Was Javert Right To Be Suspicious? Marginal Treatment Effects with Censored Data*

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July 24, 2023

Abstract

This paper presents new econometric tools to unpack the treatment effect heterogeneity of punishing misdemeanor offenses on time-to-recidivism or, more generally, of a given treatment intervention on a duration outcome. More specifically, we show how one can (point and set) identify, estimate and make inferences on the distributional, quantile, and average marginal treatment effects in setups where the treatment selection is endogenous and the outcome of interest is right-censored. We explore our proposed econometric methodology to evaluate the effect of fines and community service sentences as a form of punishment on time-to-recidivism in the State of São Paulo, Brazil, between 2010 and 2019, leveraging the as-if random assignment of judges to cases. Our results highlight substantial treatment effect heterogeneity that other tools are not able to capture. For instance, we find that people who would be punished by most judges take longer to recidivate as a consequence of the punishment, while people who would be punished only by strict judges recidivate at an earlier date than if they were not punished. This result suggests that designing sentencing guidelines that encourage strict judges to become more lenient could reduce recidivism.

Keywords: Duration Outcomes, Instrumental Variable, Alternative Sentences, Recidivism

JEL Codes: C24, C31, C36, C41, K42

*We would like to thank Fernando Ramon Machado de Andrade, Xiaohong Chen, Jesse Bruhn, Paul Goldsmith-Pinkham, Renata Hirota, Peter Hull, Toru Kitagawa, Helena Laneuville, Elizabeth Luh, Andriy Norets, Jonathan Roth, Susanne Schennach, Henrik Sigstad, Renato Caetano de Almeida Silva, Alexander Torgovitsky, Julio Trecenti, and Edward Vytlacil, seminar participants at Brown University, University of Chicago and Yale University, and the institutional support of Associação Brasileira de Jurimetria.

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To owe his life to a malefactor, to accept that debt and to repay it; to be, in spite of himself, on a level with a fugitive from justice, and to repay his service with another service; to allow it to be said to him, "Go," and to say to the latter in his turn: "Be free"; to sacrifice to personal motives duty, that general obligation, and to be conscious, in those personal motives, of something that was also general, and, perchance, superior, to betray society in order to remain true to his conscience; that all these absurdities should be realized and should accumulate upon him,—this was what overwhelmed him.

Les Misérables by Victor Hugo

1 Introduction

Understanding how different types of sanctions impact the behavior of defendants is a critical area of research in the field of Economics of Crime. For misdemeanors, which are relatively minor offenses, we know relatively little about the causal effects of prosecution on defendants' subsequent criminal justice involvement (Agan, Doleac and Harvey, 2023), and arguably even less about the effect of alternative sentences on defendants' recidivism.[1] This is a particularly important topic as a misdemeanor charge is often the point of entry for individuals to the criminal justice system. If they are convicted, they will then acquire a criminal record. This could "lower the cost" of committing other crimes or work as intended and prevent future criminal behavior. In practice, it is unclear which direction dominates, and it is likely that this varies from individual to individual. Being able to understand the types of defendants which are on either side is therefore desirable and policy-relevant.

In this article, we propose econometric tools that are tailored to highlight treatment effect heterogeneity with respect to the unobserved punishment resistance on time-to-recidivism. These tools can then be used to shed light on to whom punishments are working as intended in terms of avoiding (or postponing) recidivism. Importantly, our tools account for the fact that (i) time-to-recidivism is a duration outcome that is subject to right-censoring, i.e., not all defendants recidivate by the end of the sampling period (but may do it later on); (ii) treatment selection is endogenous, and judges are likely to have more information about the case than econometricians; (iii) individuals may be inherently heterogeneous (essential heterogeneity); (iv) one may be interested in causal effects beyond local average treatment effect parameters; (iv) distributional features of time-to-recidivism may also be relevant.

We achieve these goals by extending the marginal treatment effects (MTE) framework developed by Heckman and Vytlacil (1999, 2005) and the distributional and quantile MTE

[1] See Huttunen, Kaila and Nix (2020), Giles (2021), Klaassen (2021), Possebom (2022), and Lieberman, Luh and Mueller-Smith (2023) for some advances in this area.

extensions developed by Carneiro and Lee (2009) to setups in which the outcome variable is right-censored. The main requirement to use our tools is access to a continuous instrument such that the propensity score has full support.[2] In the context of crime economics, this instrument is usually given by the trial judge's leniency rate.

In our view, the MTE framework is particularly attractive to studying the effect of punishments on time-to-recidivism. For example, it allows one to assess the treatment effect of punishment on recidivism for defendants on a margin of indifference between being punished or not. By considering different degrees of unobserved punishment resistance, the MTE provides a detailed picture of how punishments heterogeneously affect recidivism and can be used to design better sentencing criteria and/or train judges to follow a specific protocol. For example, suppose that one finds a negatively sloped MTE function with some positive and negative effects. This would suggest that defendants who would be punished even by very lenient judges, i.e., defendants with low unobserved punishment resistance, would take more time to recidivate as a result of the punishment (punishment is working as intended). On the other hand, defendants who would be fined only by very strict judges, i.e., defendants with high unobserved punishment resistance, would recidivate sooner than if they were not punished (punishment is not effective, perhaps because of scaring effects of a criminal record). Such degree of heterogeneity is usually washed-out when using single summaries of treatment effects such as local average treatment effect (LATE) (Imbens and Angrist, 1994). However, even when one is interested in summary measures of causal effects, one can use the MTE function to construct them; see, e.g., Heckman and Vytlacil (2005), Heckman, Urzua and Vytlacil (2006). It is also interesting to mention that exploring a continuous instrument makes the definition of "complier" less clear than in the binary instrument case, which could potentially make the LATE results harder to interpret formally. The MTE does not focus on "compliers", so it is immune to this potential limitation.

Dealing with time-to-recidivism, or more generally, a duration variable that is subject to right-censoring, introduces some interesting challenges depending on the censoring mechanism. For instance, if censoring is independent of potential outcomes, we are able to point-identify the distributional marginal treatment effect (DMTE) and quantile marginal treatment effect (QMTE) functions for some but not necessarily all distribution support points or quantiles. Nonparametrically identifying the entire DMTE and QMTE functions, which are required to identify the average MTE (henceforth MTE for simplicity), is only possible if the support of the censoring variable is at least as large as the support of the duration outcome, a restriction that is sometimes strong. When this support restriction is

[2]See Brinch, Mogstad and Wiswall (2017) and Mogstad, Santos and Torgovitsky (2018) for extensions of the MTE framework that does not require this support condition.

not satisfied, one can only nonparametrically point-identify truncated MTE functions. We propose semiparametric estimators and inference procedures for the DMTE, QMTE, and (truncated) MTE functions and establish their large sample properties.

Now, if censoring is potentially dependent on the potential outcomes, point-identification of DMTE and QMTE functions is not feasible without additional assumptions and data requirements. In such cases, we still show how one can partially identify these marginal treatment effect functions and propose semiparametric estimators for these bounds. We also show how one can explore some economically-motivated restrictions on the dependence between censoring and potential outcomes to sharpen the bounds. In particular, we consider the restriction that defendants are committing fewer crimes over time, which is implied by a negative regression dependence between potential outcomes and censoring variables (Lehmann, 1966). Other types of restrictions are also possible.

In some setups, in order to bypass the challenges associated with right censored time-to-recidivism, researchers may choose to focus on recidivism within a given time frame, say two years. Although this is convenient and generically valid, the choice of cutoff is arbitrary, and it may be the case that punishment has no effect on recidivism within two years but then has an effect within two years and a half or within one year.[3] One can interpret our DMTE results as an extension of this "binarization" approach that aims to avoid choosing arbitrary cutoffs and, instead, consider recidivism within y periods for a continuum of $y \in \mathbb{R}_+$. Our QMTE and MTE results "transform" our DMTE results so the underlying treatment effects are expressed in the same units as the time-to-recidivism outcome, which can lead to additional insights. Furthermore, when a policymaker is interested in minimizing the cost of recidivism inter-temporally, they discount the cost of recidivism more strongly if the time-to-recidivism is longer. Therefore, to make more informed treatment allocations (or recommendations), the policymaker needs information on time-to-recidivism beyond whether or not a defendant recidivates within two years; see Appendix E.1 for additional details. In such cases, however, one needs to tackle the censoring problem directly. Failing to do so may lead to misleading conclusions.

We show how our causal inference tools can be used in practice by evaluating the effect of fines and community service sentences as a form of punishment on time-to-recidivism in the State of São Paulo, Brazil, between 2010 and 2019.[4] Our treated group (punished group) is

[3]In Appendix E.2, a simple example illustrates that focusing on quantile and average treatment effects for duration outcomes may provide different conclusions than focusing on short-run recidivism indicators.

[4]São Paulo is the largest state in Brazil, with a population above 41 million people according to the Brazilian Census in 2010. Moreover, analyzing the impact of judicial policies on criminal behavior in this state is relevant due to its relatively high criminality. For example, according to São Paulo Public Safety Secretary, there were 6.48 murders, 878.83 thefts, and 490.23 robberies per 100,000 inhabitants in 2020. Importantly, theft is one of the most common crimes in our sample.

the defendants who were fined or sentenced to community services, and our untreated group (unpunished group) contains defendants who were acquitted or whose cases were dismissed. To measure recidivism, we check whether the defendant's name appears in any criminal case within the sample period after the final sentence's date. More precisely, our outcome variable is the time between the final sentence and a subsequent criminal case. Since the sampling period is finite, the outcome variable is right-censored.

To deploy our proposed methodology, we need a continuous instrumental variable since we do face endogenous selection into punishment. We use the trial judge's leave-one-out rate of punishment (or "leniency rate") as an instrument for the trial judge's decision (Bhuller, Dahl, Loken and Mogstad, 2019; Agan et al., 2023). Importantly, this instrumental variable is continuous with large support, and is independent of the defendant's counterfactual criminal behavior because judges are randomly assigned to cases conditional on court districts according to state law in São Paulo. Our outcome data — time-to-recidivism — is right-censored by construction, requiring a methodology that accounts for this identification challenge.

We find that QMTE functions for $0.10, 0.25$ and 0.50 quantiles and the MTE function averaged across all court districts are heterogeneous with respect to unobserved punishment resistance, the treatment effects being sometimes positive and sometimes negative. More precisely, we find that people who would be punished by most judges (those with low punishment resistance) take longer to recidivate as a consequence of the punishment, while people who would be punished only by strict judges (high punishment resistance) recidivate at an earlier date than if they were not punished. This result suggests that designing sentencing guidelines that encourage strict judges to become more lenient could increase time-to-recidivism.

We also compare our results with methods that ignore the time-to-recidivism being right-censored. In particular, we find that using a linear MTE estimator overestimates the treatment effects across all unobserved punishment resistance variable while ignoring the censoring problem and estimating the MTE model semiparametrically lead to attenuated effects. If one were to use two-stage least squares (ignoring censoring), one would find that treatment effects are slightly negative but would not be able to highlight heterogeneity as in the MTE function.

Related literature: This article contributes to different branches of literature. Concerning its theoretical contribution, our work contributes to the literature on MTE by extending the MTE framework of Heckman and Vytlacil (1999, 2005), Heckman et al. (2006), and Carneiro and Lee (2009) to a setting with right-censored data.[5] We also contribute to

[5] The MTE framework has also been extended to settings with sample selection (Bartalotti, Kedagni and

the literature on duration outcomes; see, e.g., Khan and Tamer (2009), Frandsen (2015), Tchetgen, Walter, Vansteelandt, Martinussen and Glymour (2015), Sant'Anna (2016, 2021), Beyhum, Florens and Keilegom (2022), Delgado, Garcia-Suaza and Sant'Anna (2022). None of these papers consider MTE-type parameters as we do. Among these, the closest work to ours is Frandsen (2015), which considers the case where the censoring variable is observed and shows how one can identify distributional and quantile local treatment effects, assuming that censoring is exogenous. Our results can be interpreted as an extension of Frandsen (2015) to the MTE framework, possibly allowing for endogenous censoring.

Concerning its empirical contribution, our work is inserted in the literature about the effect of fines and community service sentences on future criminal behavior; see, e.g., Huttunen et al. (2020), Giles (2021), Klaassen (2021), Possebom (2022), and Lieberman et al. (2023). They all focus on binary variables indicating recidivism within a pre-specified period. Within these, as we build on his dataset, Possebom (2022) is the closest to ours. However, his focus is very different from ours, and he does not handle duration outcomes as we do.

This paper is organized as follows. Section 2 describes the data and explains why focusing on long-term recidivism is useful in our empirical application. Section 3 presents our structural model and discusses our identifying assumptions. Section 4 provides our identification results for the DMTE function with a right-censored outcome variable under two sets of assumptions. Moreover, Section 5 briefly explains how to semi-parametrically estimate the objects that are necessary to implement the identification strategy described in the previous section. Furthermore, Section 6 discusses the finite sample performance of our semi-parametric estimator using a Monte Carlo exercise. Finally, Section 7 discusses the empirical results, while Section 8 concludes.

This paper also contains an online supporting appendix. All proofs are detailed in Appendix A. Appendix B derives the asymptotic distribution of our semi-parametric estimators. Additional Monte Carlo exercises and empirical results can be found in Appendices C and D. Moreover, Appendix E provides two arguments that justify focusing on the MTE function of duration outcomes. Furthermore, Appendix F identify a conditional version of our target parameters under weaker assumptions than the ones used in the main text. Finally, Appendix G proposes an extra partial identification result that relies on a median independence assumption.

Possebom, 2022), misclassified treatment variables (Acerenza, Ban and Kédagni, 2021; Possebom, 2022), discrete instrumental variables (Brinch et al., 2017; Mogstad et al., 2018; Acerenza, 2022), and possibly invalid instruments (Mourifie and Wan, 2020).

2 Empirical Context and Data

In our empirical application, we answer the question: "Do alternative sentences (fines and community service) impact time-to-recidivism?". To answer this question, we collect data from all criminal cases brought to the Justice Court System in the State of São Paulo, Brazil, between January 4th, 2010, and December 3rd, 2019. In this section, we briefly explain our dataset and discuss why focusing on time-to-recidivism instead of recidivism within a pre-specified time horizon is helpful.[6]

We restricted our sample to cases that started between 2010 and 2017 to ensure that every defendant is observed for at least two years. Moreover, we focus on the criminal cases whose maximum prison sentence is less than 4 years because, according to Brazilian Law, these cases must be punished with a fine or a community service sentence. Due to this sample restriction, the most common crime types in our sample are theft and domestic violence.

In our dataset, we observe the defendant's full name, the defendant's court district, the case's starting date, the assigned trial judge's full name, the case's final ruling, the case's final ruling's date. Based on those variables, we define our outcome variable (Y ="time to recidivism"), our censoring variable (C ="number of days between the case's final ruling's date and the end of the sampling period"), our treatment variable (D ="final ruling in the case"), our instrument (Z = "trial judge's leniency rate") and our covariates (X = "full set of court district dummies").

Our treatment variable D divides the case-defendant pairs into two groups. The first group (treated) receives a punishment, i.e., its defendants were fined or sentenced to community services because they were either convicted or signed a non-prosecution agreement according to the final ruling in their case. The second group (control) did not receive a punishment, i.e., its defendants were acquitted or its cases were dismissed according to the final ruling in their case.

Our instrument Z is the trial judge's leniency rate. This variable is equal to the leave-one-out rate of punishment for each trial judge, where the defendant's own decision is excluded from this average. To do so, we only use the 525 judges who analyzed more than 20 cases during our sample period and worked in court districts with at least two judges during the sample period.

We, now, describe our definition of the observed outcome variable (Y ="number of days between the case's final ruling's date and the first recidivism event"). A defendant i in a case j recidivated if and only if defendant i's full name appears in a case $\bar{j}$ whose starting date is after case j's final sentence's date.[7] Then, we measure our outcome variable as the

[6] This dataset was originally used by Possebom (2022), who provides a detailed description of it.

[7] To match defendants' names across cases, we follow Possebom (2022) and define a fuzzy match if the

number of days between case j's final ruling's date and case $\bar{j}$'s starting date.[8] If defendant i did not recidivate, then $Y = C$.

Our covariates contain a full set of court district dummies. Since our identification strategy leverages the random allocation of judges to criminal cases, we only use districts with two or more judges during our sample period.

At the end, we impose one final restriction in our dataset: common support between the treatment and control groups. To do so, we impose that the minimum and maximum values of the instrument Z are the same across both treatment arms. Our final sample has 43,468 case-defendant pairs.

Now, we analyze the relationship between the censoring variable and the realized outcome to argue that focusing on long-term recidivism is relevant in our empirical context.

Figure 1 shows the right tail of the probability mass function (PDF) of the uncensored potential outcome (Y^*) given cohorts based on the censoring variable. We find that a non-negligible share of defendants has their first recidivism event in their fifth, sixth or seventh year after their sentence's date, implying that analyzing long-term recidivism is relevant. For this reason, we use time-to-recidivism as an outcome variable and focus on quantile marginal treatment effect parameters instead of focusing on short-term binary variables as commonly done in the empirical literature.

