Memmel's derivation assumes iid Gaussian returns.

This assumption makes no sense in the context of (semi)volatility management, though. If (semi)volatility changes over time in a persistent fashion, returns are neither iid, nor unconditionally Gaussian. Ledoit and Wolf (2008) nonetheless derive a bootstrap version of the test that allows not only for serial correlation and conditional heteroskedasticity in the returns, but also heavy tails. Accordingly, although we also report the Jobson-Korkie test, we rely mostly on the Ledoit-Wolf test in our empirical assessment.

Given our interest in downside risks, we extend both Jobson-Korkie and Ledoit-Wolf tests for Sortino ratio comparisons. The interest now lies on $\Delta_{(-)} = \mathrm{SR}_{(-),i} - \mathrm{SR}_{(-),j}$, with $\mathrm{SR}_{(-)} = \mu/\sigma_{(-)}$. As before, we stack the parameters of interest into a vector $\boldsymbol{\theta}_{(-)} = (\mu_i, \mu_j, \sigma^2_{(-),i}, \sigma^2_{(-),j})$ and then estimate both means and semivariances by their realized counterparts. To establish the asymptotic behavior of $\widehat{\boldsymbol{\Delta}}_{(-)}$, we assume as in Barndorff-Nielsen et al. (2008) that asset prices follow a Brownian semimartingale process:

$$P_{k,t} = \int_0^t a_{k,s}\,\mathrm{d}s + \int_0^t \sigma_{k,s}\,\mathrm{d}W_{k,s},$$

where $a_{k,s}$ is a locally bounded, predictable drift process and $\sigma_{k,s}$ is càdlàg volatility process, for asset $k \in \{i, j\}$.

In the case of iid Gaussian returns, it follows that $\sigma_k = \sigma_{k,s}$ is constant over time and that $\mathrm{var}(\hat{\sigma}^2_{(-)}) = (5/4)\sigma^4$. In addition, normality also implies that $\mathrm{cov}(\sigma^2_{(\cdot),i}, \sigma^2_{(\cdot),j}) = \mathrm{cov}(\sigma^2_i, \sigma^2_j)/4 = \sigma^2_{ij}/2$, in view not only that $\sigma^2_{(-),k} = \sigma^2_{(+),k} = \sigma^2_k/2$ for $k = i, j$ but also that $\mathrm{cov}(\sigma^2_i, \sigma^2_j) = \mathrm{cov}(\sigma^2_{(+),i}, \sigma^2_{(+),j}) + \mathrm{cov}(\sigma^2_{(+),i}, \sigma^2_{(-),j}) + \mathrm{cov}(\sigma^2_{(-),i}, \sigma^2_{(+),j}) + \mathrm{cov}(\sigma^2_{(-),i}, \sigma^2_{(-),j})$. As such, the covariance matrix of $\hat{\boldsymbol{\theta}}_{(-)}$ reads

$$\boldsymbol{\Omega}_{(-)} = \begin{pmatrix} \sigma^2_i & \sigma_{ij} & 0 & 0 \\ \sigma_{ij} & \sigma^2_j & 0 & 0 \\ 0 & 0 & (5/4)\sigma^4_i & \sigma^2_{ij}/2 \\ 0 & 0 & \sigma^2_{ij}/2 & (5/4)\sigma^4_j \end{pmatrix}.$$

Applying the delta method then yields that, under iid Gaussian returns, the asymptotic variance of $\widehat{\Delta}_{(-)}$ is $\frac{1}{T}\left(\frac{\sigma^2_i}{\sigma_{(-),i})^2} - \frac{2\sigma_{ij}}{\sigma_{(-),i}\sigma_{(-),j}} + \frac{\sigma^2_j}{\sigma^2_{(-),j}} + \frac{5\mu^2_i\sigma^4_i}{16\sigma^6_{(-),i}} - \frac{\mu_i\mu_j\sigma^2_{ij}}{4\sigma^3_{(-),i}\sigma^3_{(-),j}} + \frac{5\mu^2_j\sigma^4_j}{16\sigma^6_{(-),j}}\right)$. To relax the iid Gaussian assumption, it suffices to follow the block-bootstrap procedure of Ledoit and Wolf (2008).[4]

For completeness, we entertain three scenarios for the above tests. The first is as in Moreira and Muir (2017), without any restriction on leverage or on the use of ex-post information. The second restricts leverage to at most two, in order to avoid unfeasible investment positions. The third uses only real-time information to set the value of the scaling constants, apart from constraining leverage as in the second.

[4] A small Monte Carlo study shows the asymptotic validity of both Jobson-Korkie and Ledoit-Wolf tests for Sortino ratios. These results are available upon request.

Our third assessment rests on testing the difference in the squared maximum Sharpe ratio that we can attain from two nonnested factor models, in order to determine which model yields smaller pricing errors. Barillas et al. (2020) extend the asymptotic theory of Barillas and Shanken (2017) to accommodate nonnested models and nontraded factors, under the assumption of jointly stationary and ergodic returns with finite fourth moments.[5] For a given set of factors F, the square of the maximum-attainable Sharpe ratio is $\boldsymbol{\mu}_F' \boldsymbol{\Sigma}_F^{-1} \boldsymbol{\mu}_F$, where $\boldsymbol{\mu}_F$ is the vector of average returns and $\boldsymbol{\Sigma}_F$ is the covariance matrix of the factors. The test then gauges the differences in squared Sharpe ratios. In particular, we assess whether the squared maximum-attainable Sharpe ratios of the Fama-French three- and five-factor models increase once we replace the original factors with their managed counterparts.