3 Econometric Framework

In this section, we explain our theoretical framework. We analyze a threshold-crossing model (Heckman and Vytlacil, 2005) with a duration outcome (Frandsen, 2015; Sant'Anna, 2016; Delgado et al., 2022):

$$D = \mathbf{1}\left\{P\left(Z, C\right) \geqslant V\right\}, \tag{1}$$

$$Y^* = Y^*\left(1\right) \cdot D + Y^*\left(0\right) \cdot \left(1 - D\right), \tag{2}$$

$$Y = \min\left\{Y^*, C\right\}. \tag{3}$$

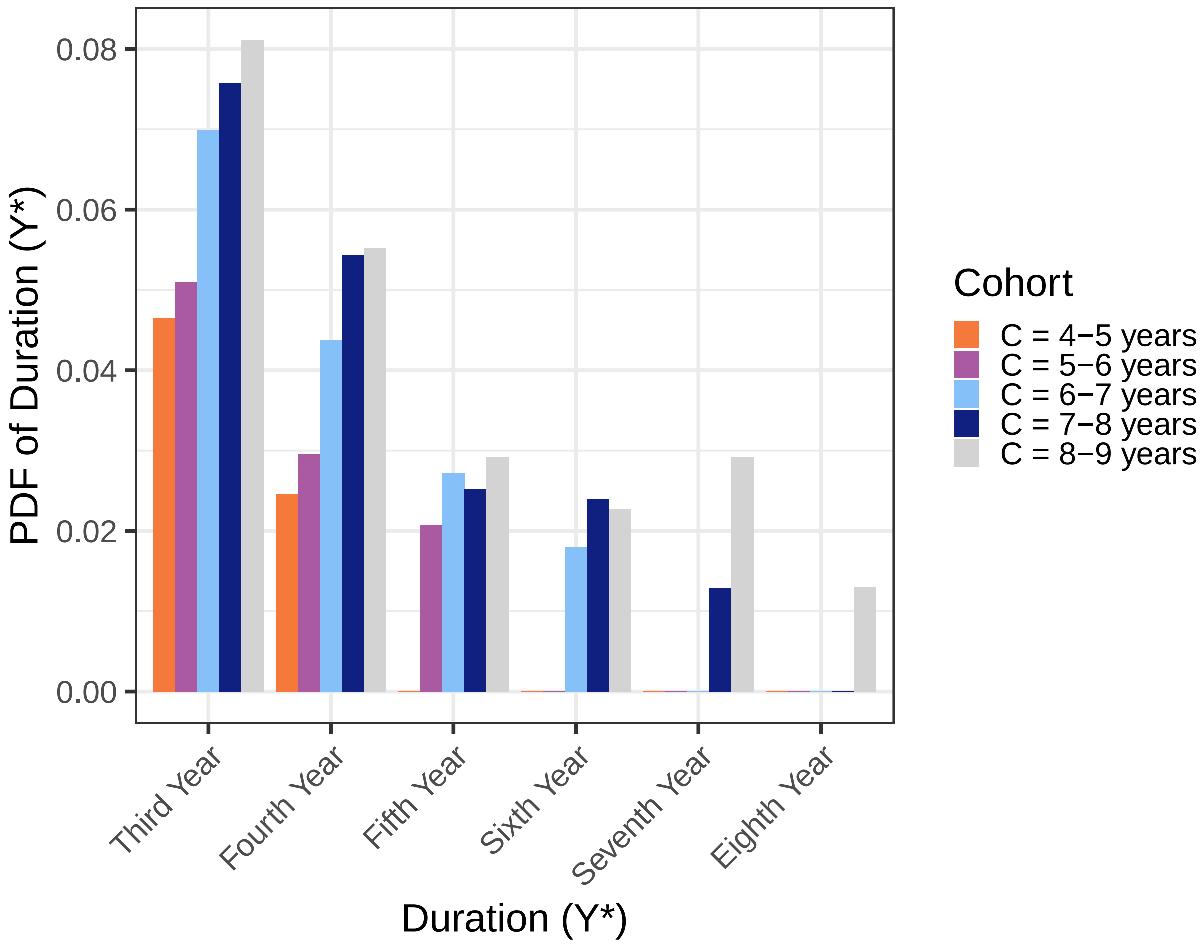
Variable Z is an observable instrumental variable with support given by an open set $\mathcal{Z} \subset \mathbb{R}$. In our empirical example, it measures the trial judge's leniency rate.

Variable C is the censoring variable. In our empirical application, it captures the length of time between the defendant's sentence date and the end of our sampling period. Since we

similarity between full names in two different cases is greater than or equal to 0.95 using the Jaro–Winkler similarity metric.

[8] Case $\bar{j}$ can be about any type of crime, including more severe crimes whose maximum sentence is greater than four years, while case j has to about a crime whose maximum sentence is at most 4 years.

Figure 1: PDF of the Uncensored Outcome given the Defendant's Cohort: $\mathbb{P}\left[y_1 \leqslant Y^* \leqslant y_2 | C\right]$



Notes: This figure shows the right tail of the probability mass function (PDF) of the uncensored potential outcome (Y^*) given cohorts based on the censoring variable. Each color denotes a different cohort: orange denotes defendants who are observed for at least four years and at most five years during our sampling period, purple denotes defendants who are observed for at least five years and at most six years, light blue denotes defendants who are observed for at least six years and at most seven years, dark blue denotes defendants who are observed for at least seven years and at most eight years, and gray denotes defendants who are observed for at least eight years and at most nine years. These conditional PDFs are evaluated at six bins of the uncensored potential outcome (e.g., "third year = between 730 days and 1095 days" or "fourth year = between 1095 days and 1460"), and these evaluation points are denoted in the x-axis. The y-axis denotes the value of the PDF.

stop observing all defendants on the same date, the censoring variable C varies only because of the sentence's date. Let $\mathcal{C}$ denote the support of C.

Function $P\colon \mathcal{Z} \times \mathcal{C} \to \mathbb{R}$ is unknown and captures the willingness to take the treatment for each value of Z and C. In our empirical application, it captures the trial judge's punishment criteria and it allows trial judges to update their punishment criteria over time (Bhuller and Sigstad, 2022) by including C as an argument.[9] Let $\mathcal{P}$ denote the support of $P(Z, C)$.

Variable V is a latent heterogeneity term and captures the unobserved treatment resistance. In our empirical application, it captures the amount of criminal evidence in the defendant's favor.

Variable D is the treatment status. In our empirical application, it captures whether the defendant received some type of punishment — a fine or community service sentence imposed by a non-prosecution agreement or a conviction — in her case's final ruling. Note that Equation (1) models how the agent self-selects into treatment and imposes monotonicity (Imbens and Angrist, 1994; Vytlacil, 2002).

Note that Equation (1) allows the treatment status to depend on the value of C. In other words, our "complier group" depends on the value of C. Differently from us, Frandsen (2015) does not condition the group of compliers on the value of the censoring variable, implicitly imposing that $D(z, c) = D(z)$. In this sense, we generalize Frandsen's (2015) framework.

Variable Y^* is the uncensored outcome variable. $Y^*(0)$ and $Y^*(1)$ are the potential uncensored outcomes that depend on the treatment status. In our empirical application, it captures the length of time between the defendant's sentence date and her next criminal case's starting date.

Finally, variable Y is the censored outcome variable. In our empirical application, it captures the length of time between the defendant's sentence's date and the earliest of two dates: her next criminal case's starting date or the end of our sampling period.

The researcher observes only the vector (Y, C, D, Z), while $Y^*(0)$, $Y^*(1)$, Y^* and V are latent variables. For simplicity, we drop exogenous covariates from the model and focus on the case with a single instrument. All results derived in the paper hold conditionally on covariates and can be extended to the case with multiple instruments.

Following Heckman and Vytlacil (2005) and Frandsen (2015), we impose five assumptions. These assumptions are sufficient to identify the distributional marginal treatment effect and some quantile marginal treatment effects when the outcome variable is right-censored. After presenting these five assumptions, we also propose a weaker restriction (Assumption 6) that is sufficient to partially identify the same treatment effect parameters. Moreover, to identify

[9] In other empirical applications, the censoring variable C may be a post-treatment variable. In these cases, it is better to eliminate C from the propensity score function.

the marginal treatment effect, we impose two extra restrictions (Assumptions 7 and 8) that restrict the support of the uncensored potential outcomes.

Assumption 1 (Random Assignment)**.** *Conditional on C, the latent variables $Y^*(0)$, $Y^*(1)$ and V are independent of the instrument Z, i.e.,*

$$Z \perp\!\!\!\perp (Y^*(0), Y^*(1), V) | C.$$

Assumption 1 is an exogeneity assumption and is common in the literature about instrumental variables with censored outcomes (Frandsen, 2015; Sant'Anna, 2016; Delgado et al., 2022). In our empirical application, this assumption holds conditional on the court district because, in the State of São Paulo, Brazil, trial judges are randomly assigned to cases within each court district.

Note also that Assumption 1 allows the instrument to depend on the censoring variable. In our empirical application, this flexibility is useful because the trial judge's punishment rate may depend on the case's sentence date if judges who entered the Judiciary more recently are more lenient than judges who retired at the beginning of our sampling period.

Assumption 2 (Propensity Score is Continuous)**.** *Conditional on C, $P(z, c)$ is a nontrivial function of z and the random variable $P(Z, c)| C = c$ is absolutely continuous with support given by an interval $\mathcal{P} := [\underline{p}, \overline{p}] \subseteq [0, 1]$ for any $c \in \mathcal{C}$.*[10]

Assumption 2 is a rank condition, intuitively imposing that the instrument is locally relevant. In addition, we implicitly assume that the support of the propensity score does not vary with the value of C. In our application, this implicit assumption is plausible because the judges are mostly the same over time.

Assumption 3 (V is continuous)**.** *The distribution of the latent heterogeneity variable V conditional on C is absolutely continuous with respect to the Lebesgue measure.*

Assumption 3 is a regularity condition that allows us to normalize the marginal distribution of $V| C$ to be the standard uniform. Consequently, the propensity score $\mathbb{P}[D = 1| Z = z, C = c]$ satisfies $P(z, c) = \mathbb{P}[D = 1| Z = z, C = c]$ for any $z \in \mathcal{Z}$ and $c \in \mathcal{C}$. Moreover, this normalization implies that V is independent of C.

Assumption 4 (Positive Mass)**.** *Conditional on C, all treatment groups exist, i.e., $\mathbb{P}[D = d| C = c] \in (0, 1)$ for any $d \in \{0, 1\}$ and any $c \in \mathcal{C}$.*

[10]The assumption that $\mathcal{P}$ is an interval is made for notational simplicity. All the proofs can be easily extended to the case where $\mathcal{P}$ is a set with a non-empty interior.

Assumption 4 is a regularity condition. It extends the standard positive mass assumption in the policy evaluation literature to the setting with a duration outcome.

Assumption 5 (Random Censoring)**.** *The censoring variables are independent of the uncensored potential outcomes given the latent heterogeneity V, i.e.,*

$$C \perp\!\!\!\perp \left(Y^*(0), Y^*(1)\right) \middle| V.$$

Assumption 5 is an exogeneity assumption and is common in the literature about duration outcomes (Frandsen, 2015; Sant'Anna, 2016; Delgado et al., 2022). When combined with Assumption 3, Assumption 5 implies that C is unconditionally independent of the uncensored potential outcomes, i.e., $C \perp\!\!\!\perp \left(Y^*(0), Y^*(1)\right)$. In our empirical application, this restriction imposes that the case's sentence date is independent of the defendant's decision to commit another crime in the future.

Importantly, Assumption 5 imposes that controlling for V accounts for all sources of endogeneity coming through the censoring variable. This assumption can be restrictive since endogeneity might still be present once controlling for the latent heterogeneity. If the researcher believes that this assumption is too strong in a particular application, she can use an alternative assumption that is sufficient to partially identify the distributional marginal treatment effect and some quantile marginal treatment effects when the outcome variable is right-censored.

This alternative assumption restricts the relationship between the latent heterogeneity, the censoring variable and the potential outcomes.

Assumption 6 (Censoring Independence and Regression Dependence)**.** *Conditional on V, the potential outcomes are negatively regression dependent on the censoring variable, i.e., $\mathbb{P}\left[Y^*(d) \leqslant y \middle| C = \tilde{c}, V = v\right] \geqslant \mathbb{P}\left[Y^*(d) \leqslant y \middle| C = c, V = v\right]$ for any $d \in \{0, 1\}$, any $v \in (0, 1)$ and any $(c, \tilde{c}) \in \mathcal{C}^2$ such that $c \leqslant \tilde{c}$.*

In our empirical application, Assumption 6 imposes that the potential outcomes of more recent cases first-order stochastically dominate the potential outcomes of older cases.[11] Intuitively, this restriction imposes that defendants are committing fewer crimes over time and is plausible given that the state of São Paulo became safer during our sampling period.

Assumptions 1-5 and Assumptions 1-4 and 6 are sufficient to identify the distributional marginal treatment effect and some quantile marginal treatment effects when the outcome variable is right-censored. However, to identify the marginal treatment effect, we need to impose two support restrictions: Assumptions 7 and 8.

[11] For more information on the definition of regression dependence and other concepts of statistical dependence, see Lehmann (1966).

Assumption 7 (Finite Moments)**.** *Conditional on C, the potential outcome variables have finite first moments, i.e.,* $\mathbb{E}\left[|Y(d)|\right| V = v, C = c] < \infty$ *for any* $d \in \{0,1\}$*, any* $v \in [0,1]$ *and any* $c \in \mathcal{C}$*.*

Assumption 7 is a regularity condition that allows us to apply standard integration theorems and ensures that average treatment effects are well-defined.

Assumption 8 (Support Restriction)**.** *The support of the uncensored potential outcomes is smaller than the support of the censoring variable, i.e.,* $\gamma_C = +\infty$ *or* $\gamma_d < \gamma_C$ *for any* $d \in \{0,1\}$*, where* $\gamma_C := \inf\left\{c \in \overline{\mathbb{R}} \colon \mathbb{P}\left[C \leqslant c\right] = 1\right\}$ *and* $\gamma_d := \inf\left\{y \in \overline{\mathbb{R}} \colon \mathbb{P}\left[Y^*(d) \leqslant y\right] = 1\right\}$ *for any* $d \in \{0,1\}$*.*

Assumption 8 restricts the support of the uncensored potential outcomes to be smaller than the support of the censoring variable. In our empirical application, this assumption imposes that all defendants recidivate within 10 years, which is the longest observation period in our sample. Formally, this restriction imposes that $\gamma_d < \gamma_C = 10$ years for any $d \in \{0,1\}$. If a researcher believes that this assumption is implausible, she cannot identify the marginal treatment effect function and must focus on some quantile marginal treatment effect function.

4 Identification

In this section, we, first, define our parameters of interest. Second, in Subsection 4.1, we impose Assumptions 1-5, 7 and 8 to point-identify our target parameters by imposing that the censoring variable is independent of the potential outcomes. Then, in Subsection 4.2, we replace Assumption 5 with Assumption 6 and partially identify the target parameters by imposing that the potential outcomes are negatively regression dependent on the censoring variable.

Our target parameters are the Distributional Marginal Treatment Response functions:

$$DMTR_d\left(y, v\right) := \mathbb{P}\left[Y^*(d) \leqslant y\right| V = v\right]. \tag{4}$$

for any $d \in \{0,1\}$, $y < \gamma_C$ and $v \in [0,1]$.

If we can identify these functions, we can also identify the Quantile Marginal Treatment Response functions,

$$QMTR_d\left(\tau, v\right) := \inf\{y \colon \mathbb{P}\left[Y^*(d) \leqslant y\right| V = v\right] \geqslant \tau\} \tag{5}$$

for any $d \in \{0,1\}$ and $\tau \in [0, \overline{\tau}_d(v))$, where $\overline{\tau}_d(v) := DMTR_d\left(\gamma_C, v\right)$ for any $d \in \{0,1\}$.

Using these objects, we can also identify the Distributional Marginal Treatment Effect function,

$$DMTE\left(y,v\right) := DMTR_1\left(y,v\right) - DMTR_0\left(y,v\right), \tag{6}$$

the Quantile Marginal Treatment Effect function,

$$QMTE\left(\tau,v\right) := QMTR_1\left(\tau,v\right) - QMTR_0\left(\tau,v\right), \tag{7}$$

and the Marginal Treatment Effect function,

$$MTE\left(v\right) := \mathbb{E}\left[Y^*\left(1\right) - Y^*\left(0\right)\middle| V = v\right] = \int_0^1 QMTE\left(\tau,v\right)\, d\tau. \tag{8}$$

Note that, when analyzing the impact of judicial decisions on recidivism, many author (Agan et al., 2023; Bhuller et al., 2019; Giles, 2021; Huttunen et al., 2020; Klaassen, 2021; Possebom, 2022) focus on distributional impacts (Equation (6)) for small values of y (short term analysis). In this paper, we advocate for moving beyond this short term horizon and focusing on quantile or average treatment effects of duration outcomes (Equations (7) and (8)). In Appendix E.2, we numerically exemplify why focusing on duration outcomes may provide more information than the standard approach in crime economics.