Finally, we complement our cross-sectional analysis by testing for first-order stochastic dominance in the empirical distributions of Sharpe and Sortino ratios of the anomaly portfolios and ETFs. More specifically, we carry out Barrett and Donald's test (2003) to check whether each (semi)volatility-management strategy displays first-order stochastic dominance with respect to their unmanaged counterparts. The plots of the empirical distributions are also very helpful to understand how each (semi)volatility management strategy behaves as we move from the lowest to highest performing factors, anomaly portfolios and exchange-traded funds.

3 How does risk timing fare empirically?

To build managed portfolios, we estimate the realized partial variances at month t using daily returns on factors and anomaly portfolios and 1-minute ETF returns within month t. To estimate the scaling factor using only real-time information, we burn in the first 24 months for initialization purposes. We then expand the estimation window as we move along the sample so as to include the entire past. In what follows, we mainly discuss our findings for the feasible portfolios that restrict attention only to real-time information and truncate the magnitude of the investment position.

The first panel of Table 3 reports the OLS estimates of the spanning-regression alphas for the sample of factor returns, with p-values based on heteroskedasticity-and-autocorrelation consistent (HAC) standard errors. The second and third panels document respectively the differences in Sharpe and Sortino ratios between managed and unmanaged factors, which we assess using both Jobson-Korkie and Ledoit-Wolf tests. There are two patterns that strike the eyes. First, a statistically nonzero alpha in the span-

[5] They also show how to control for the small-sample bias in the estimation of the squared Sharpe ratio of nontraded factors.

ning regression does not automatically translate into significant changes in risk-adjusted portfolio. There are many managed factors with positive alphas that entail lower Sharpe ratios than their unmanaged counterparts. Second, it is much harder to find significant evidence of better performance with direct comparisons than with spanning regressions. Along the same lines, the Ledoit-Wolf bootstrap-based test sets a much higher bar than the Jobson-Korkie test based on the iid Gaussian assumption.

In line with Moreira and Muir (2017), we find that volatility management yields significantly positive alphas for MKT, HML, MOM, RMW, BAB, IA, ROE, and EG. However, once we focus on the direct comparisons, volatility-managed factors do not perform so well. Timing volatility helps improve the Sharpe ratio of the momentum factor at the 5% significance level, as well as the Sortino ratios of MOM and ROE with p-values only slightly above 5%. Timing downside variance as in Wang and Yan (2021) fails to produce alpha for only two factors in the spanning regressions (CMA and IA). In contrast, we find little evidence that it ameliorates the risk-adjusted performances of the factors, except for RMW and BAB. Timing skewness has the worst performance in the spanning regressions, failing to produce significantly positive alphas for MOM, CMA, IA and EG. It nonetheless enhances the risk-adjustment performance of three factors: HML, RMW and BAB. Timing both skewness and downside variance seems to payoff, though. It fails to produce alpha only for MOM and CMA, whereas it yields at the 5% significance level higher Sharpe ratios for HML, RMW and BAB, and higher Sortino ratios for HML, RMW, BAB and ROE.

Altogether, timing second moments does not work so well for MKT, SMB, CMA, IA and EG, whereas only volatility management improves on MOM. This justifies *ex-post* why Barroso and Santa-Clara (2015) focus exclusively on timing momentum. At any rate, timing both skewness and downside variance yields the best overall performance for managed factors.

We now turn our attention to the broader set of test portfolios. Table 4 documents how (semi)volatility management performs relative to just buying and holding anomaly portfolios and exchange-traded funds. The spanning regressions yield significantly positive alphas for 142 anomaly portfolios (out of 207) when timing downside risk. This figures slightly decreases to 138 if we time both skewness and downside risk, but drops drastically to 110 and 102 if we time only total volatility or skewness, respectively. The direct comparisons of Sharpe and Sortino ratios reveal similar relative performances, with $f_{(-)}$ and $f_{(\sqrt{+}/-)}$ easily outclassing both $f_{(\sigma)}$ and $f_{(+/-)}$. The Jobson-Korkie test indicates a very small advantage to timing only downside variance, which improves the Sharpe ratio of 63 anomaly portfolios and the Sortino ratio of 89 portfolios, relative to timing both skewness and downside variance (63 and 86, respectively). The more realistic assessment based on bootstrap flips the advantage to timing both downside variance and skewness: 34 against 30 and 38 against 35, respectively.

We next examine the performance of timing (partial) second moments for 71 ETF returns in excess over the 1-month T-bill rate. The motivation is twofold. First, our realized measures of partial variances are much more precise due to the access to 1-minute ETF returns. Second, investors can easily trade ETFs as opposed to the above risk factors and anomaly portfolios. Unfortunately, Table 4 reveals that (semi)volatility management does not perform as well as one would expect from our previous findings. Timing both skewness and downside variance yields once more the largest number of significantly positive alphas (17) and differences in Sharpe and Sortino ratios (17 and 19, respectively). Timing skewness and kurtosis performs at par with $f_{(\sigma)}$ and $f_{(-)}$, whereas timing skewness performs the worst of all.

For the sake of brevity, we omit the results for the less realistic settings without leverage restrictions and using the full sample for the estimation of the scaling factor. A very brief summary is as follows. If we do not restrict attention to real-time information, the risk-adjusted performance worsens very slightly across the board. If we keep leverage unchecked, investment positions become impractically large in magnitude, resulting in extremely unrealistic alphas in the spanning regressions.