4.1 Point-Identification of the DMTR Function

Before point-identifying the $DMTR$, we state a lemma that will be used to derive our main identification results.

Lemma 4.1. *If Assumptions 1-4 hold, then*

$$\mathbb{P}\left[Y \leqslant y, D = 1\middle| P\left(Z,C\right) = p, C = y+\delta\right] = \int_0^p \mathbb{P}\left[Y^*(1) \leqslant y\middle| C = y+\delta, V = v\right] dv \tag{9}$$

and

$$\mathbb{P}\left[Y \leqslant y, D = 0\middle| P\left(Z,C\right) = p, C = y+\delta\right] = \int_p^1 \mathbb{P}\left[Y^*(0) \leqslant y\middle| C = y+\delta, V = v\right] dv \tag{10}$$

for any $y < \gamma_C$, $p \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y+\delta \in \mathcal{C}$.

If Assumption 5 holds too, then

$$\mathbb{P}\left[Y \leqslant y, D = 1\middle| P\left(Z,C\right) = p, C = y+\delta\right] = \int_0^p \mathbb{P}\left[Y^*(1) \leqslant y\middle| V = v\right] dv \tag{11}$$

and

$$\mathbb{P}\left[Y \leqslant y, D=0 \mid P(Z,C)=p, C=y+\delta\right]=\int_{p}^{1} \mathbb{P}\left[Y^{*}(0) \leqslant y \mid V=v\right] dv \tag{12}$$

for any $y<\gamma_C$, $p \in \mathcal{P}$ *and* $\delta \in \mathbb{R}_{++}$ *such that* $y+\delta \in \mathcal{C}$.

Proof. See Appendix A.1. ■

Now, we state our main result: point-identification of the $DMTR$ functions.

Proposition 4.1. *If Assumptions 1-5 hold, then*

$$DMTR_d(y,p)=(2\cdot d-1)\cdot \int_{\mathcal{D}} \frac{\partial \mathbb{P}\left[Y \leqslant y, D=d \mid P(Z,C)=p, C=y+\delta\right]}{\partial p} d\delta$$

for any $d \in \{0,1\}$, $y<\gamma_C$ *and* $p \in \mathcal{P}$, *where* $\mathcal{D}:=\{\delta \in \mathbb{R}_{++}: y+\delta \in \mathcal{C}\}$.

Proof. See Appendix A.2. ■

Moreover, the identification of some quantile marginal treatment effect functions is an immediate consequence of Proposition 4.1. We state this result as a corollary for convenience.

Corollary 4.1. *If Assumptions 1-5 hold, then* $QMTE(\tau,p)$ *is identified for any* $p \in \mathcal{P}$ *and* $\tau \in [0,\overline{\tau}(p))$, *where* $\overline{\tau}(p):=\min\{\overline{\tau}_0(p),\overline{\tau}_1(p)\}$ *and* $\overline{\tau}_d(p):=DMTR_d(\gamma_C,p)$ *for any* $d \in \{0,1\}$.

When we impose Assumptions 7 and 8, the identification of the marginal treatment effect function is also an immediate consequence of Proposition 4.1. We state this result as a corollary for convenience.

Corollary 4.2. *If Assumptions 1-5, 7 and 8 hold, then* $MTE(p)$ *is identified for any* $p \in \mathcal{P}$.

4.2 Partial Identification under Regression Dependence

In some empirical applications, Assumption 5 may be implausible. Alternatively, the researcher can restrict the dependence between the censoring variable and the latent heterogeneity. Imposing Assumption 6, we can partially identify the $DMTR$ functions.

Proposition 4.2. *If Assumptions 1-4 and 6 hold, then*

$$\begin{aligned} &DMTR_d(y,p)\\ &\quad \in \left[\max_{\delta\in\mathcal{D}}\left\{\mathbb{P}(y+\delta \leqslant C)\cdot(2\cdot d-1)\cdot\frac{\partial \mathbb{P}\left[Y \leqslant y, D=d \mid P(Z,C)=p, C=y+\delta\right]}{\partial p}\right\},\right.\\ &\quad\quad \left.\min_{\delta\in\mathcal{D}}\left\{\begin{array}{c}\mathbb{P}(C \leqslant y)+\mathbb{P}(y+\delta \leqslant C)+\mathbb{P}(y \leqslant C \leqslant y+\delta)\\ \cdot(2\cdot d-1)\cdot\frac{\partial \mathbb{P}\left[Y \leqslant y, D=d \mid P(Z,C)=p, C=y+\delta\right]}{\partial p}\end{array}\right\}\right]\end{aligned}$$

for any $d \in \{0,1\}$, $y < \gamma_C$ *and* $p \in \mathcal{P}$, *where* $\mathcal{D} := \{\delta \in \mathbb{R}_{++} : y + \delta \in \mathcal{C}\}$.

Proof. See Appendix A.3. ■

Moreover, partial identification of some quantile marginal treatment effect functions is an immediate consequence of Proposition 4.2. We state this result as a corollary for convenience.

Corollary 4.3. *If Assumptions 1-4 and 6 hold, then* $QMTE(\tau, p)$ *is partially identified for any* $p \in \mathcal{P}$ *and* $\tau \in [0, \overline{\tau}(p))$, *where* $\overline{\tau}(p) := \min\{\overline{\tau}_0(p), \overline{\tau}_1(p)\}$ *and* $\overline{\tau}_d(p) := DMTR_d(\gamma_C, p)$ *for any* $d \in \{0,1\}$.

When we impose Assumptions 7 and 8, partial identification of the marginal treatment effect function is also an immediate consequence of Proposition 4.2. We state this result as a corollary for convenience.

Corollary 4.4. *If Assumptions Assumptions 1-4 and 6-8 hold, then* $MTE(p)$ *is partially identified for any* $p \in \mathcal{P}$.

If the researcher believes that even Assumption 6 is too strong in a particular application, she can use only Assumptions 1-4 to identify a conditional version of our target parameters. We discuss this possibility in Appendix F.

5 Estimation and Inference

In this section, we explain how to semi-parametrically estimate the DMTE, QMTE and MTE functions based on the identification results described in Propositions 4.1 and 4.2. In Subsection 5.1, we propose semi-parametric point-estimation and inference procedures for the DMTE, QTE and MTE functions under Assumptions 1-5, 7 and 8 Finally, in Subsection 5.2, we briefly discuss set-estimation and inference procedures under Assumptions 1-4 and 6-8.

5.1 Semi-parametric Point-Estimation and Inference

In this section, we propose semi-parametric estimation and inference procedures for the DMTE, QTE and MTE functions under Assumptions 1-5, 7 and 8. In Subsection 5.1.1, we describe our semi-parametric estimator, while, in Subsection 5.1.2, we construct point-wise confidence intervals using the Bayesian bootstrap. We prove the consistency of our semi-parametric estimator and derive its asymptotic distribution in Appendix B.

5.1.1 Semi-parametric Estimation

We assume that we observe a sample $\{Y_i, C_i, D_i, Z_i, X_i\}_{i=1}^{N}$, where X_i is a scalar covariate.[12]

Our semi-parametric estimators can be described in 9 steps.

1. Estimate the propensity score $P\colon \mathcal{Z} \times \mathcal{C} \rightarrow [0,1]$ using a semi-parametric series estimator,[13]
$$\widehat{P}_i = \widehat{\mathbb{E}}\left[D_i | Z_i, C_i, X_i\right] = \widehat{\alpha}_0 + \widehat{\alpha}_X \cdot X_i + \widehat{\alpha}_C \cdot C_i + \sum_{l=1}^{L} \widehat{\alpha}_Z^l \cdot Z_i^l, \tag{13}$$
where $L \in \mathbb{N}$.

2. Define a grid of values for the duration outcome Y and the censoring variable C, $\{y_k\}_{k=0}^{K}$ such that $y_k > y_{k-1}$ for any $k \in \{1, \ldots, K\}$ and $K \in \mathbb{N}$.

3. For each $k \in \{0, \ldots, K\}$ and each $d \in \{0,1\}$, estimate the conditional distribution function of $Y \cdot \mathbf{1}\{D = d\}$ given $P(Z,C)$ and C using a logit specification,
$$\begin{aligned}\widehat{\Gamma}_{d,k}\left(P_i, C_i\right) &:= \widehat{\mathbb{E}}\left[\mathbf{1}\left\{Y_i \leqslant y_k, D_i = d\right\} | P_i, C_i\right] \\ &= \frac{\exp\left\{\widehat{\beta}_{0,d,k} + \widehat{\beta}_{X,d,k} \cdot X_i + \widehat{\beta}_{C,d,k} \cdot C_i + \widehat{\beta}_{P,d,k} \cdot \widehat{P}_i\right\}}{1 + \exp\left\{\widehat{\beta}_{0,d,k} + \widehat{\beta}_{X,d,k} \cdot X_i + \widehat{\beta}_{C,d,k} \cdot C_i + \widehat{\beta}_{P,d,k} \cdot \widehat{P}_i\right\}}.\end{aligned} \tag{14}$$

4. For each $k \in \{0, \ldots, K\}$ and each $d \in \{0,1\}$, estimate the derivative of the conditional distribution function of $Y \cdot \mathbf{1}\{D = d\}$ given $P(Z,C)$ and C, and multiply it by -1 if $d = 0$:
$$\widehat{\gamma}_{d,k}\left(p, c\right) := (2 \cdot d - 1) \cdot \frac{\exp\left\{\widehat{\beta}_{0,d,k} + \widehat{\beta}_{X,d,k} \cdot X_i + \widehat{\beta}_{C,d,k} \cdot C_i + \widehat{\beta}_{P,d,k} \cdot \widehat{P}_i\right\}}{\left(1 + \exp\left\{\widehat{\beta}_{0,d,k} + \widehat{\beta}_{X,d,k} \cdot X_i + \widehat{\beta}_{C,d,k} \cdot C_i + \widehat{\beta}_{P,d,k} \cdot \widehat{P}_i\right\}\right)^2} \cdot \widehat{\beta}_{P,d,k},$$
where $p \in \mathcal{P}$ and $c \in \mathcal{C}$.

[12] We use a scalar covariate for ease of notation. Our semi-parametric can be extended to include a vector of covariates in a straightforward way.

[13] We could instead propose a semiparametric logit approach:
$$\widehat{\mathbb{E}}\left[D_i | Z_i, C_i, X_i\right] = \frac{\exp\{\widehat{\alpha}_0 + \widehat{\alpha}_X \cdot X_i + \widehat{\alpha}_C \cdot C_i + \sum_{l=1}^{L} \widehat{\alpha}_Z^l \cdot Z_i^l\}}{1 + \exp\{\widehat{\alpha}_0 + \widehat{\alpha}_X \cdot X_i + \widehat{\alpha}_C \cdot C_i + \sum_{l=1}^{L} \widehat{\alpha}_Z^l \cdot Z_i^l\}},$$
where we approximate the non-parametric component (the relationship between Z and D) of the propensity score with a polynomial series. Our regularity conditions would still hold in this case since the first-stage estimator still enters linearly in the second stage. Similarly, a non-parametric first stage using a series estimator, $\widehat{\mathbb{E}}\left[D_i | Z_i, C_i, X_i\right] = \sum_{j=0}^{J} \sum_{k=0}^{K} \sum_{l=0}^{L} \widehat{\alpha}_{l,k,j} \cdot Z_i^l X_i^k C_i^j$, would be valid.

5. For each $k \in \{0, \ldots, K\}$ and each $d \in \{0, 1\}$, estimate $DMTR_d(y_k, p)$ by averaging $\widehat{\gamma}_{d,k}(p, c)$ over values of c such that $c > y_k$,

$$\widehat{DMTR}_d(y_k, p) := \begin{cases} 0 & \text{if } k = 0 \\ \dfrac{\sum_{r=k+1}^{K} \widehat{\gamma}_{d,k}(p, y_r)}{K - k} & \text{if } 0 < k < K \\ 1 & \text{if } k = K \end{cases}.$$

6. For each $k \in \{0, \ldots, K\}$, estimate $DMTE(y_k, p)$ using

$$\widehat{DMTE}(y_k, p) := \widehat{DMTR}_1(y_k, p) - \widehat{DMTR}_0(y_k, p).$$

7. For each $d \in \{0, 1\}$ and any $\tau \in [0, 1]$, estimate $QMTR_d(\tau, p)$ by inverting $DMTR_d(\cdot, p)$,

$$\widehat{QMTR}_d(\tau, p) := \min_{k \in \{0, \ldots, K\}} \left\{ y_k : \widehat{DMTR}_d(y_k, p) \geq \tau \right\}.$$

8. For each $\tau \in [0, 1]$, estimate $QMTE(\tau, p)$ using

$$\widehat{QMTE}(\tau, p) := \widehat{QMTR}_1(\tau, p) - \widehat{QMTR}_0(\tau, p).$$

9. Given $S \in \mathbb{N}$ and a grid $\{\tau_1, \ldots, \tau_S\} \subset [0, 1]$, estimate $MTE(p)$ using

$$\widehat{MTE}(p) := \frac{\sum_{s=1}^{S} \widehat{QMTE}(\tau_s, p)}{S}.$$

The estimators described above are consistent and asymptotically normal according to Appendix B.

5.1.2 Inference: Bayesian Bootstrap

To construct point-wise confidence intervals around the DMTE, QTE and MTE functions, we can use the Bayesian bootstrap according to the following procedure:

1. Compute the estimators for DMTE, QTE and MTE according to the steps described in Subsection 5.1.1.

2. Generate $B \in \mathbb{N}$ vectors of size N with weights ω_{ib} for individual i in vector b, where $E(\omega_{ib}) = 1$ and $Var(\omega_{ib}) = 1$.[14]

[14]For example, you can use $\omega_{ib} \sim Exp(1)$.

3. Repeat Step 1 for the B vectors each one weighting differently each individual.

4. Store the estimated $DMTE_b$, QTE_b and MTE_b.

5. Use the empirical distribution of $DMTE_b$, QTE_b and MTE_b for constructing confidence intervals.

5.2 Estimation and Inference for a Partially Identified DMTE and MTE functions

The bounds in Proposition 4.2 can be implemented using methods similar to the methods described in Subsection 5.1.1. The main difference between the estimators of the bounds and the point-estimators (Subsection 5.1.1) is that, when estimating the bounds, we take either the maximum or the minimum over values of c in Step 5 instead of taking the mean.

Consequently, these estimators will converge in probability to the bounds in Proposition 4.2. Furthermore, an asymptotically valid bootstrap procedure can be used to build confidence intervals for the entire identified set, such as those constructed by Manski and Nagin (1998).

6 Monte Carlo Simulation: Assumptions 1-5, 7 and 8

In this section, we study the finite sample performance of the point-estimator proposed in Subsection 5.1 when Assumptions 1-5, 7 and 8 are valid. In Appendix C, we study the finite sample performance of the estimators of the bounds (Proposition 4.2) when Assumptions 1-4 and 6-8 are valid.

To ensure that Assumptions 1-5, 7 and 8 are valid in this simulation, we use the following data-generating process (DGP):

$$\begin{aligned}
V &\sim Unif\,[0,1] \\
C &\sim Exp\,(1) \\
Z &\sim Unif\,[0,1] \\
D &= \mathbf{1}\left\{\frac{\exp\left(-3+6\cdot Z+\alpha\cdot C\right)}{1+\exp\left(-3+6\cdot Z+\alpha\cdot C\right)} \geqslant V\right\} \\
Y^*\,(0) &\sim Exp\,(1) \\
Y^*\,(1) &= Y^*\,(0)+0.5+V \\
Y^* &= D\cdot Y^*\,(1)+(1-D)\cdot Y^*\,(0) \\
Y &= \min\left\{Y^*, C\right\},
\end{aligned} \tag{15}$$

where V, C, Z and $Y^*\,(0)$ are mutually independent and $\alpha \in \{-1, 0\}$.

Moreover, for every simulated data set, we use the same sample size, $N = 40,000$ and the same grid for Y and C, $\{0, 0.25, 0.5, \ldots, 7\}$. Furthermore, we simulate $B = 1,000$ data sets.

Note that, in this DGP, the marginal treatment effect function, $MTE\colon\, [0,1] \to \mathbb{R}$, is given by

$$MTE\,(v) = 0.5 + v \text{ for any } v \in [0,1]\,.$$

We also need to define the target parameters of our Monte Carlo simulation. Our first set of target parameters are the values of this function evaluated at $v \in \mathcal{V} := \{0, 0.1, \ldots 0.9, 1\}$. Moreover, we target the average treatment effect,

$$ATE := \int_0^1 MTE\,(v)\; dv = 1,$$

because it is common to use the MTE to compute other treatment effect parameters.

Furthermore, we estimate the marginal treatment effect function using our semi-parametric estimator ($\widehat{MTE}$, Subsection 5.1) with $\{\tau_1, \ldots, \tau_S\} = \{0, 0.01, \ldots, 0.99, 1\}$ in Step 9 and $L = 3$ in Equation (13).

Using this estimator, we also estimate a discrete approximation for the average treatment effect, $\widehat{ATE} := \frac{\sum_{v \in \mathcal{V}} \widehat{MTE}\,(v)}{|\mathcal{V}|}$.

We want to analyze the finite sample properties of our estimator. To do so, we report its average relative bias,

$$\mathbb{E}\left[\frac{\widehat{MTE}(v) - MTE(v)}{MTE(v)}\right] \text{ and } \mathbb{E}\left[\frac{\widehat{ATE} - ATE}{ATE}\right], \tag{16}$$

and mean squared error,

$$\mathbb{E}\left[\left(\widehat{MTE}(v) - MTE(v)\right)^2\right] \text{ and } \mathbb{E}\left[\left(\widehat{ATE} - ATE\right)^2\right], \tag{17}$$

for $v \in \mathcal{V}$.

For comparison, we also estimate the MTE function using two naive estimators that ignore censoring.

The first estimator (Naive Parametric) estimates the propensity score using a cubic polynomial of the instrument only. It also imposes that the true MTE function is linear, estimating the reduced-form outcome equation using the level of the censored outcome and a quadratic polynomial of the propensity score (Cornelissen, Dustmann, Raute and Schonberg, 2016, Appendix B.2). Although this estimator ignores censoring, it has the advantage of correctly imposing a linear functional form for the true MTE function. This parametric assumption may improve its performance if the censoring problem is not severe.

The second estimator (Naive Nonparametric) uses the level of the censored outcome and a nonparametric LIV estimator (Cornelissen et al., 2016, Appendix B.1). To do so, it uses a locally quadratic estimator of the derivative of the reduced-form outcome equation with an Epanechnikov kernel (Calonico, Cattaneo and Farrell, 2019). Since this flexible estimator requires a sufficiently large number of observations around each value of the propensity score and our DGP does not produce many observations with extreme propensity scores, the Naive Nonparametric Estimator cannot estimate $MTE(0)$, $MTE(0.1)$, $MTE(0.9)$ and $MTE(1)$ reliably. Consequently, the ATE estimator associated with the Naive Nonparametric Estimator averages only over $\widehat{MTE}(0.2), \widehat{MTE}(0.3), \ldots, \widehat{MTE}(0.8)$.

Table 1 reports the average relative bias of all three estimators. The first row of the table defines the value of α (Equation (15)) that is used to generate the data in each one of the $B = 1,000$ Monte Carlo repetitions. The second row of the table defines which estimator is used to estimate the MTE function. Each cell reports the estimated average relative bias (Equation (16)) when targeting the parameter described in the first column.

Our first result is that our estimator's average relative bias is larger for small and large values of v regardless of the values of α. In comparison, the naive parametric estimator's average relative bias has a similar magnitude regardless of the values of v. Consequently, our estimator has a smaller bias than the naive estimator for values of v that are close to

Table 1: Average Relative Bias

	$\alpha = -1$			$\alpha = 0$		
	Ours	Naive Parametric	Naive Nonparametric	Ours	Naive Parametric	Naive Nonparametric
$MTE\,(0)$	0.99	-0.88	–	2.73	-0.56	–
$MTE\,(0.1)$	0.54	-0.83	–	1.94	-0.60	–
$MTE\,(0.2)$	0.19	-0.80	-0.61	1.32	-0.63	-0.78
$MTE\,(0.3)$	-0.10	-0.77	-0.52	0.77	-0.66	-0.57
$MTE\,(0.4)$	-0.35	-0.76	-0.51	0.24	-0.67	-0.65
$MTE\,(0.5)$	-0.59	-0.74	-0.57	-0.34	-0.69	-0.79
$MTE\,(0.6)$	-0.83	-0.73	-0.64	-0.81	-0.70	-0.96
$MTE\,(0.7)$	-1.10	-0.72	-0.65	-1.07	-0.71	-1.03
$MTE\,(0.8)$	-1.40	-0.71	0.02	-1.21	-0.72	-0.76
$MTE\,(0.9)$	-1.61	-0.70	–	-1.29	-0.73	–
$MTE\,(1)$	-1.89	-0.69	–	-1.33	-0.73	–
ATE	-0.83	-0.74	-0.48	-0.33	-0.69	-0.81

Note: The first row of the table defines the value of α (Equation (15)) that is used to generate the data in each one of the $B = 1,000$ Monte Carlo repetitions. The second row of the table defines which estimator is used to estimate the MTE function. Each cell reports the estimated average relative bias (Equation (16)) when targeting the parameter described in the first column. Some cells are empty because the Naive Nonparametric Estimator cannot estimate $MTE\,(0)$, $MTE\,(0.1)$, $MTE\,(0.9)$ and $MTE\,(1)$ reliably due to the fact that there exist few observations with extreme propensity score values.

0.5. This phenomenon is not surprising because, given the functional form of Equation (15), most observations in each Monte Carlo sample have propensity scores between 0.3 and 0.6, allowing our estimator to estimate $MTE\,(v)$ more precisely for values of v that are close to 0.5.

The second result is that our estimator's average relative bias for the ATE is smaller than its bias for the MTE function. This finding is due to the fact that the positive bias of $\widehat{MTE}\,(v)$ for small values of v is compensated by the negative bias of $\widehat{MTE}\,(v)$ for large values of v. This phenomenon explains why our estimator performs better than the naive estimators when $\alpha = 0$ and our target parameter is the ATE.

The third result is that the average relative bias is larger when $\alpha = -1$ for most of our estimators. This finding is not surprising because, when $\alpha = -1$, most observations in each Monte Carlo sample have smaller propensity scores, substantially increasing the bias when targeting the MTE function for large values of v.

Finally, the performance of the naive nonparametric estimator is erratic. When $\alpha = -1$, it performs better than the naive parametric estimator when targeting the MTE function and it has the best performance when targeting the ATE. However, when $\alpha = 0$, it frequently presents the worst performance of all estimators when targeting the MTE function and it has the worst performance when targeting the ATE.

Table 2 reports the mean squared error of our estimator and of the two naive estimators. The first row of the table defines the value of α (Equation (15)) that is used to generate the data in each one of the $B = 1,000$ Monte Carlo repetitions. The second row of the table defines which estimator is used to estimate the MTE function. Each cell reports the estimated mean squared error (Equation (17)) when targeting the parameter described in the first column.

Table 2: Mean Squared Error (MSE)

	$\alpha = -1$			$\alpha = 0$		
	Ours	Naive Parametric	Naive Nonparametric	Ours	Naive Parametric	Naive Nonparametric
$MTE(0)$	0.25	0.19	–	1.86	0.08	–
$MTE(0.1)$	0.11	0.25	–	1.36	0.13	–
$MTE(0.2)$	0.02	0.31	0.69	0.85	0.20	1.32
$MTE(0.3)$	0.01	0.38	0.36	0.38	0.28	0.44
$MTE(0.4)$	0.10	0.46	0.37	0.05	0.37	0.45
$MTE(0.5)$	0.35	0.55	0.46	0.12	0.47	0.77
$MTE(0.6)$	0.83	0.64	0.85	0.80	0.59	1.23
$MTE(0.7)$	1.73	0.74	1.84	1.65	0.73	1.88
$MTE(0.8)$	3.33	0.85	422.22	2.49	0.87	2.50
$MTE(0.9)$	5.10	0.97	–	3.27	1.03	–
$MTE(1)$	8.08	1.09	–	3.98	1.21	–
ATE	0.70	0.55	8.87	0.11	0.47	0.69

Note: The first row of the table defines the value of α (Equation (15)) that is used to generate the data in each one of the $B = 1,000$ Monte Carlo repetitions. The second row of the table defines which estimator is used to estimate the MTE function. Each cell reports the estimated mean squared error (Equation (17)) when targeting the parameter described in the first column. Some cells are empty because the Naive Nonparametric Estimator cannot estimate $MTE(0)$, $MTE(0.1)$, $MTE(0.9)$ and $MTE(1)$ reliably due to the fact that there exist few observations with extreme propensity score values.

Its results are similar to the ones in Table 1. Nevertheless, there are two new findings in Table 2. First, our estimators' Mean Squared Error is small when targeting the ATE and $\alpha = 0$. Second, the naive nonparametric estimator performs much worse than the other two estimators, particularly when targeting the ATE.

Analyzing all results jointly, we conclude that our semi-parametric estimator performs better than or similarly to competing estimators that ignore the censored nature of the outcome variable.

7 Empirical Application

In our empirical application, we answer the question: "Do alternative sentences (fines and community service) impact time-to-recidivism?". In Subsection 7.1, we provide key descriptive statistics while, in Subsection 7.2, we describe the results of our empirical analysis.

7.1 Descriptive Statistics

In this subsection, our descriptive analysis has two goals. First, we show that ignoring endogenous self-selection into treatment may lead to conclusions that conflict with an analysis that addresses endogeneity. Second, we discuss the validity of two of our identifying assumptions: Random Censoring (Assumption 5) and the support restriction (Assumption 8).

Table 3 shows the outcome's mean, 1st decile, 1st quartile and median for all defendants, for the defendants who were punished (treated group), and for the defendants who were not punished (control group). It also shows the sample size of each one of these three groups. The comparison between the treated and control groups suggests that being punished slightly harms defendants. However, this naive comparison ignores endogenous selection-into-treatment, right-censoring and heterogeneous treatment effects and is not fully supported by our empirical results in Subsection 7.2.

Table 3: Descriptive Statistics — Outcome Variable

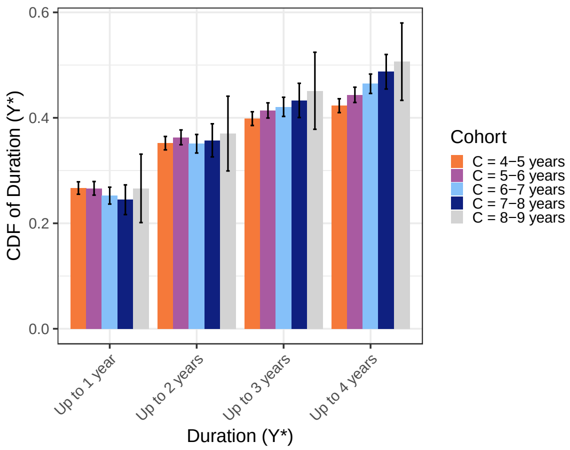
	Unconditional	Treated Group	Control Group
Mean	1,081	1,047	1,116
1st Decile	77	69	86
1st Quartile	364	321	430
Median	1082	1047	1127
Number of Observations	43,468	22,060	21,408

Note: The treated group receives a punishment, i.e., its defendants were fined or sentenced to community services because they were either convicted or signed a non-prosecution agreement. The control group did not receive a punishment, i.e., its defendants were acquitted or its cases were dismissed. The outcome variable measures the number of days between the case's final ruling's date and the first recidivism event if the defendant recidivates or the number of days between the case's final ruling's date and the end of the sampling period if the defendant did not recidivate. An observation is a case-defendant pair.

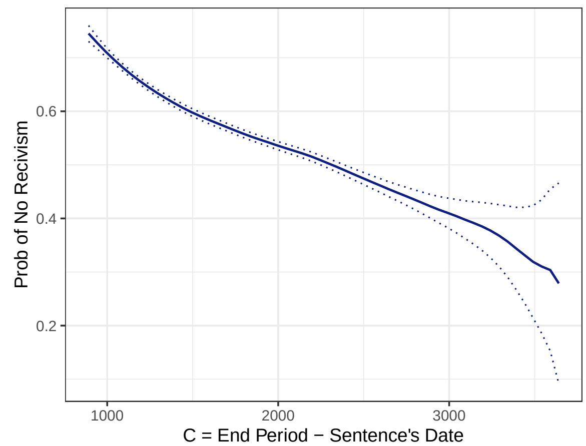
Figure 2 provides two ways to assess the validity of our identifying assumptions.

Subfigure 2a shows the cumulative distribution function (CDF) of the uncensored potential outcome (Y^*) given cohorts based on the censoring variable. Taking into account the sampling uncertainty, this result suggests that the censoring variable may be independent of the potential outcomes as implied by Assumption 5. More clearly, this figure suggests that the potential outcomes are negatively regression dependent on the censoring variable as imposed by Assumption 6.

Figure 2: Descriptive Statistics for the Uncensored Outcome (Y^*) and the Censoring variable (C)



(a) CDF of the Uncensored Outcome given the Defendant's Cohort: $\mathbb{P}[Y^* \leqslant y|\, C]$



(b) Probability of No Recidivism during the Sampling Period: $\mathbb{P}[Y^* > C|\, C]$

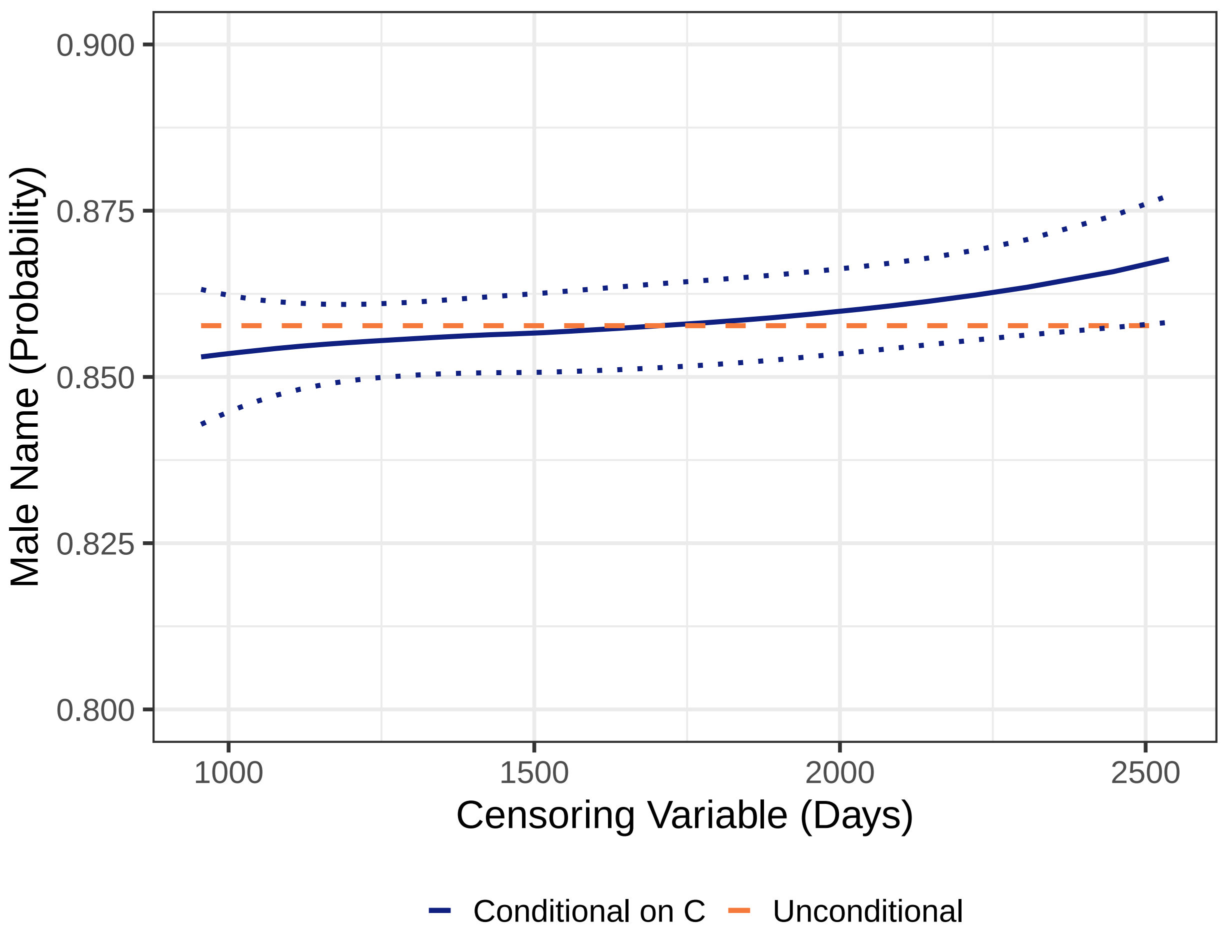
Notes: Subfigure 2a shows the cumulative distribution function (CDF) of the uncensored potential outcome (Y^*) given cohorts based on the censoring variable. Each color denotes a different cohort: orange denotes defendants who are observed for at least four years and at most five years during our sampling period, purple denotes defendants who are observed for at least five years and at most six years, light blue denotes defendants who are observed for at least six years and at most seven years, dark blue denotes defendants who are observed for at least seven years and at most eight years, and gray denotes defendants who are observed for at least eight years and at most nine years. These conditional CDFs are evaluated at four values of the uncensored potential outcome (one, two, three or four years), and these evaluation points are denoted in the x-axis. The y-axis denotes the value of the CDF, while black lines denote point-wise 99%-confidence intervals around the values of the CDF.