Lastly, we complement the above time-series analyses with some cross-sectional tests. Table 5 documents the t-statistics of the difference in the squared maximum Sharpe ratio we can attain from managed and unmanaged Fama-French three- and five-factor models. We compute the t-statistics for nonnested models with nontraded factors, as in Barillas et al. (2020). Although every timing strategy helps reduce pricing errors, and hence improve the maximum-attainable Sharpe ratio, these enhancements are significant only for $f_{(-)}$ and $f_{(\sqrt{\mp}/-)}$. Table 6 reports pairwise tests of first-order stochastic dominance (Barrett and Donald, 2003). Given that these tests are noncommutative and do not account for local dominance, we check whether the risk-adjusted performances of managed portfolios stochastically dominate those of the unmanaged portfolios, and vice-versa. We find no evidence at the usual significance levels that unmanaged portfolios dominates any of the (semi)volatility- managed portfolios. In contrast, we find relatively strong evidence that $f_{(\sigma)}$, $f_{(-)}$ and $f_{(\sqrt{\mp}/-)}$ stochastically dominate the unmanaged portfolio for both anomaly portfolios and exchange-traded funds.

Perhaps more important than the stochastic dominance analysis between managed and unmanaged portfolios is to understand which anomaly portfolios and ETFs each timing strategy seems to improve. Figures 3 and 4 report the cross-sectional distribution of Sharpe and Sortino ratios of the managed and unmanaged anomaly portfolios, respectively. They reveal that timing only skewness helps only the worst-performing anomaly portfolios, whereas volatility management works best with the top-performing anomaly portfolios. Timing both skewness and downside volatility also enhances the Sharpe ratios of the worst-performing portfolios, while working as well as timing only downside volatility in the bulk of the distribution. In addition, timing risk seems to have a stronger impact on Sortino ratios than on Sharpe ratios. Figures 4 and 5 unveil a slightly different pattern for exchange-traded funds. Timing skewness helps only funds with negative Sharpe ratios, whereas the remaining timing strategies perform best for funds with positive Sharpe ratios.

To sum up, our empirical investigation reproduces well enough the extant analyses in the literature: e.g., the spanning regressions in Moreira and Muir (2017) and the direct comparisons in Cederburg et al. (2020) and Wang and Yan (2021). In addition, it contributes by entertaining novel timing strategies that aim to control for higher-order (partial) moments. Our results are quite encouraging, especially if we scale risk exposures by both skewness and downside volatility. Despite the methodological differences, this seems in line with Bianchi et al.'s (2022) evidence that adjusting for conditional skewness improves the risk-adjusted performance of the volatility-managed MOM portfolio.

4 Conclusion

It is still subject to ongoing debate what are the benefits, if any, of volatility timing on active portfolio management. This work contributes to the literature by entertaining timing strategies that target different combinations of partial second moments to control for higher-order moments. It also innovates in assessing how (semi)volatility management performs for exchange-traded funds. Our empirical analysis evince that volatility management does not enhance the risk-adjusted performance of the vast majority of risk factors, anomaly portfolios, and exchange-traded funds. Our timing strategy that controls for both downside volatility and skewness performs more robustly across the board, particularly improving the Sharpe and Sortino ratios of the worst-performing portfolios.

We still need to check how trading costs affect our results, as well as to extend our analysis in two directions. The first is to consider multiple rather than individual tests as in DeMiguel et al. (2021). The second is to propose a more general timing strategy that explicitly accounts for jumps. As it stands, we deal with jumps only implicitly by looking either at the downside-to-upside variance ratio or at the partial variance below the first quartile.

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Table 1: Descriptive statistics for the monthly returns on the factors

We describe the set of factors we consider by reporting the number of monthly time-series observations in 'size', sample period in columns 'start' and 'end', annualized average, minimum, and maximum returns, annualized standard deviation in 'volatility', annualized downside deviation in 'semivolatility', skewness, and excess kurtosis. MKT is the Fama-French market factor, as measured by the excess returns on the market portfolio; SMB is the Fama-French size factor, as measured by the small-minus-big mimicking portfolio; HML is the Fama-French value factor, as measured by the high-minus-low mimicking portfolio; MOM is the momentum factor, as measured by the winners-minus-losers portfolio; RMW is the Fama-French profitability factor, as measured by the robust-minus-weak mimicking portfolio; CMA is the Fama-French investment factor, as measured by the conservative-minus-aggressive mimicking portfolio; BAB is the betting-against-beta factor; IA is the investment q-factor; ROE is the profitability q-factor; and EG is the expected growth q-factor.

factor	size	start	end	mean	minimum	maximum	volatility	semivolatility	skewness	excess kurtosis
MKT	1,139	07/1926	05/2021	8.26%	-29.13%	38.85%	18.52%	12.27%	0.17	7.60
SMB	1,139	07/1926	05/2021	2.45%	-16.82%	36.70%	11.02%	6.59%	1.88	18.82
HML	1,139	07/1926	05/2021	4.09%	-13.96%	35.46%	12.17%	6.89%	2.07	18.24
MOM	1,133	01/1927	05/2021	7.66%	-52.27%	18.36%	16.30%	12.48%	-2.96	26.82
RMW	695	07/1963	05/2021	3.10%	-18.48%	13.38%	7.55%	4.91%	-0.30	11.95
CMA	695	07/1963	05/2021	3.17%	-6.86%	9.56%	6.93%	4.19%	0.32	1.53
BAB	1,086	12/1930	05/2021	8.21%	-21.95%	18.65%	11.29%	7.44%	-0.68	7.02
IA	648	01/1967	12/2020	4.01%	-7.16%	9.24%	6.55%	3.93%	0.15	1.22
ROE	648	01/1967	12/2020	6.07%	-14.46%	10.38%	8.87%	5.98%	-0.91	5.72
EG	648	01/1967	12/2020	9.65%	-9.72%	11.51%	6.85%	3.51%	0.11	3.95

Table 2: List of exchange-traded funds in our sample

For each ETF ticker in the sample, we display its description and sample period. The column 'size' reports the number of monthly time-series observations, whereas 'start' and 'end' document the first and last month of the sample.