Subfigure 2b shows the probability that a defendant does not recidivate during our sampling period given the value of her censoring variable. This nonparametric function was estimated using a local linear regression with an Epanechnikov kernel based on Calonico et al. (2019). The bandwidth was optimally selected according to the IMSE criterion. The dotted lines are robust bias-corrected 95%-confidence intervals.

Subfigure 2b shows the probability that a defendant does not recidivate during our sampling period given the value of her censoring variable. Conditioning on the defendants who stay the longest in our sample (large values of C), we still find a 30% probability that they do not recidivate during the observation period. This result suggests that our support restriction (Assumption 8) may not be valid in this context. Although this result does not invalidate the analysis of the quantile marginal treatment effect, it implies that the marginal treatment effect estimates should be interpreted carefully.

Another way to assess the validity of the random censoring assumption is to analyze the relationship between the censoring variable and an excluded covariate — having a typically male name according to the Brazilian 2010 Census (*R package* `genderBR`). Figure 3 shows the probability of having a typically male name given the defendant's censoring variable (dark blue line). We find that, regardless of the censoring variable, this probability is close to the unconditional share of male names (orange line). Consequently, there is indirect and suggestive evidence that our random censoring restriction (Assumption 5) is valid.

Figure 3: Probability of having a typically male name given the defendant's censoring variable: $\mathbb{P}[\text{Male Name}|C]$



Notes: The solid dark blue line shows the probability that a defendant has a typically male name given the value of her censoring variable. This nonparametric function was estimated using a local linear regression with an Epanechnikov kernel based on Calonico et al. (2019). The bandwidth was optimally selected according to the IMSE criterion. The dotted dark blue lines are robust bias-corrected 99%-confidence intervals. The dashed orange line is the unconditional probability of having a typically male name.

7.2 Empirical Results

We start by presenting the results of the first stage regression in our empirical analysis. In our model, the treatment variable D ("final ruling") is a function of the instrument Z ("trial judge's punishment rate"), the censoring variable C, and court district fixed effects. Following Subsection 5.1.1, we use a polynomial series to approximate the propensity score and report the estimated coefficients of a quadratic model in Table 4. Note that our instrument is strong according to the F-statistic of the first stage regression. This result implies that Assumption 2 is valid.

Table 4: First Stage Results

	Z	Z^2	C
Coefficient	0.66***	0.10	0.00***
Clusterized S.E.	(0.23)	(0.21)	(0.00)
F-statistic	817		

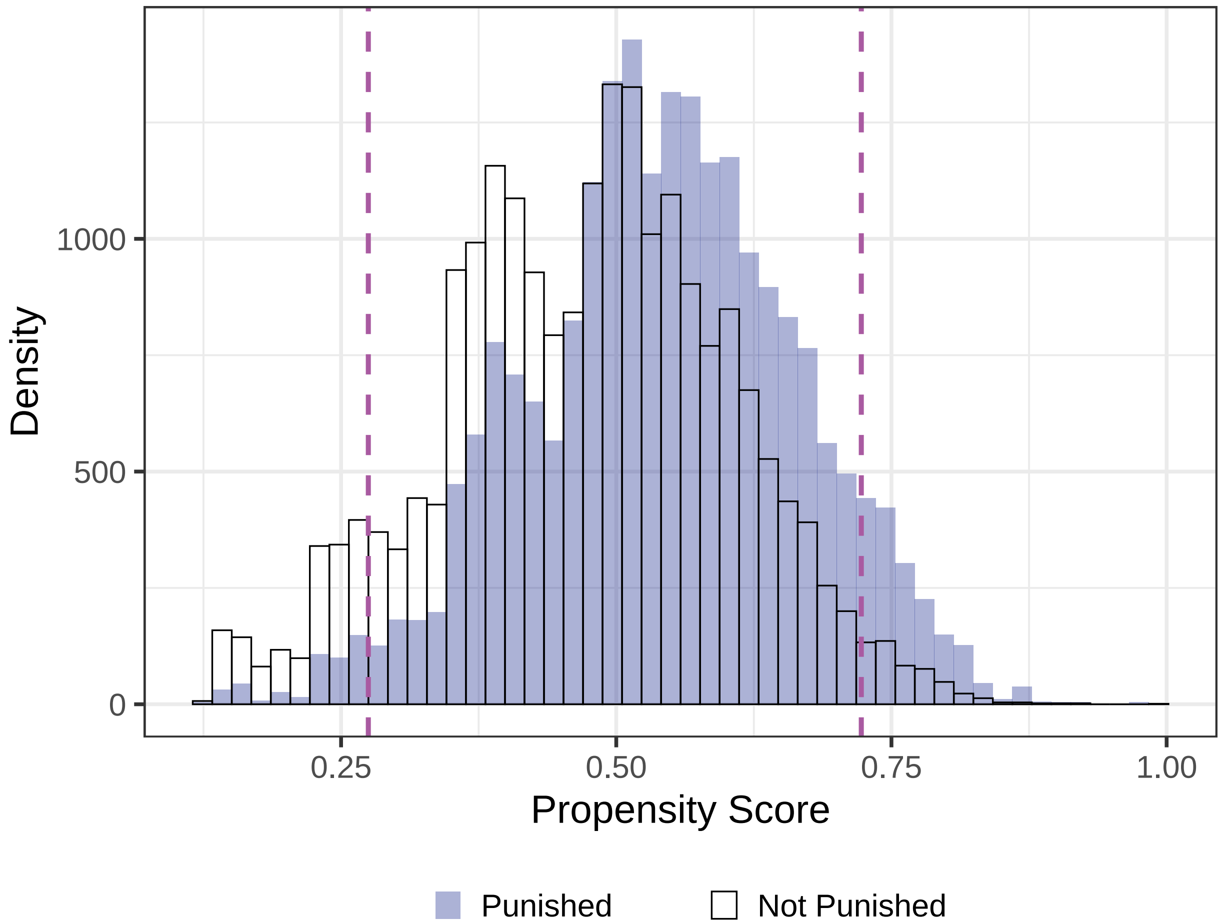
Note: The left-hand side variable is our treatment variable, i.e., D ="punished according to the final ruling in the case". The standard errors are clusterized at the court district level. The third line reports the F-Statistic of a hypothesis test whose null is that the coefficients associated with Z and Z^2 are equal to zero. The first stage regression control for court district fixed effects.

We also report the distribution of the estimated propensity score in Figure 4. The blue histogram shows the distribution of the estimated propensity score given that defendant was punished (treated group) while the white histogram shows the distribution of the estimated propensity score given that defendant was not punished (control group). We find that most defendants have a probability of being punished around 50%. However, some defendants are very unlikely to be punished (estimated propensity score around 30%) and others are very likely to be punished (estimated propensity score around 70%). These widely spread propensity score distributions are positive for identification and estimation because they allow us to discuss QMTE and MTE functions evaluated at many different points of the latent heterogeneity variable.

The vertical lines denote the unconditional 5^{th} and 95^{th} percentiles of the estimated propensity score. When discussing our results about the QMTE and MTE functions, we only report the estimates for latent heterogeneity values between these two percentiles. We do so to avoid extrapolation bias and to ensure the validity of Assumption 2.

To estimate the DMTE, QMTE and MTE functions in our empirical application, we need to control for court district fixed effects. Consequently, we estimate 193 district-specific functions for each one of our treatment effect parameters (Subsection 5.1.1). To summarize

Figure 4: Distribution of the estimated propensity score given treatment status



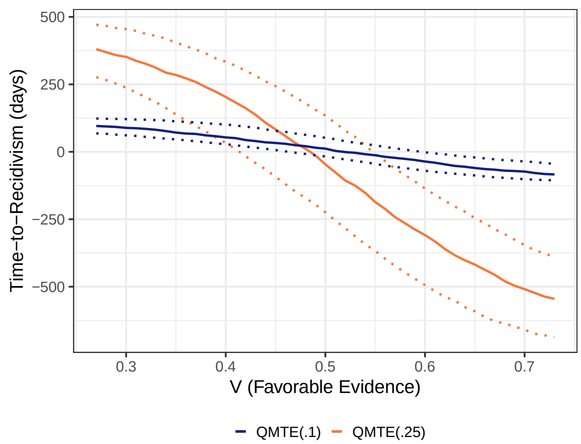
Notes: The blue histogram shows the distribution of the estimated propensity score given that defendant was punished (treated group). The white histogram shows the distribution of the estimated propensity score given that defendant was not punished (control group). The vertical lines denote the unconditional 5^{th} and 95^{th} percentiles of the estimated propensity score.

our results, we average these functions over court districts using the proportion of cases per court district as weights.

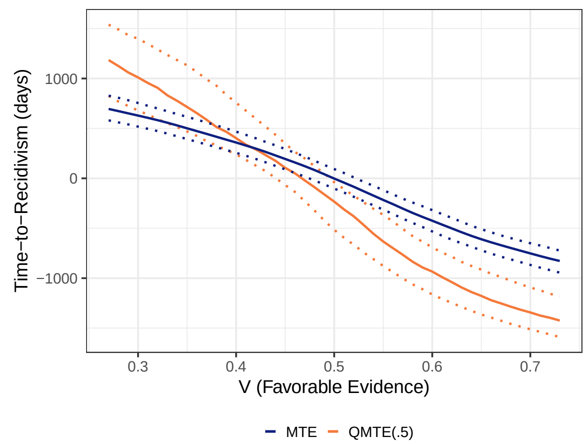
First, in Figure 5, we report the semi-parametrically estimated average QMTE functions for the first decile, the first quartile and the median, and the semi-parametrically estimated average MTE function. These results are based on Proposition 4.1 and its corollaries, imposing Assumptions 1-5 when we focus on the QMTE functions, and Assumptions 1-5, 7 and 8 when we focus on the MTE function. As a caveat, we recall that the support restriction may be implausible according to the results in Subfigure 2b. For this reason, the results related to the MTE function should be interpreted cautiously.

We find that all target functions are positive for small values of V and negative for large

Figure 5: Estimated QMTE and MTE functions



(a) Estimated $QMTE(.1, \cdot)$ and $QMTE(.25, \cdot)$



(b) Estimated $QMTE(.5, \cdot)$ and $MTE(\cdot)$

Notes: Solid lines are the point-estimates for the target functions indicated in the legend of each subfigure. These results are based on Proposition 4.1 and its corollaries. To compute these average functions, we estimate one function for each court district using our semi-parametric estimator (Subsection 5.1.1) and, then, we average across court districts using the proportion of cases per court district as weights. Moreover, the dotted lines are point-wise 90%-confidence intervals. These confidence intervals were computed using the Bayesian bootstrap clusterized at the court district level (Subsection 5.1.2).

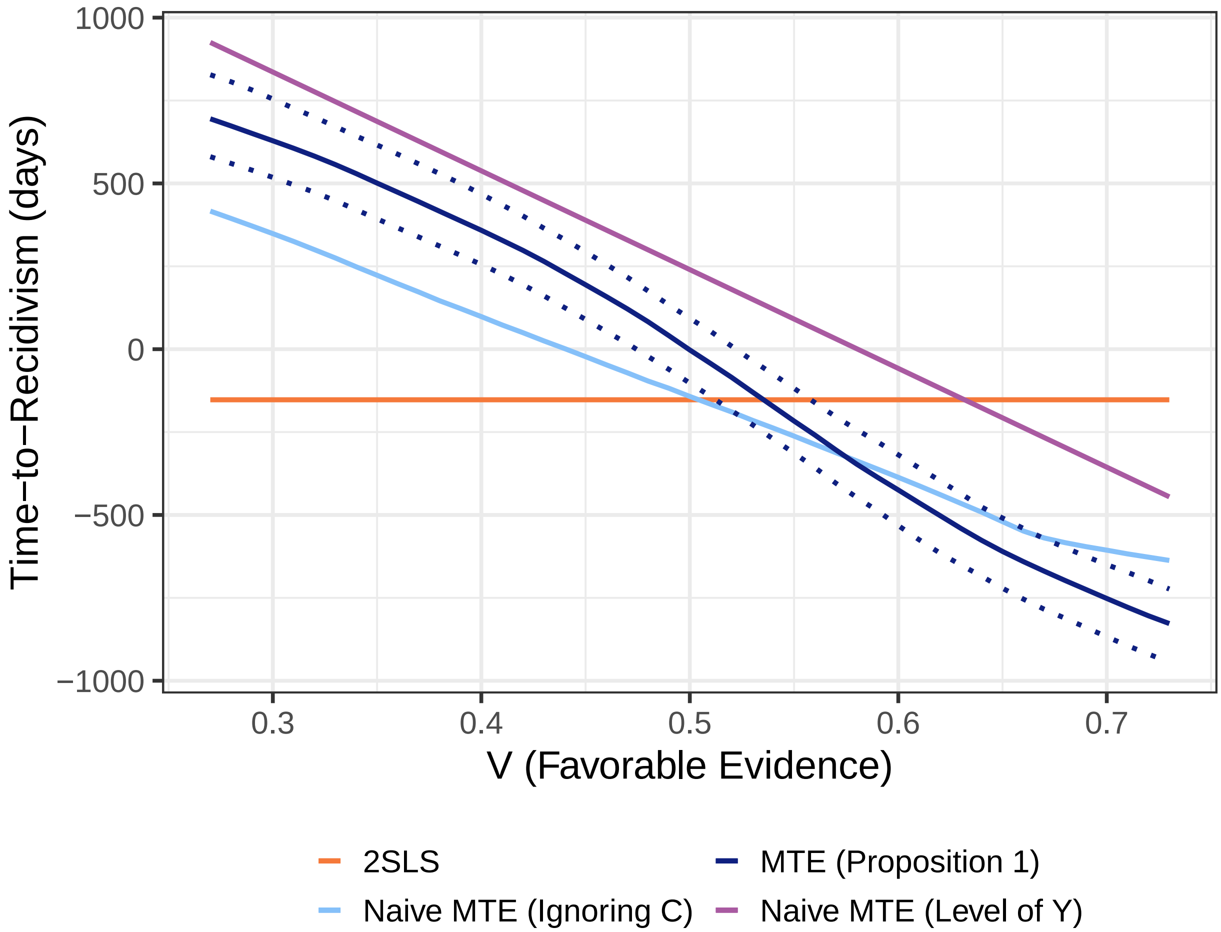
values of V. More precisely, people who would be punished by most judges recidivate later because they had to pay a fine or do community service (treatment is working as intended), while people who would be punished only by strict judges recidivate at an earlier date because of the treatment. These results suggest that designing sentencing guidelines that encourage strict judges to become more lenient could increase time-to-recidivism.

Second, our methodology also accounts for sample uncertainty (Subsection 5.1.2). In Figure 6, the dotted blue lines represent point-wise 90%-confidence intervals around our estimator for the average MTE function (Subsection 5.1.1). We find that the average MTE function is statistically significant for small and large values of V. Consequently, even accounting for sample uncertainty, we conclude that more lenient sentencing guidelines could increase time-to-recidivism.

Third, in Figure 6, we focus on the semi-parametrically estimated average MTE function (solid dark blue line) and compare it against naive estimates of the average MTE function and the estimated 2SLS estimand. Differently from our approach, these estimates ignore that the outcome variable is right-censored and provide different conclusions when compared against our proposed estimator.

The light blue line is the estimated average MTE function when we use the semi-

Figure 6: Estimated Treatment Effect Parameters



Notes: The solid dark blue line is the estimated average MTE function based on Corollary 4.2 and our semi-parametric estimator (Subsection 5.1.1). To compute it, we estimate one MTE function for each court district using our semi-parametric estimator (Subsection 5.1.1) and, then, we average across court districts using the proportion of cases per court district as weights. Moreover, the dotted dark blue lines are point-wise 90%-confidence intervals. These confidence intervals were computed using the Bayesian bootstrap clusterized at the court district level (Subsection 5.1.2). The orange line is the two-stage least square (2SLS) estimate based on a regression of the censored outcome variable on treatment and court district fixed effects using the judge's punishment rate as the instrument. The purple line is the estimated average MTE function based on a parametric estimator (Cornelissen et al., 2016, Appendix B.2) that imposes a linear MTE curve and directly uses the level of the censored outcome variable. The light blue line is the estimated average MTE function when we use the semi-parametric estimator proposed in Subsection 5.1.1, but we do not control for the censoring variable.

parametric estimator proposed in Subsection 5.1.1, but we do not control for the censoring variable. We find that this estimator attenuates the effect of the correctly estimated average MTE function (dark blue line).

The purple line is the estimated average MTE function based on a parametric estimator (Cornelissen et al., 2016, Appendix B.2) that imposes a linear MTE curve and directly uses the level of the censored outcome variable. We find that this estimator is upward biased, exacerbating the benefits of being punished with a fine or community service.

The orange line is the two-stage least square (2SLS) estimate based on a regression of the censored outcome variable on treatment and court district fixed effects using the trial judge's punishment rate as the instrument. We find that this estimator does not capture the rich heterogeneity behind the treatment effects of fines and community service. In particular, the 2SLS estimate suggests a small and negative effect, ignoring that the treatment increases time-to-recidivism for some defendant types.

Finally, if the researcher does not find Assumption 5 credible, she can set identify the MTE function under Assumptions 1-4 and 6-8 using Corollary 4.4. We report the estimated bounds in Appendix D and highlight that they are too wide to draw any conclusions.

8 Conclusion

In this paper, we identify the distributional marginal treatment effect (DMTE), the quantile marginal treatment effect (QMTE) and the marginal treatment effect (MTE) functions when the outcome variable is right-censored. To do so, we extend the MTE framework (Heckman et al., 2006; Carneiro and Lee, 2009) to scenarios with duration outcomes. In this section, we discuss in which contexts our proposed methodology can be used and deepen our empirical discussion.

Our methodology can be applied to many empirical problems that face two simultaneous identification challenges: endogenous selection into treatment and right-censored data. In our empirical application, we focus on the effect of a fine on defendants' time-to-recidivism. In this case, judges observe more information than the econometrician when making their decisions and time-to-recidivism is a right-censored variable. In labor economics, the same identification challenges appear when analyzing the effect of receiving unemployment benefits on unemployment spells. Moreover, in the health sciences, when studying the effect of a drug on survival time, a researcher has to address both identification problems too.[15]

Concerning its empirical contribution, our work is inserted in the literature about the effect of fines and community service sentences on future criminal behavior. Five recent papers in this field were written by Huttunen et al. (2020), Giles (2021), Klaassen (2021), Possebom (2022), Lieberman et al. (2023). All of them focus on binary variables indicating recidivism within a pre-specified time period. Huttunen et al. (2020) and Giles (2021)

[15]The effect of unemployment benefits is discussed by Krueger and Meyer (2002), Chetty (2008) and Delgado et al. (2022). Medical treatments are analyzed by Sullivan, Zwaag, El-Zeky, Ramanathan and Mirvis (1993), Spiegel (2002) and Trinquart, Jacot, Conner and Porcher (2016).

find that this type of punishment increases the probability of recidivism in Finland and Milwaukee (a city in the State of Wisconsin in the U.S.), respectively. Klaassen (2021) finds that alternative sentences decrease the probability of recidivism in North Carolina (a state in the U.S.). Possebom (2022) finds that this type of punishment has a small and statistically insignificant effect on the probability of recidivism in São Paulo, Brazil. Finally, Lieberman et al. (2023) analyze five American states and find that court fees have no impact on recidivism.

Differently from these five papers, our outcome variable is time-to-recidivism. Using a continuous outcome instead of binary indicators allows for a finer analysis of the heterogeneous effects of fines and community service sentences on future criminal behavior and may conciliate the conflicting results in the previous literature. For example, we find that this type of punishment increases time-to-recidivism for some individuals while decreasing it for other individuals. If the first type of individual is more common in North Carolina than in Milwaukee and Finland, our focus on essential heterogeneity may shed light on these conflicting results.

Moreover, Possebom (2022), who uses the same dataset as ours, finds that fines and community service sentences have a small and statistically insignificant effect on the probability of recidivism in São Paulo, while we find a significant and richly heterogeneous effect of this type of punishment on time-to-recidivism. This difference suggests that the time dimension captured by our time-to-recidivism variable is relevant in the decision process of defendants and should be taken into account when discussing recidivism. Furthermore, this result suggests caution in the application of the standard MTE framework when analyzing a duration variable or a relevant terminal time problem.

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Supporting Information
(Online Appendix)

A Proofs of the main results

A.1 Proof of Lemma 4.1

Fix $y < \gamma_C$, $p \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$. To prove Equation (9), note that

$$\begin{aligned}
&\mathbb{P}\left[Y \leqslant y, D = 1 \middle| P\left(Z, C\right) = p, C = y + \delta\right] \\
&\quad = \mathbb{E}\left[\mathbf{1}\left\{Y \leqslant y\right\}\mathbf{1}\left\{P\left(Z, C\right) \geqslant V\right\} \middle| P\left(Z, C\right) = p, C = y + \delta\right] \\
&\quad\quad \text{by Equation (1)} \\
&\quad = \mathbb{E}\left[\mathbf{1}\left\{Y^*(1) \leqslant y\right\}\mathbf{1}\left\{p \geqslant V\right\} \middle| P\left(Z, y + \delta\right) = p, C = y + \delta\right] \\
&\quad\quad \text{because } Y_1^* \text{ is not censored when } C > y \\
&\quad = \int_0^1 \mathbb{E}\left[\mathbf{1}\left\{Y^*(1) \leqslant y\right\}\mathbf{1}\left\{p \geqslant v\right\} \middle| P\left(Z, y + \delta\right) = p, C = y + \delta, V = v\right] dv \\
&\quad\quad \text{by the Law of Iterated Expectations and Assumption 3} \\
&\quad = \int_0^1 \mathbf{1}\left\{p \geqslant v\right\} \mathbb{E}\left[\mathbf{1}\left\{Y^*(1) \leqslant y\right\} \middle| P\left(Z, y + \delta\right) = p, C = y + \delta, V = v\right] dv \\
&\quad = \int_0^p \mathbb{E}\left[\mathbf{1}\left\{Y^*(1) \leqslant y\right\} \middle| P\left(Z, y + \delta\right) = p, C = y + \delta, V = v\right] dv \\
&\quad = \int_0^p \mathbb{P}\left[Y^*(1) \leqslant y \middle| C = y + \delta, V = v\right] dv \\
&\quad\quad \text{by Assumption 1.}
\end{aligned}$$

We can prove Equation (10) analogously.

To prove Equation (11), observe that

$$\begin{aligned}
&\mathbb{P}\left[Y \leqslant y, D = 1 \middle| P\left(Z, C\right) = p, C = y + \delta\right] \\
&\quad = \int_0^p \mathbb{P}\left[Y^*(1) \leqslant y \middle| C = y + \delta, V = v\right] dv \\
&\quad = \int_0^p \mathbb{P}\left[Y^*(1) \leqslant y \middle| V = v\right] dv \\
&\quad\quad \text{by Assumption 5.}
\end{aligned}$$

A.2 Proof of Proposition 4.1

Fix $y < \gamma_C$, $p \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$.

First, note that Equations (11) and (12) imply that

$$\frac{\partial \mathbb{P}\left[Y \leqslant y, D = 1 \middle| P\left(Z, C\right) = p, C = y + \delta\right]}{\partial p} = \mathbb{P}\left[Y^*(1) \leqslant y \middle| V = p\right] \tag{A.1}$$

and

$$\frac{\partial \mathbb{P}\left[Y \leqslant y, D = 0 \middle| P\left(Z, C\right) = p, C = y + \delta\right]}{\partial z} = -\mathbb{P}\left[Y^*(0) \leqslant y \middle| V = p\right] \tag{A.2}$$

according to the Leibniz Integral Rule.

Combining Equations (4) and (A.1)-(A.2), we prove that

$$DMTR_d\left(y, p\right) = \left(2 \cdot d - 1\right) \cdot \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \middle| P\left(Z, C\right) = p, C = y + \delta\right]}{\partial p}$$

for any $d \in \{0, 1\}$.

Since the last equation holds for any $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$, we have that

$$DMTR_d\left(y, p\right) = \left(2 \cdot d - 1\right) \cdot \int_{\mathcal{D}} \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \middle| P\left(Z, C\right) = p, C = y + \delta\right]}{\partial p} d\delta$$

for any $d \in \{0, 1\}$, where $\mathcal{D} := \{\delta \in \mathbb{R}_{++} : y + \delta \in \mathcal{C}\}$.

A.3 Proof of Proposition 4.2

Fix $d \in \{0, 1\}$, $y < \gamma_C$, $p \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$.

Note that Equations (9) and (10) imply that

$$\frac{\partial \mathbb{P}\left[Y \leqslant y, D = 1 \middle| P\left(Z, C\right) = p, C = y + \delta\right]}{\partial p} = \mathbb{P}\left[Y^*(1) \leqslant y \middle| C = y + \delta, V = p\right] \tag{A.3}$$

and

$$\frac{\partial \mathbb{P}\left[Y \leqslant y, D = 0 \middle| P\left(Z, C\right) = p, C = y + \delta\right]}{\partial p} = -\mathbb{P}\left[Y^*(0) \leqslant y \middle| C = y + \delta, V = p\right] \tag{A.4}$$

according to the Leibniz Integral Rule.

Combining the last two equations, we have that

$$\mathbb{P}\left[Y^*(d) \leqslant y \middle| C = y + \delta, V = p\right] = \left(2 \cdot d - 1\right) \cdot \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \middle| P\left(Z, C\right) = p, C = y + \delta\right]}{\partial p}. \tag{A.5}$$

Moreover, observe that:

$$\begin{aligned}
&\mathbb{P}\left[Y^*(d) \leqslant y \middle| V = p\right] \\
&\quad = \int \mathbb{P}\left[Y^*(d) \leqslant y \middle| C = \tilde{c}, V = p\right] f_{C|V}\left(\tilde{c}|p\right) d\tilde{c} \\
&\qquad \text{by the Law of Iterated Expectations} \\
&\quad = \int \mathbb{P}\left[Y^*(d) \leqslant y \middle| C = \tilde{c}, V = p\right] f_{C}\left(\tilde{c}\right) d\tilde{c} \\
&\qquad \text{because } V \perp\!\!\!\perp C \text{ by Assumption 3} \\
&\quad = \int_0^y \mathbb{P}\left[Y^*(d) \leqslant y \middle| C = \tilde{c}, V = p\right] f_{C}\left(\tilde{c}\right) d\tilde{c} \\
&\qquad + \int_y^{y+\delta} \mathbb{P}\left[Y^*(d) \leqslant y \middle| C = \tilde{c}, V = p\right] f_{C}\left(\tilde{c}\right) d\tilde{c} \\
&\qquad + \int_{y+\delta}^{+\infty} \mathbb{P}\left[Y^*(d) \leqslant y \middle| C = \tilde{c}, V = p\right] f_{C}\left(\tilde{c}\right) d\tilde{c},
\end{aligned}$$

implying, by Assumption 6, that

$$\begin{aligned}
\mathbb{P}\left[Y^*(d) \leqslant y \middle| V = p\right] \leqslant & \, \mathbb{P}(C \leqslant y) + \mathbb{P}(y + \delta \leqslant C) \\
& + \mathbb{P}(y \leqslant C \leqslant y + \delta)\mathbb{P}\left[Y^*(d) \leqslant y \middle| C = y + \delta, V = p\right]
\end{aligned} \tag{A.6}$$

and

$$\mathbb{P}\left[Y^*(d) \leqslant y \middle| V = p\right] \geqslant \mathbb{P}(y + \delta \leqslant C)\mathbb{P}\left[Y^*(d) \leqslant y \middle| C = y + \delta, V = p\right] \tag{A.7}$$

Thus, combining Equations (A.6) and (A.7) with Equation (A.5), we have that

$$\begin{aligned}
& DMTR_d\left(y, p\right) \\
& \quad \in \Bigg[\mathbb{P}(y + \delta \leqslant C) \cdot (2 \cdot d - 1) \cdot \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \middle| P\left(Z, C\right) = p, C = y + \delta\right]}{\partial p}, \\
& \qquad \mathbb{P}(C \leqslant y) + \mathbb{P}(y + \delta \leqslant C) \\
& \qquad\quad + \mathbb{P}(y \leqslant C \leqslant y + \delta) \cdot (2 \cdot d - 1) \cdot \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \middle| P\left(Z, C\right) = p, C = y + \delta\right]}{\partial p} \Bigg].
\end{aligned}$$

Since the bounds above hold for any $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$, we have that

$$
\begin{aligned}
&DMTR_d(y,p) \\
&\quad \in \left[\max_{\delta \in \mathcal{D}} \left\{ \mathbb{P}(y+\delta \leqslant C) \cdot (2 \cdot d - 1) \cdot \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \middle| P(Z,C) = p, C = y + \delta \right]}{\partial p} \right\}, \right. \\
&\qquad \left. \min_{\delta \in \mathcal{D}} \left\{ \begin{array}{c} \mathbb{P}(C \leqslant y) + \mathbb{P}(y+\delta \leqslant C) + \mathbb{P}(y \leqslant C \leqslant y+\delta) \\ \cdot (2 \cdot d - 1) \cdot \dfrac{\partial \mathbb{P}\left[Y \leqslant y, D = d \middle| P(Z,C) = p, C = y + \delta \right]}{\partial p} \end{array} \right\} \right],
\end{aligned}
$$

where $\mathcal{D} := \{\delta \in \mathbb{R}_{++} \colon y + \delta \in \mathcal{C}\}$.

B Semi-parametric Estimation: Consistency and Asymptotic Normality

Although identification does not rely on any parametric assumption, some of them aid the estimation procedure. Covariates are easily incorporated when semi-parametric assumptions are made and the curse of dimensionality is avoided. Additionally, semi-parametric assumptions demand less data. In this section, we follow Rothe (2009) closely but adapt his setting for the case where the link function is known instead of unknown. For the rest of the section, we assume an $i.i.d$ sample. In this context, we introduce the following assumption:

Assumption B.1 (Semiparametric CDF)**.** *Let* $\mathbb{P}[Y \leqslant y, D = d| P, C, X] = G(\beta_{0,d,y}+\beta_{1,d,y}C+\beta_{2,d,y}P+\beta_{3,d,y}X)$*, where* $G()$ *is a known link function up to a finite dimensional vector (such as the logistic link), which is continuously differentiable in the index. Let* $G'(.)$ *be the derivative of* $G(.)$*, which is continuous.*

For the sake of exposition, let $W_{y,d} = 1\{Y \leqslant y, D = d\}$, $H = \{1, C, P, X\}, \hat{H} = \{1, C, \hat{P}, X\}$, $H_p = \{1, C, p, X\}$, $\beta_{d,y} := (\beta_{0,d,y}, \beta_{1,d,y}, \beta_{2,d,y}, \beta_{3,d,y})$ for any y and $d \in \{0, 1\}$. Taking the derivative with respect to P for $G(\cdot)$ for both $W_{y,1}$ and $W_{y,0}$, we get the $DMTE(y, p)$ as

$$DMTR_1(y, p) - DMTR_0(y, p) = G'(\beta_{1,y}H)\beta_{2,1,y} - G'(\beta_{0,y}H)\beta_{2,1,y}$$

If P was known, it would be easy to estimate the DMTE as in the parametric part.

Since P is not known, we can estimate P in a semi-parametric first stage, and obtain estimates for $\beta_{d,y}$ from the following maximum-likelihood procedure.[16] We focus on $d = 1$ for the sake of exposition and denote the semi-parametric first-stage estimates by $\hat{P}$. Define

$$Ln(\beta_{1,y}, \hat{P}) = \max_{\beta_{1,y}} \frac{1}{N} \sum_i W_{y,1,i} log[G(\beta_{1,y}\hat{H}_i)] + (1 - W_{y,1,i}) log[1 - G(\beta_{1,y}\hat{H}_i)] \tag{B.1}$$

with solution $\hat{\beta}_{1,y}(\hat{P})$. If P was known, we could use the following unfeasible standard maximum likelihood procedure:

$$Ln(\beta_{1,y}, P) = \max_{\beta_{1,y}} \frac{1}{N} \sum_i W_{y,1,i} log[G(\beta_{1,y}H_i)] + (1 - W_{y,1,i}) log[1 - G(\beta_{1,y}H_i)] \tag{B.2}$$

with solution $\hat{\beta}_{1,y}(P)$.

To analyze our semi-parametric estimator (Equation (B.1)), we need to ensure that the unfeasible estimator in Equation (B.2) is well-behaved. To do so, we impose the following

[16] In the semi-parametric first stage, we can estimate P using a standard series estimator.

assumption:

Assumption B.2 (Unfeasible Likelihood)**.** *The maximum likelihood estimator of Equation* (B.2)*, follows standard regularity conditions from Newey and McFadden (1994) for consistency and asymptotic normality.*

Assumption B.2 ensures that standard parametric inference could be performed if P was observed, implying that $\hat{\beta}_{1,y}(P) \xrightarrow{p} \beta_{1,y}$. Since G is the logistic link, the result is standard.

To ensure that our semi-parametric estimator is consistent and derive its asymptotic distribution, we need to impose that our propensity score estimator converges sufficiently fast and satisfy some regularity conditions. To do so, we follow Rothe (2009) and impose the following assumption.