ticker	ETF	size	start	end
DBC	PowerShares DB Commodity Index Fund	133	02/2006	02/2017
DIA	SPDR Dow Jones Industrial Average ETF	224	07/1998	02/2017
DUST	Direxion Daily Gold Miners Bear 3x Shares	75	12/2010	02/2017
DVY	iShares Dow Jones Select Dividend Index Fund	160	11/2003	02/2017
DXD	ProShares UltraShort Dow30	128	07/2006	02/2017
EDC	Direxion Daily Emerging Markets Bull 3X Shares	99	12/2008	02/2017
EDZ	Direxion Daily Emerging Markets Bear 3x Shares	99	12/2008	02/2017
EEM	iShares MSCI Emerging Markets Index Fund	167	04/2003	02/2017
EFA	iShares MSCI EAFE	171	12/2002	02/2017
ERX	Direxion Daily Energy Bull 3x Shares	100	11/2008	02/2017
ERY	Direxion Daily Energy Bear 3x Shares	100	11/2008	02/2017
EWJ	iShares MSCI Japan Index Fund	224	07/1998	02/2017
EWT	iShares MSCI Taiwan Index Fund	171	12/2002	02/2017
EWW	iShares MSCI Mexico Index Fund	171	12/2002	02/2017
EWY	iShares MSCI South Korea Index Fund	171	12/2002	02/2017
EWZ	iShares MSCI Brazil Index Fund	171	12/2002	02/2017
FAS	Direxion Daily Financial Bull 3x Shares	100	11/2008	02/2017
FAZ	Direxion Daily Financial Bear 3x Shares	100	11/2008	02/2017
FXI	iShares FTSE/Xinhua China 25 Index Fund	149	10/2004	02/2017
GDX	Market Vectors Gold Miners ETF	130	05/2006	02/2017
GDXJ	Market Vectors Junior Gold Miners ETF	88	11/2009	02/2017
GLD	SPDR Gold Shares	148	11/2004	02/2017
HYG	iShares iBoxx $ High Yield Corporate Bond Fund	119	04/2007	02/2017
IBB	iShares Nasdaq Biotechnology Index Fund	171	12/2002	02/2017
IJR	iShares S&P SmallCap 600 Index Fund	171	12/2002	02/2017
IVV	iShares S&P 500 Index Fund	171	12/2002	02/2017
IWD	iShares Russell 1000 Value Index Fund	171	12/2002	02/2017
IWF	iShares Russell 1000 Growth Index Fund	171	12/2002	02/2017
IWM	iShares Russell 2000 Index Fund	201	06/2000	02/2017
IWN	iShares Russell 2000 Value Index Fund	171	12/2002	02/2017
IWO	iShares Russell 2000 Growth Index Fund	171	12/2002	02/2017
JNK	SPDR Barclays Capital High Yield Bond ETF	111	12/2007	02/2017
KBE	SPDR KBW Bank ETF	136	11/2005	02/2017
KRE	SPDR KBW Regional Banking ETF	129	06/2006	02/2017
MDY	SPDR S&P MidCap 400 ETF	217	02/1999	02/2017
NUGT	Direxion Daily Gold Miners Bull 3x Shares	75	12/2010	02/2017
OIH	Market Vectors Oil Services ETF	193	02/2001	02/2017
PFF	iShares S&P U.S. Preferred Stock Index Fund	120	03/2007	02/2017
QID	ProShares UltraShort QQQ	128	07/2006	02/2017
QLD	ProShares Ultra QQQ	129	06/2006	02/2017
QQQ	PowerShares QQQ	216	03/1999	02/2017
RSX	Market Vectors Russia ETF	119	04/2007	02/2017
SCO	ProShares UltraShort DJ-UBS Crude Oil	100	11/2008	02/2017
SDS	ProShares UltraShort S&P500	128	07/2006	02/2017
SH	ProShares Short S&P500	129	06/2006	02/2017
SLV	iShares Silver Trust	131	04/2006	02/2017
SMH	Market Vectors Semiconductor ETF	202	05/2000	02/2017
SPLV	PowerShares S&P 500 Low Volatility Portfolio	70	05/2011	02/2017
SPXL	Direxion Daily S&P 500 Bull 3x Shares	100	11/2008	02/2017
SPXS	Direxion Daily S&P 500 Bear 3x Shares	100	11/2008	02/2017
SPY	SPDR S&P 500	224	07/1998	02/2017
SSO	ProShares Ultra S&P500	129	06/2006	02/2017
TLT	iShares Lehman 20+ Year Treasury Bond Fund	176	07/2002	02/2017
TNA	Direxion Daily Small Cap Bull 3x Shares	100	11/2008	02/2017
TWM	ProShares UltraShort Russell2000	122	01/2007	02/2017
TZA	Direxion Daily Small Cap Bear 3x Shares	100	11/2008	02/2017
UCO	ProShares Ultra DJ-UBS Crude Oil	100	11/2008	02/2017
UNG	United States Natural Gas Fund	119	04/2007	02/2017
USO	United States Oil Fund	131	04/2006	02/2017
VXX	iPath S&P 500 VIX Short-Term Futures ETN	98	01/2009	02/2017
XHB	SPDR S&P Homebuilders ETF	133	02/2006	02/2017
XIV	VelocityShares Daily Inverse VIX Short-Term ETN	76	11/2010	02/2017
XLB	Materials Select Sector SPDR Fund	171	12/2002	02/2017
XLF	Financial Select Sector SPDR Fund	218	01/1999	02/2017
XLI	Industrial Select Sector SPDR Fund	171	12/2002	02/2017
XLK	Technology Select Sector SPDR Fund	171	12/2002	02/2017
XLP	Consumer Staples Select Sector SPDR Fund	171	12/2002	02/2017
XLY	Consumer Discretionary Select Sector SPDR Fund	171	12/2002	02/2017
XME	SPDR S&P Metals and Mining ETF	129	06/2006	02/2017
XOP	SPDR S&P Oil & Gas Exploration & Production ETF	129	06/2006	02/2017
XRT	SPDR S&P Retail ETF	129	06/2006	02/2017

Table 3: Performance of (semi)volatility management for factor returns

The first panel displays the OLS estimates of the spanning regression alphas, with their HAC standard errors within parentheses. We respectively denote by $*$, $**$ and $***$ significance at the 10%, 5% and 1% levels. The second and third panels report direct comparisons of Sharpe and Sortino ratios, respectively. For each (semi)volatility management proposal, we document the difference in risk-adjusted performance (namely, Δ and $\Delta_{(-)}$), with the p-values of the Jobson-Korkie and Ledoit-Wolf tests within parentheses and brackets, respectively. We highlight the significance of the Ledoit-Wolf test using asterisks, as before. The description of the factors is as in Table 1.

	MKT	SMB	HML	MOM	RMW	CMA	BAB	IA	ROE	EG
spanning-regression alpha estimates										
$f_{(\sigma)}$	3.20**	-0.07	1.79**	7.00***	1.38***	0.17	3.40***	1.18**	3.78***	2.90***
	(0.0247)	(0.9224)	(0.0366)	(0.0000)	(0.0097)	(0.6964)	(0.0008)	(0.0191)	(0.0000)	(0.0000)
$f_{(-)}$	2.91**	1.32**	3.37***	4.38**	2.48***	0.02	5.36***	0.51	4.05***	2.86***
	(0.0288)	(0.0447)	(0.0001)	(0.0107)	(0.0001)	(0.9153)	(0.0000)	(0.1061)	(0.0000)	(0.0001)
$f_{(+/-)}$	1.86*	1.26*	3.13***	0.56	2.84***	0.14	4.64***	0.46	2.34***	0.52
	(0.0876)	(0.0776)	(0.0000)	(0.7478)	(0.0005)	(0.4633)	(0.0000)	(0.1358)	(0.0004)	(0.3666)
$f_{(\sqrt{+}/-)}$	2.91**	1.49**	3.87***	2.67	2.68***	0.11	5.79***	0.47*	3.34***	1.86***
	(0.0137)	(0.0258)	(0.0000)	(0.1361)	(0.0001)	(0.4988)	(0.0000)	(0.0984)	(0.0000)	(0.0036)
Sharpe ratio differences Δ										
$f_{(\sigma)}$	0.07	-0.07	0.05	0.39**	0.16	-0.09	0.12	0.07	0.37	0.14
	(0.3847)	(0.3667)	(0.5228)	(0.0000)	(0.1657)	(0.3481)	(0.1178)	(0.4588)	(0.0008)	(0.1849)
	[0.5217]	[0.4177]	[0.7215]	[0.0247]	[0.2918]	[0.3764]	[0.2512]	[0.4963]	[0.1193]	[0.9987]
$f_{(-)}$	0.05	0.08	0.20	0.13	0.31*	-0.30	0.26**	-0.07	0.41	0.01
	(0.5130)	(0.3724)	(0.0082)	(0.1498)	(0.0094)	(0.0511)	(0.0007)	(0.5900)	(0.0009)	(0.9638)
	[0.6895]	[0.3911]	[0.1159]	[0.4777]	[0.0873]	[0.0799]	[0.0280]	[0.6436]	[0.6889]	[0.9880]
$f_{(+/-)}$	0.00	0.06	0.15*	-0.13	0.28**	-0.22	0.23***	-0.06	0.14	-0.26
	(0.9941)	(0.4296)	(0.0155)	(0.1525)	(0.0019)	(0.1397)	(0.0001)	(0.6002)	(0.1908)	(0.0103)
	[1.0000]	[0.5296]	[0.0646]	[0.4977]	[0.0460]	[0.8881]	[0.0100]	[0.6289]	[0.5750]	[0.2159]
$f_{(\sqrt{+}/-)}$	0.06	0.11	0.22**	0.02	0.32**	-0.24	0.31***	-0.05	0.34	-0.09
	(0.4057)	(0.1778)	(0.0004)	(0.8153)	(0.0021)	(0.1166)	(0.0000)	(0.6379)	(0.0031)	(0.4174)
	[0.4963]	[0.2805]	[0.0240]	[0.9154]	[0.0273]	[0.3284]	[0.0020]	[0.7089]	[0.1972]	[0.9960]
Sortino ratio differences $\Delta_{(-)}$										
$f_{(\sigma)}$	0.09	-0.14	0.10	0.74*	0.35	-0.17	0.22	0.15	1.10*	0.98
	(0.4688)	(0.2647)	(0.5096)	(0.0000)	(0.0737)	(0.3094)	(0.0998)	(0.3923)	(0.0000)	(0.0650)
	[0.6835]	[0.3804]	[0.7275]	[0.0553]	[0.2998]	[0.4117]	[0.3438]	[0.4917]	[0.0566]	[0.9567]
$f_{(-)}$	0.07	0.16	0.45	0.24	0.78**	-0.51	0.65**	-0.05	1.51	0.77
	(0.5785)	(0.3125)	(0.0039)	(0.0615)	(0.0011)	(0.0405)	(0.0000)	(0.8419)	(0.0001)	(0.1676)
	[0.7695]	[0.4290]	[0.1093]	[0.4943]	[0.0466]	[0.1552]	[0.0213]	[0.9067]	[0.1692]	[0.5516]
$f_{(+/-)}$	0.16	0.13	0.68**	-0.16	0.87***	-0.34	0.67***	0.04	0.67	-0.17
	(0.2124)	(0.3624)	(0.0013)	(0.1567)	(0.0002)	(0.1872)	(0.0000)	(0.8729)	(0.0073)	(0.6660)
	[0.9554]	[0.6069]	[0.0120]	[0.5716]	[0.0087]	[0.8421]	[0.0067]	[0.9167]	[0.1912]	[0.8688]
$f_{(\sqrt{+}/-)}$	0.21	0.19	0.65**	0.06	0.87***	-0.38	0.84***	-0.01	1.22**	0.41
	(0.0869)	(0.1748)	(0.0001)	(0.6446)	(0.0002)	(0.1537)	(0.0000)	(0.9762)	(0.0001)	(0.3944)
	[0.2818]	[0.3884]	[0.0120]	[0.8581]	[0.0073]	[0.4557]	[0.0027]	[0.9887]	[0.0393]	[0.9900]