Assumption B.3 (First stage assumptions)**.** *Let* $\hat{P}$ *satisfy:*

1. $\hat{P}_i - P_i = \frac{1}{N}\sum_j w_n(Z_i, C_i, X_i, Z_j, C_j, X_j)\phi_j + r_{in}$ *with* $\max_i ||r_{in}|| = o_p(N^{-\frac{1}{2}})$ *and* $\max_i |\hat{P}_i - P_i| = o_p(N^{-\frac{1}{4}})$ *where* $\phi_j = \phi(D_j, Z_j, C_j, X_j)$ *is an influence function with* $E\left[\phi_j|Z_j, C_j, X_j\right] = 0$ *and* $E\left[\phi_j^2|Z_j, C_j, X_j\right] \leqslant \infty$ *and weights* $w_n(Z_i, C_i, X_i, Z_j, C_j, X_j) = o(N)$.

2. *There exists a space* $\mathcal{P}$ *such that* $\mathbb{P}(\hat{P} \in \mathcal{P}) \to 1$ *and the integral between 0 and infinity with respect to the radius of the log of the covering number with respect to the* l_∞ *norm of the class of functions* $\mathcal{P}$ *is finite.*

Assumption B.3 is a high-level condition on the estimator. The first part states that the estimator admits a certain asymptotic expansion, whereas the second part requires the estimator to take values in some well-behaved function space with probability approaching 1.[17]

To ensure consistency of the feasible estimator, we need to prove asymptotic equivalence between the solution of Equations (B.1) and (B.2). Then, by Assumption B.2, we get consistency of the feasible semi-parametric estimator.

Note that

$$
\begin{aligned}
&\sup_{\beta_{1,y}} |Ln(\beta_{1,y}, \hat{P}) - Ln(\beta_{1,y}, P)| \\
&\quad \leqslant \left[\inf_{\beta_{1,y}} \min_i \{G(\beta_{1,y}\hat{H}_i), G(\beta_{1,y}H_i), 1 - G(\beta_{1,y}\hat{H}_i), 1 - G(\beta_{1,y}H_i)\}\big(\sup_{\beta_{1,y}} \max_i |G(\beta_{1,y}\hat{H}_i) - G(\beta_{1,y}H_i)|\big)\right] \\
&\quad \leqslant \left[O(1)\big(\sup_{\beta_{1,y}} \max_i |G(\beta_{1,y}\hat{H}_i) - G(\beta_{1,y}H_i)|\big)\right] \\
&\quad = o_p(1),
\end{aligned}
$$

[17] A standard series estimator satisfies Assumption B.3.

where the first inequality can be derived using standard algebraic manipulations. Moreover, the second inequality holds because $G(\cdot) \in (0, 1)$. Furthermore, note that $G(\cdot)$ is continuous and $\max_i |\hat{H}_i - H_i|$ due to Assumption B.3 converges, implying that $\max_i |G(\beta_{1,y}\hat{H}_i) - G(\beta_{1,y}H_i)|$ converges due to the continuous mapping theorem. Finally, since the supremum over $\beta_{1,y}$ in the third line is also continuous, we can apply the continuous mapping theorem again to prove the last equality.

Furthermore, $Ln(\beta_{1,y}, P)$ is a standard parametric likelihood, implying that it converges uniformly in $\beta_{1,y}$ to its expectation (Newey and McFadden, 1994, Lemma 2.4). Formally, we have that

$$\sup_{\beta_{1,y}} |Ln(\beta_{1,y}, P) - L(\beta_{1,y})| = o_p(1)$$

where $L(\beta_{1,y}) = E(Ln(\beta_{1,y})) = E(W_{y,1,i} log(G(\beta_{1,y}H)) + (1 - W_{y,1,i}) log(1 - G(\beta_{1,y}H))]))$ is a non-random function that is continuous in $\beta_{1,y}$. Taken together, it follows from the triangle inequality that

$$\sup_{\beta_{1,y}} |Ln(\beta_{1,y}, \hat{P}) - L(\beta_{1,y})| = o_p(1)$$

implying that $\hat{\beta}_{1,y}(P)$ is consistent whenever $L(\beta_{1,y})$ attains a unique maximum at the true value of the parameter, which is the case by our identification results and Assumption B.1.

As a consequence, consistency of our feasible semi-parametric estimator follows from Theorem 2.1 by Newey and McFadden (1994) via Assumption B.2.

Now, we derive the asymptotic distribution of our semi-parametric estimator in Equation B.1. Let $Ln(\beta_{1,y}, \hat{P}_i)_\beta, Ln(\beta_{1,y}, P_i)_\beta, L(\beta_{1,y}, P_i)_\beta$ be the derivative with respect to β of the individuals feasible log-likelihood, unfeasible log-likelihood and true log-likelihood respectively (the score). Define similarly the second order derivative.

From a standard second order Taylor expansion of the semi-parametric log likelihood around $\beta_{1,y}$, we have that

$$\sqrt{N}(\hat{\beta}_{1,y}(\hat{P}) - \beta_{1,y}) = \left[\frac{1}{N}\sum_i Ln(\bar{\beta}_{1,y}, \hat{P}_i)_{\beta,\beta}\right]^{-1} \sqrt{N}\frac{1}{N}\sum_i Ln(\beta_{1,y}, \hat{P}_i)_\beta, \qquad \text{(B.3)}$$

where $\bar{\beta}_{1,y}$ is between the estimated and true values. By the first part of Assumption B.3

and the consistency of $\hat{\beta}_{1,y}(\hat{P})$, we know that,

$$\left[\frac{1}{N}\sum_i Ln(\bar{\beta}_{1,y},\hat{P}_i)_{\beta,\beta}\right]^{-1} \xrightarrow{p} E(L(\beta_{1,y},P_i)_{\beta,\beta})^{-1} =: \Sigma.$$

Now, we focus on the last term in Equation (B.3):

$$\begin{aligned}
\sum_i Ln(\beta_{1,y},\hat{P}_i)_\beta &= \sum_i W_{y,1,i}\frac{\partial log[G(\beta_{1,y}\hat{H}_i)]}{\partial\beta} + (1-W_{y,1,i})\frac{\partial log[1-G(\beta_{1,y}\hat{H}_i)]}{\partial\beta} \\
&= \sum_i W_{y,1,i}\begin{bmatrix} \frac{G'(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}{G(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)} \\ \frac{G'(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}{G(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}C_i \\ \frac{G'(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}{G(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}\hat{P}_i \\ \frac{G'(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}{G(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}X_i \end{bmatrix} \\
&+ (1-W_{y,1,i})\begin{bmatrix} \frac{-G'(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}{1-G(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)} \\ \frac{-G'(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}{1-G(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}C_i \\ \frac{-G'(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}{1-G(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}\hat{P}_i \\ \frac{-G'(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}{1-G(\beta_{0,1,y}+\beta_{1,1,y}C_i+\beta_{2,1,y}\hat{P}_i+\beta_{3,1,y}X_i)}X_i \end{bmatrix}
\end{aligned}$$

Considering the path $P_e = (1-e)P + e[\hat{P}-P]$, we take the path-wise derivative of $\sum_i Ln(\beta_{1,y},P_i)_\beta$ at direction $\hat{P}-P$ (the derivative of the submodel P_e evaluated at $e=0$). This object is denoted by $\sum_i Ln(\beta_{1,y},P_i)_{\beta,P_i}$ and is equal to

$$\begin{aligned}
\sum_i Ln(\beta_{1,y},P_i)_{\beta,P_i} &= \sum_i W_{y,1,i}\begin{bmatrix} \frac{G''(\beta_{1,y}H_i)G(\beta_{1,y}H_i)-G'(\beta_{1,y}H_i)^2}{G(\beta_{1,y}H_i)^2}\beta_{2,1,y}[\hat{P}_i-P_i] \\ \frac{G''(\beta_{1,y}H_i)G(\beta_{1,y}H_i)-G'(\beta_{1,y}H_i)^2}{G(\beta_{1,y}H_i)^2}C_i\beta_{2,1,y}[\hat{P}_i-P_i] \\ [\frac{G''(\beta_{1,y}H_i)G(\beta_{1,y}H_i)-G'(\beta_{1,y}H_i)^2}{G(\beta_{1,y}H_i)^2}P_i\beta_{2,1,y}+\frac{G'(\beta_{1,y}H_i)}{G(\beta_{1,y}H_i)}][\hat{P}_i-P_i] \\ \frac{G''(\beta_{1,y}H_i)G(\beta_{1,y}H_i)-G'(\beta_{1,y}H_i)^2}{G(\beta_{1,y}H_i)^2}X_i\beta_{2,1,y}[\hat{P}_i-P_i] \end{bmatrix} \\
&+ \sum_i(1-W_{y,1,i})\begin{bmatrix} \frac{-G''(\beta_{1,y}H_i)[1-G(\beta_{1,y}H_i)]-G'(\beta_{1,y}H_i)^2}{[1-G(\beta_{1,y}H_i)]^2}\beta_{2,1,y}[\hat{P}_i-P_i] \\ \frac{-G''(\beta_{1,y}H_i)[1-G(\beta_{1,y}H_i)]-G'(\beta_{1,y}H_i)^2}{[1-G(\beta_{1,y}H_i)]^2}C_i\beta_{2,1,y}[\hat{P}_i-P_i] \\ [\frac{-G''(\beta_{1,y}H_i)[1-G(\beta_{1,y}H_i)]-G'(\beta_{1,y}H_i)^2}{[1-G(\beta_{1,y}H_i)]^2}P_i\beta_{2,1,y}+\frac{-G'(\beta_{1,y}H_i)}{1-G(\beta_{1,y}H_i)}][\hat{P}_i-P_i] \\ \frac{-G''(\beta_{1,y}H_i)[1-G(\beta_{1,y}H_i)]-G'(\beta_{1,y}H_i)^2}{[1-G(\beta_{1,y}H_i)]^2}X_i\beta_{2,1,y}[\hat{P}_i-P_i] \end{bmatrix}
\end{aligned} \tag{B.4}$$

We also define $E\left[Ln(\beta_{1,y},P_i)_{\beta,P_i}\right]$ analogously.

With these results in hand, we go back to Equation (B.3) and expanding around the deviations of the true first stage:

$$\sqrt{N}(\hat{\beta_{1,y}}(\hat{P})-\beta_{1,y}) = \Sigma\sqrt{N}\left(\frac{1}{N}\sum_i Ln(\beta_{1,y},P_i)_\beta + \frac{1}{N}\sum_i Ln(\beta_{1,y},P_i)_{\beta,P_i}\right) + o_p(1) \tag{B.5}$$

where $\sum_i \frac{1}{N} Ln(\beta_{1,y}, P_i)_\beta$ is the usual estimate of the score, which has mean 0. Thus, if we can show that the second term also has mean 0, the asymptotic normality of our semi-parametric estimator follows by a standard multivariate CLT for the vector $\left[\frac{1}{N}\sum_i Ln(\beta_{1,y}, P_i)_\beta, \frac{1}{N}\sum_i Ln(\beta_{1,y}, P_i)_{\beta,P_i}\right]$.

Since all the components of Equation (B.4) have a similar structure, we can focus on one of them and the results are symmetric for the rest. Consider, $\frac{1}{N}\sum_i W_{y,1,i} \frac{G''(\beta_{1,y}H_i)G(\beta_{1,y}H_i) - G'(\beta_{1,y}H_i)^2}{G(\beta_{1,y}H_i)^2} \beta_{2,1,y}[\hat{P}_i - P_i]$. For notation simplicity, let $\frac{G''(\beta_{1,y}H_i)G(\beta_{1,y}H_i) - G'(\beta_{1,y}H_i)^2}{G(\beta_{1,y}H_i)^2} \beta_{2,1,y} =: A(\beta_{1,y}H_i)$. Note that

$$\begin{aligned}
\frac{1}{N}\sum_i W_{y,1,i} A(\beta_{1,y}H_i)[\hat{P}_i - P_i] &= \frac{1}{N^2}\sum_i\sum_j w_n(Z_i, C_i, Z_j, C_j) W_{y,1,i} A(\beta_{1,y}H_i)\phi_j + o_p(N^{-\frac{1}{2}}) \\
&= \frac{1}{N}\sum_i E(w_n(Z_i, C_i, Z, C) E(W_{y,1,i} A(\beta_{1,y}H_i)|H, Z, C)|Z_i, C_i)\phi_i + o_p(N^{-\frac{1}{2}})
\end{aligned}$$

where the first equality is due to Assumption B.3 and the second equality is due to the U-statistics Hajek projection.

Now, by a standard law of large numbers, we have that

$$\begin{aligned}
&\frac{1}{N}\sum_i E(w_n(Z_i, C_i, Z, C) E(W_{y,1,i} A(\beta_{1,y}H_i)|H, Z, C)|Z_i, C_i)\phi_i + o_p(N^{-\frac{1}{2}}) \\
&\xrightarrow{p} E[E(w_n(Z_i, C_i, Z, C) E(W_{y,1,i} A(\beta_{1,y}H_i)|H, Z, C)|Z_i, C_i)\phi_i] \\
&= E[E[E(w_n(Z_i, C_i, Z, C) E(W_{y,1,i} A(\beta_{1,y}H_i)|H, Z, C)|Z_i, C_i)\phi_i|Z_i, C_i]] \\
&= E[E(w_n(Z_i, C_i, Z, C) E(W_{y,1,i} A(\beta_{1,y}H_i)|H, Z, C)|Z_i, C_i) E[\phi_i|Z_i, C_i]] \\
&= 0
\end{aligned}$$

where the last equality is due to Assumption B.3. Thus, a standard CLT assures asymptotic normality of the estimator for the parametric part.

The previous display thus implies that for $\beta_y = [\beta_{1,y}, \beta_{0,y}]$: $(\hat{\beta}_y(\hat{P}) - \beta_y) = O_p(N^{-\frac{1}{2}})$ and $\sqrt{N}(\hat{\beta}_y \hat{P} - \beta_y) = N(0, V_\beta)$. Then, by the continuous mapping theorem and our asymptotic equivalence result, we have that

$$\begin{aligned}
&\sqrt{N}\begin{bmatrix} \widehat{DMTR}_1(y,p) - DMTR_1(y,p)) \\ \widehat{DMTR}_0(y,p) - DMTR_0(y,p)) \end{bmatrix} \\
&:= \sqrt{N}\begin{bmatrix} G'(\hat{\beta}_{1,y}\hat{H})\hat{\beta}_{2,1,y} - G'(\beta_{1,y}H)\beta_{2,1,y} \\ G'(\hat{\beta}_{0,y}\hat{H})\hat{\beta}_{2,0,y} - G'(\beta_{0,y}H)\beta_{2,0,y} \end{bmatrix} \\
&= N(0, \begin{bmatrix} V_{DMTR_1(y,p)} & Cov_{DMTR_1(y,p),DMTR_0(y,p)} \\ Cov_{DMTR_1(y,p),DMTR_0(y,p)} & V_{DMTR_0(y,p)} \end{bmatrix}). \qquad \text{(B.6)}
\end{aligned}$$

Then,

$$\sqrt{N}(\widehat{DMTE}(y,p) - DMTE(y,p))$$
$$:= \sqrt{N}\Bigg[G'(\hat{\beta}_{1,y}\hat{H})\hat{\beta}_{2,1,y} - G'(\hat{\beta}_{0,y}\hat{H})\hat{\beta_{2,1,y}} - G'(\beta_{1,y}H)\beta_{2,1,y} + G'(\beta_{0,y}H)\beta_{2,1,y}\Bigg]$$
$$= N(0, V_{DMTE_{p,y}}). \tag{B.7}$$

where $V_{DMTE_{p,y}} = \begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix} V_{DMTR_1(y,p)} & Cov_{DMTR_1(y,p),DMTR_0(y,p)} \\ Cov_{DMTR_1(y,p),DMTR_0(y,p)} & V_{DMTR_0(y,p)}\end{bmatrix}\begin{bmatrix}1 \\ -1\end{bmatrix}$

Following Frandsen (2015), we can recover the asymptotic distribution of the $QMTE(\tau,p)$ via the $QMTR_d(\tau,p)$. The $QMTR_d$ are Hadamard differentiable functions of the $DMTR_d$ functions with Jacobian $J \equiv \begin{pmatrix} -f_{Y*(1)|V}(QMTR_1(\tau,p)|p)^{-1} & 0 \\ 0 & -f_{Y*(0)|V}(QMTR_0(\tau,p)|p)^{-1}\end{pmatrix}$. Consequently, by a functional delta method (van der Vaart, van der Vaart and Wellner, 1996, van der Vaart, 1998), we have that

$$\sqrt{N}\begin{bmatrix}\widehat{QMTR}_1(\tau,p) - QMTR_1(\tau,p)) \\ \widehat{QMTR}_0(\tau,p) - QMTR_0(\tau,p))\end{bmatrix}$$
$$:== N\left(0, \begin{bmatrix} V_{QMTR_1(\tau,p)} & Cov_{QMTR_1(\tau,p),QMTR_0(\tau,p)} \\ Cov_{QMTR_1(\tau,p),QMTR_0(\tau,p)} & V_{QMTR_0(\tau,p)}\end{bmatrix}\right). \tag{B.8}$$

where

$$\begin{bmatrix} V_{QMTR_1(\tau,p)} & Cov_{QMTR_1(\tau,p),QMTR_0(\tau,p)} \\ Cov_{QMTR_1(\tau,p),QMTR_0(\tau,p)} & V_{QMTR_0(\tau,p)}\end{bmatrix} =$$
$$J^T\begin{bmatrix} V_{DMTR_1(y_1,p)} & Cov_{DMTR_1(y_1,p),DMTR_0(y_2,p)} \\ Cov_{DMTR_1(y_1,p),DMTR_0(y_2,p)} & V_{DMTR_0(y_2,p)}\end{bmatrix}J,$$

where y_1 is the value of y in the distribution of $Y_1^*|\,V$ such that $QMTR_1(\tau,p)|p)$ is the corresponding τ quantile and y_2 is the value in the distribution of $Y_1^*|\,V$ such that $QMTR_0(\tau,p)|p)$ is the corresponding τ quantile. Then, similarly to the DMTE, we can get:

$$\sqrt{N}(\widehat{QMTE}(\tau,p) - QMTE(\tau,p)) = N(0, V_{QMTE_{p,\tau}}). \tag{B.9}$$

where the definition of $V_{QMTE_{p,\tau}}$ is analogous to the one of $V_{DMTE_{p,y}}$.