Table 4: Timing risk of a broader set of test portfolios

We report the number of anomaly portfolios and exchange-traded funds for which we find significantly *positive* alphas and differences in Sharpe and Sortino ratios at the 5% significance level. For the direct comparisons of risk-adjusted performances, we employ both Jobson-Korkie (JK) and Ledoit-Wolf (LW) tests.

	alpha	Δ		$\Delta_{(-)}$	
		JK	LW	JK	LW
207 anomaly portfolios					
$f_{(\sigma]}$	110	51	25	64	24
$f_{(-)}$	142	63	30	89	35
$f_{(+/-)}$	102	44	19	63	25
$f_{(\sqrt{+}/-)}$	138	63	34	86	38
71 exchange-traded funds					
$f_{(\sigma]}$	14	9	1	13	0
$f_{(-)}$	11	6	1	10	0
$f_{(+/-)}$	3	3	1	3	0
$f_{(\sqrt{+}/-)}$	17	17	3	19	2
$f_{(q)}$	12	9	0	12	0

Table 5: Comparison of the pricing errors of the Fama-French factor models, with and without timing

We report the t-statistics of the difference in the squared maximum Sharpe ratios that we can attain from managed and unmanaged Fama-French factor models. We compute the t-statistics for the case of nonnested models with nontraded factors (Barillas et al., 2020). Asymptotic-valid critical values rest on the standard Gaussian distribution. We denote by $*$, $**$ and $***$ significance at the 10%, 5% and 1% levels.

	timing strategy			
	$f_{(\sigma)}$	$f_{(-)}$	$f_{(+/-)}$	$f_{(\sqrt{+}/-)}$
Fama-French three-factor model	1.17	1.86**	0.65	1.82**
Fama-French five-factor model	0.23	1.79**	1.05	2.00***

Table 6: Stochastic dominance between managed and unmanaged portfolios

We report Barrett and Donald's (2003) test of first-order stochastic dominance in the cross-section distribution of Sharpe ratios of managed and unmanaged portfolios. We denote by $*$, $**$ and $***$ significance at the 10%, 5% and 1% levels, whose asymptotic-valid critical values are respectively 1.073, 1.2239 and 1.5174.

	timing strategy				
	$f_{(\sigma)}$	$f_{(-)}$	$f_{(+/-)}$	$f_{(\sqrt{+}/-)}$	$f_{(q)}$
anomaly portfolios					
dominates	1.08*	1.62***	0.74	1.67***	
dominated	0.34	0.05	0.49	0.20	
exchange-traded funds					
dominates	1.35**	1.18*	0.34	1.27**	0.93
dominated	0.25	0.25	0.42	0.25	0.17

Figure 1: Descriptive statistics of the monthly returns on anomaly portfolios
We report their box plots for the average returns (mean), volatility (vol), and downside semivolatility (semivol) in percentage per annum, as well skewness and excess kurtosis.

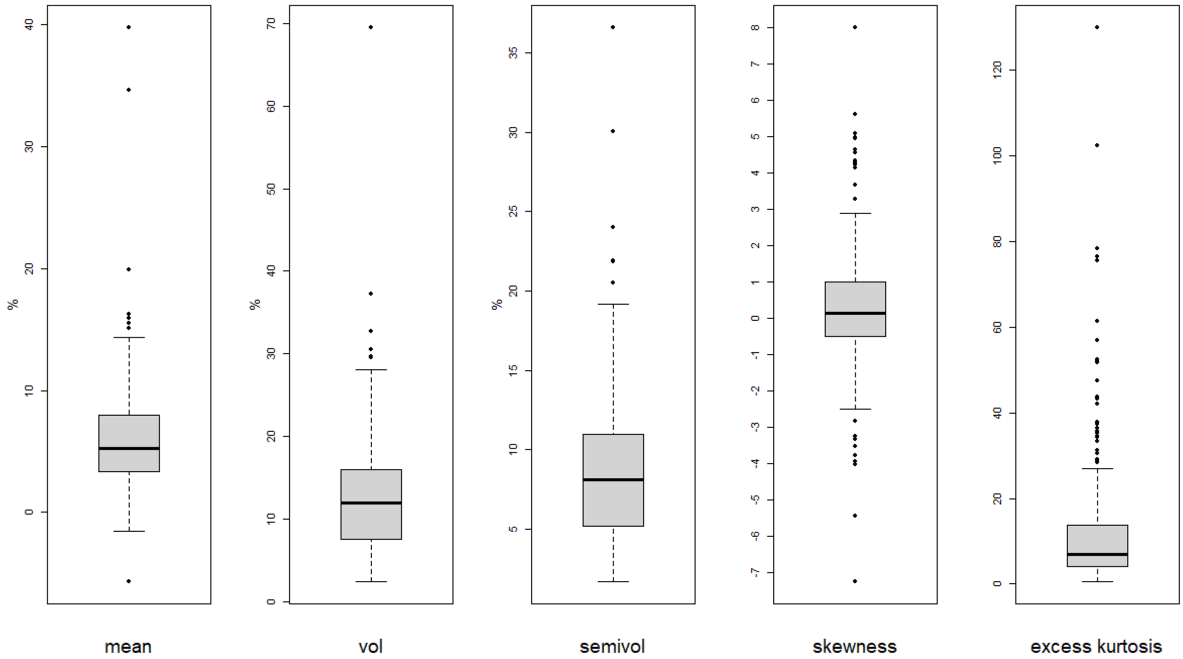

Figure 2: Descriptive statistics of the monthly returns on exchange-traded funds
We report their box plots for the average returns (mean), volatility (vol), and downside semivolatility (semivol) in percentage per annum, as well skewness and excess kurtosis.

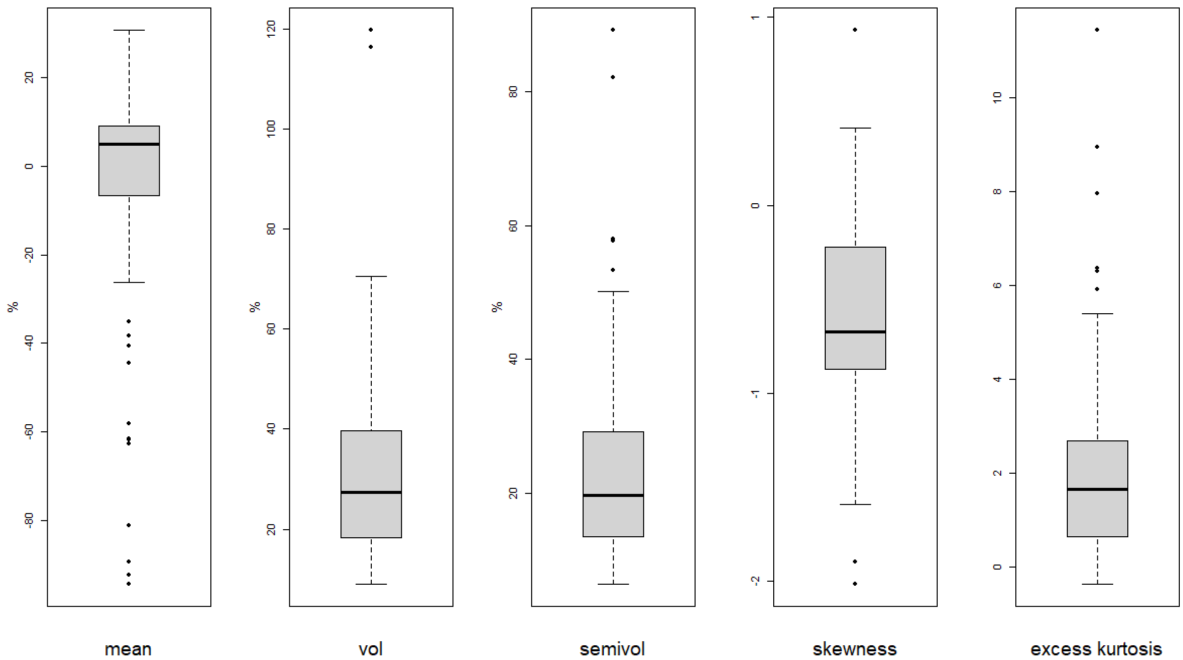

Figure 3: Distribution of the Sharpe ratios of the managed and unmanaged anomaly portfolios
We report the empirical distribution of the Sharpe ratios of the 207 anomaly portfolios for the different timing strategies.

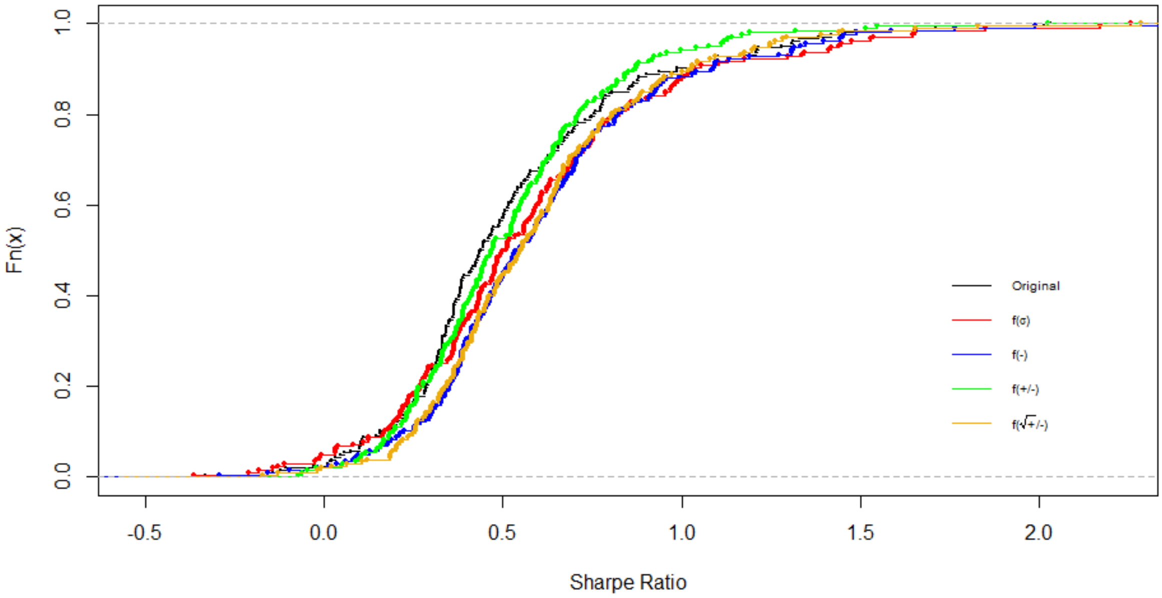

Figure 4: Distribution of the Sortino ratios of the managed and unmanaged anomaly portfolios
We report the empirical distribution of the Sortino ratios of the 207 anomaly portfolios for the different timing strategies.

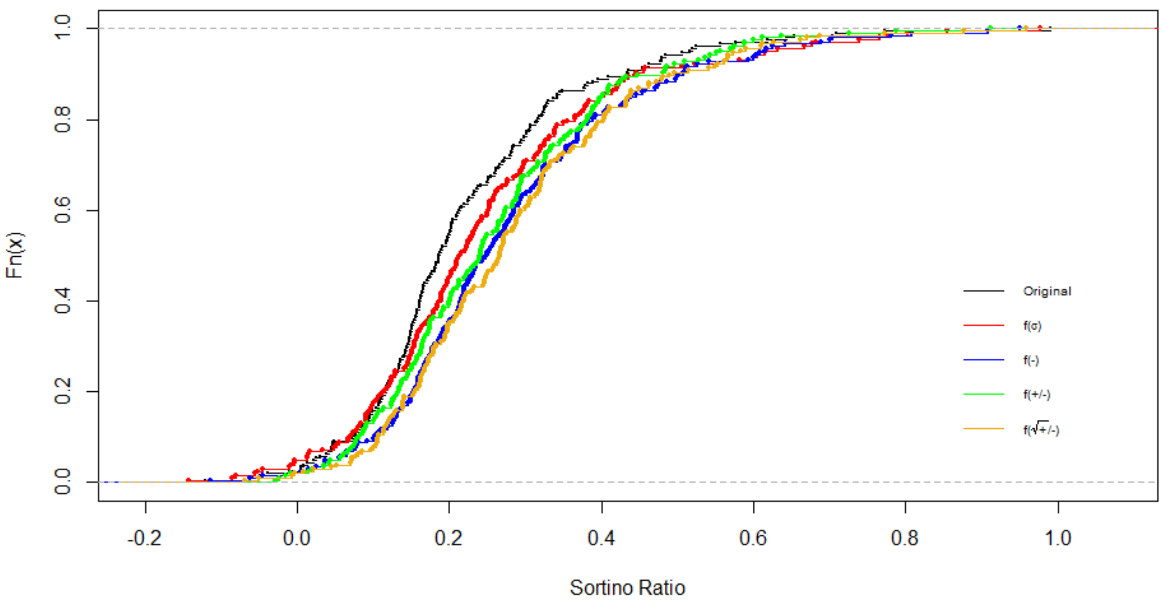

Figure 5: Distribution of the Sharpe ratios of the managed and unmanaged exchange-traded funds
We report the empirical distribution of the Sharpe ratios of the 71 exchange-traded portfolios for the different timing strategies.

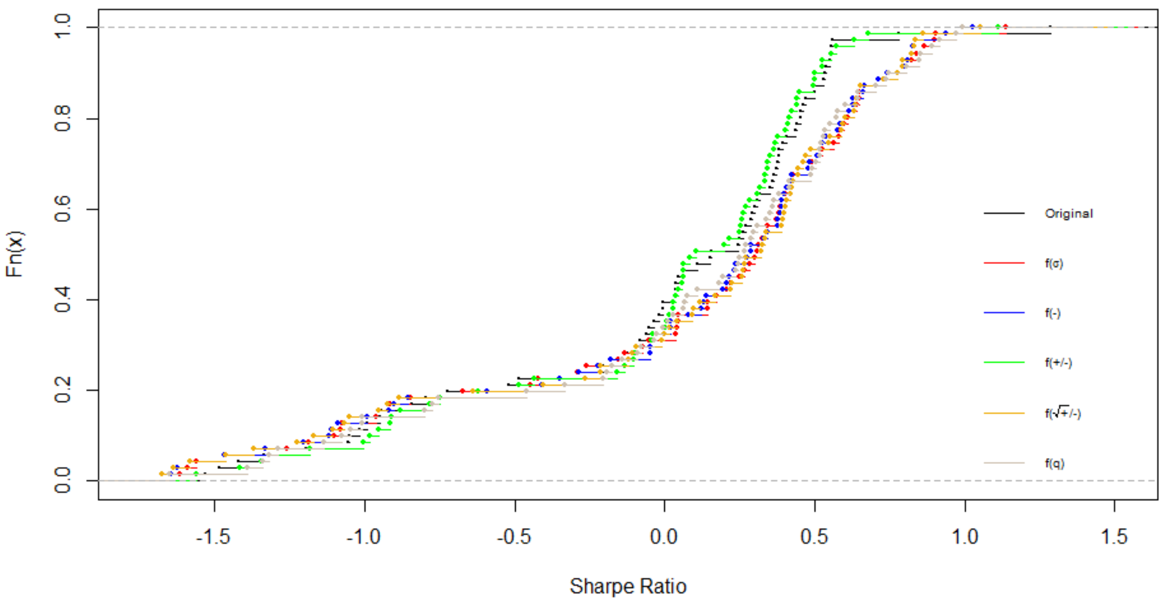

Figure 6: Distribution of the Sortino ratios of the managed and unmanaged exchange-traded funds
We report the empirical distribution of the Sortino ratios of the 71 exchange-traded portfolios for the different timing strategies.