By recalling that $MTE(p) = \int_0^1 QMTE(\tau,p)d\tau$ and the fact that we just provided asymptotic normality for $QMTE(\tau, p)$ we can recover the distribution of the $MTE(p)$ following Masten, Poirier and Zhang (2020).

At this point, it is worth to be specific about the definition of Hadamard differentiability and how it connects to the QTE and the MTE.

Definition 1. Let $\phi : D \to E$ where D, E are Banach spaces. Say ϕ is Hadamard differentiable at $\theta \in D$ if $\exists \phi'_\theta : D \to E$, $\forall h \in D$, if $t \to 0$, $||h_t - h|| \to 0$, then:

$$\left|\left|\frac{\phi(\theta + th_t) - \phi(\theta)}{t} - \phi'_\theta(h)\right|\right|_E \to 0$$

In our context, we set $D = \mathcal{C}([0,1],[0,1])$ and $E = \mathcal{R}$,i.e., D is the space of continuous functions where the first component refers to τ and the second one to v. Then, we know that $\frac{QMTE(\theta_1+th_{1t},\theta_2+th_{2t})-QMTE(\theta_1,\theta_2)}{t} \longrightarrow_{||||_R} QMTE'_{\theta_1,\theta_2}(h_1,h_2)$ and $\frac{QMTE(\tau,\theta_2+th_{2t})-QMTE(\tau,\theta_2)}{t} \longrightarrow_{||||_R} QMTE'_{\theta_2}(h_2)$, where $||||_R$ denotes the norm of convergence. Furthermore, we have that

$$\frac{MTE(\theta_2 + th_{2t}) - MTE(\theta_2)}{t} = \int_0^1 \frac{QMTE(\tau, \theta_2 + th_{2t}) - QMTE(\tau, \theta_2)}{t} d\tau,$$

which, under the conditions for the dominated convergence theorem, implies that

$$\begin{aligned} \frac{MTE(\theta_2 + th_{2t}) - MTE(\theta_2)}{t} &= \int_0^1 \frac{QMTE(\tau, \theta_2 + th_{2t}) - QMTE(\tau, \theta_2)}{t} d\tau \\ &\to \int_0^1 QMTE'_{\theta_2}(h_2) d\tau \equiv MTE'_{\theta_2}(h_2). \end{aligned}$$

Consequently, the MTE is Hadamard differentiable and we can apply the functional delta method again to get the asymptotic Gaussian distribution of the MTE.

C Monte Carlo Simulation: Assumptions 1-4 and 6-8

In this appendix, we study the finite sample performance of the estimator of the bounds (Proposition 4.2) when Assumptions Assumptions 1-4 and 6-8 are valid. In Section 6, we study the finite sample performance of the semi-parametric point-estimator proposed in Subsection 5.1 when Assumptions 1-5, 7 and 8 are valid.

To ensure that Assumptions Assumptions 1-4 and 6-8 are valid in this simulation, we use the following data generating process (DGP):

$$\begin{aligned}
V &\sim Unif\,[0,1] \\
C &\sim Unif\,[0,7] \\
Z &\sim Unif\,[0,1] \\
D &= \mathbf{1}\left\{ \frac{\exp\left(-3 + 6 \cdot Z + \alpha \cdot C\right)}{1 + \exp\left(-3 + 6 \cdot Z + \alpha \cdot C\right)} \geqslant V \right\} \\
Y^*(0) &\sim Unif\,[0,5] \\
Y^*(1) &= Y^*(0) + 1 + V - \frac{C}{7} \\
Y^* &= D \cdot Y^*(1) + (1 - D) \cdot Y^*(0) \\
Y &= \min\left\{Y^*, C\right\},
\end{aligned} \tag{C.1}$$

where V, C, Z and $Y^*(0)$ are mutually independent and $\alpha \in \{-1, 0\}$.

Moreover, for every simulated data set, we use the same sample size, $N = 10,000$, the same grid for Y, $\{0, 0.25, 0.5, \ldots, 7\}$, and the same grid for C, $\{2.5, 2.75, \ldots, 4.25, 4.5\}$. Furthermore, we simulate $B = 10,000$ data sets.

Note that, in this DGP, the marginal treatment effect function — $MTE\colon [0,1] \to \mathbb{R}$ — is given by

$$MTE\,(v) = 0.5 + v \text{ for any } v \in [0,1]\,.$$

We also need to define the target parameters of our Monte Carlo simulation. Our first set of target parameters are the values of this function evaluated at $v \in \mathcal{V} := \{0, 0.1, \ldots 0.9, 1\}$. Moreover, we target the average treatment effect,

$$ATE := \int_0^1 MTE\,(v)\; dv = 1,$$

because it is common to use the MTE to compute other treatment effect parameters.

Furthermore, we estimate the bounds around the marginal treatment effect function using $\{\tau_1, \ldots, \tau_S\} = \{0, 0.01, \ldots, 0.99, 1\}$ in Step 9 of Subsection 5.1.1 and $L = 3$ in Equation (13).

To estimate the bounds around the ATE, we take the mean of the bounds around $MTE(v)$ for $v \in \mathcal{V}$.

We want to analyze the finite sample properties of our semi-parametric estimator. To do so, we compute the probability that our bounds contain the true treatment effect parameters and their average length.

Panel A in Table C.1 reports the probability that our bounds contain the true treatment effect parameters. The second row of the table defines the value of α (Equation (C.1)) that is used to generate the data in each one of the $B = 10,000$ Monte Carlo repetitions. Each cell in Panel A reports the probability that the bounds contain the true parameter described in the rows.

Table C.1: Probability that the Bounds Contain the True Parameter and their Average Length

	Panel A: $\mathbb{P}[contain]$		Panel B: Average Length	
	$\alpha = -1$	$\alpha = 0$	$\alpha = -1$	$\alpha = 0$
$MTE(0)$	1	1	7.27	6.82
$MTE(0.1)$	1	1	6.74	6.54
$MTE(0.2)$	1	1	5.55	6.17
$MTE(0.3)$	1	1	3.07	5.80
$MTE(0.4)$	0	1	2.03	5.49
$MTE(0.5)$	0	1	2.23	5.34
$MTE(0.6)$	0	1	2.46	5.30
$MTE(0.7)$	0	1	2.69	5.38
$MTE(0.8)$	0	1	2.90	5.56
$MTE(0.9)$	0	1	3.08	5.86
$MTE(1)$	0	1	3.15	6.15
ATE	1	1	3.74	5.86

Note: The second row of the table defines the value of α (Equation (C.1)) that is used to generate the data in each one of the $B = 10,000$ Monte Carlo repetitions. Each cell in Panel A reports the probability that the bounds contain the true parameter described in the rows. Each cell in Panel B reports the estimated average length of the estimated bounds around the parameters described in the rows.

We highlight that a large share of the bounds contain the true treatment effect parameter with probability one. In particular, the bounds with $\alpha = 0$ always contain the true treatment effect parameter.

The bounds with $\alpha = -1$ do not perform so well. In particular,these bounds never contain the $MTE(v)$ for any $v \in \{0.4, 0.5, \ldots, 1\}$. Despite this phenomenon, the bounds for the ATE are still able to contain the ATE with probability one.

Panel B in Table C.1 reports the bounds' average length. The second row of the table defines the value of α (Equation (C.1)) that is used to generate the data in each one of the $B = 10,000$ Monte Carlo repetitions. Each cell in Panel B reports the estimated average length of the estimated bounds around the parameters described in the rows.

Our first result is that the $MTE(v)$ bounds are very wide for small values of v. This phenomenon suggests that our bounds may be uninformative in some empirical applications. It also explains why these bounds always contain the true treatment effect parameter in Panel A.

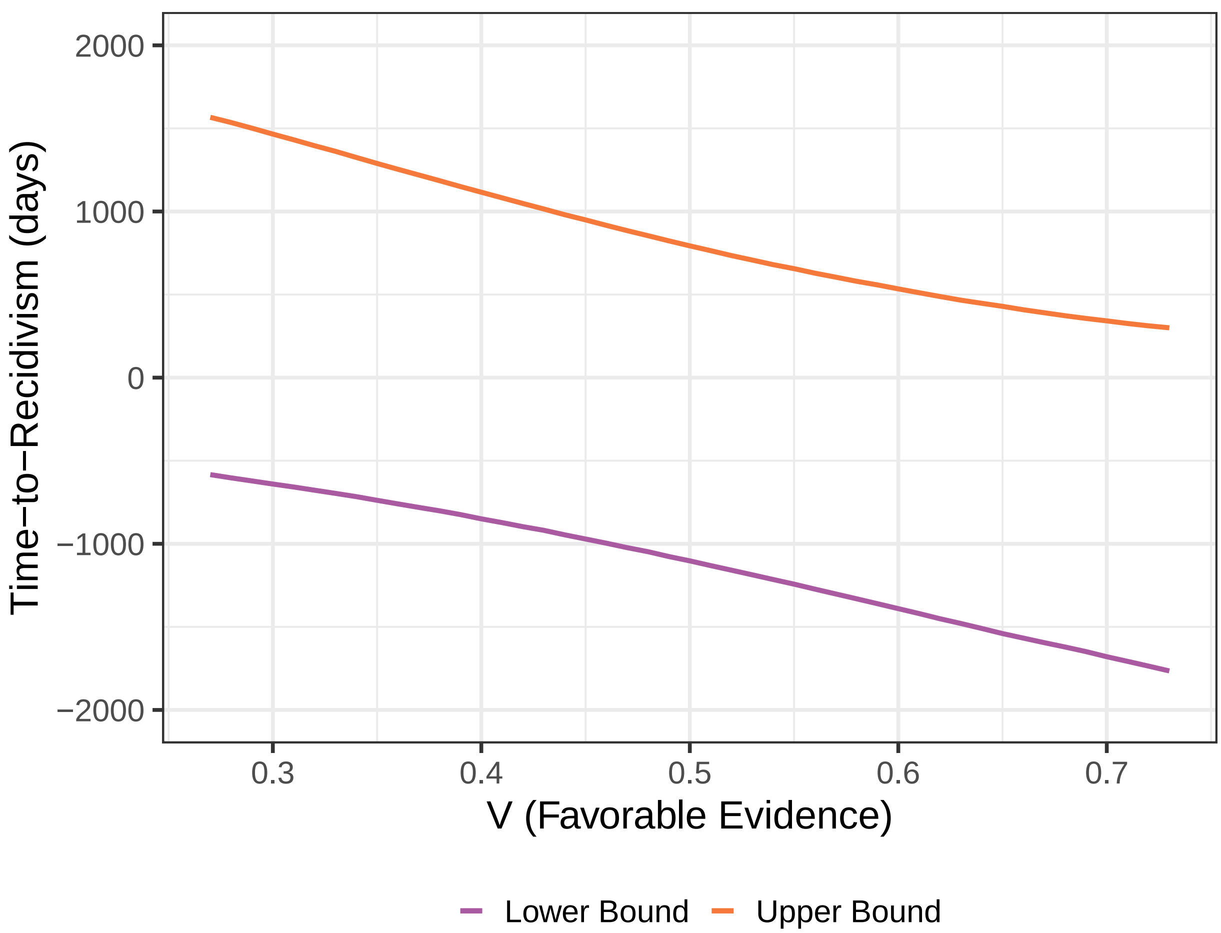
Our second result is that $MTE(v)$ bounds are very short for some values of v when $\alpha = -1$. These bounds' short width explain why they never contain the true treatment effect parameter in Panel A.

Our final result is that the ATE bounds are usually shorter than the MTE bounds, but still wide. For this reason, these bounds contain the true ATE with probability one in all scenarios as shown in Table C.1.

D Additional Empirical Results

We report, in Figure D.1, the semi-parametrically estimated bounds around the average MTE function. To compute them, we estimate bounds around the MTE function for each court district and, then, average across court districts using the proportion of cases per court district as weights. The orange line is the estimated upper bound while the purple line is the estimated lower bound. These results are based on Corollary 4.4 and Assumptions 1-4 and 6-8. We note that they are too wide to be informative.

Figure D.1: Bounds around the Average MTE of Punishment on Time-to-Recidivism — Corollary 4.4



Notes: The orange line is the estimated upper bound while the purple line is the estimated lower bound.

E Relevance of MTE for Duration Outcomes

In this appendix, we justify focusing on the marginal treatment effect (MTE) for duration outcomes using two arguments.

In Appendix E.1, we develop a theoretical model with a policymaker who selects a treatment assignment rule that minimizes the cost of recidivism for the target population of defendants.

In Appendix E.2, we provide a simple example where the treatment benefits most agents in our population. In this example, our proposed focus on quantile treatment effects for duration outcomes correctly highlights that this treatment is beneficial to society. However, focusing on short-time horizons as usually done in the crime economics literature leads to the opposite conclusion.

E.1 Theoretical Justification of Relevance of MTE for Duration Outcomes

Following Kitagawa and Tetenov (2018), the policymaker has to choose a treatment rule that determines whether individuals with variables $W = \{Z, V, C\}$ in our target population will be assigned to the treatment group or the control group. The policymaker chooses non-randomized treatment rules that are described by decision sets $G \subset \mathcal{W}$, where $\mathcal{W}$ is the support of W. These decision sets determine the group of individuals $\{W \in G\}$ to whom treatment is assigned. We denote the collection of candidate treatment rules by $\mathcal{G} = \{G \subset \mathcal{W}\}$.

The goal of the policymaker in our context is to select a treatment assignment rule that minimizes the cost of recidivism for the target population of defendants. Assuming that the policymaker discounts cost inter-temporally, she chooses the treatment rule that maximizes Y^* for each individual in the target population.

Specifically, we impose that the policymaker chooses the decision set $G \in \mathcal{G}$ that minimizes

$$K(G) := \mathbb{E}\left[\ln\left\{b^{[Y^*(1)\cdot \mathbf{1}\{W \in G\} + Y^*(0)\cdot \mathbf{1}\{W \notin G\}]} \cdot k\right\}\right]$$

where $k \in \mathbb{R}_{++}$ is the fixed cost of recidivism and $b \in (0, 1)$ is the policymaker's discount rate. Rearranging the last equation, we find that

$$\begin{aligned} K(G) &= \ln\{b\} \cdot \mathbb{E}\left[Y^*(1) \cdot \mathbf{1}\{W \in G\} + Y^*(0) \cdot \mathbf{1}\{W \notin G\}\right] + \ln\{k\} \\ &= \ln\{b\} \cdot \mathbb{E}\left[(Y^*(1) - Y^*(0)) \cdot \mathbf{1}\{W \in G\}\right] + \ln\{b\} \cdot \mathbb{E}\left[Y^*(0)\right] + \ln\{k\} \end{aligned}$$

Consequently, the policymaker's problem is equivalent to

$$\max_{G \in \mathcal{G}} \mathbb{E}\left[\left(Y^*(1) - Y^*(0)\right) \cdot \mathbf{1}\{W \in G\}\right].$$

Moreover, note that

$$\begin{aligned}
&\mathbb{E}\left[\left(Y^*(1) - Y^*(0)\right) \cdot \mathbf{1}\{W \in G\}\right] \\
&\quad = \mathbb{E}\left[\mathbb{E}\left[\left(Y^*(1) - Y^*(0)\right) \cdot \mathbf{1}\{W \in G\} \middle| V, Z, C\right]\right] \\
&\qquad \text{by the Law of Iterated Expectations} \\
&\quad = \mathbb{E}\left[\mathbb{E}\left[\left(Y^*(1) - Y^*(0)\right) \middle| V, Z, C\right] \cdot \mathbf{1}\{W \in G\}\right] \\
&\quad = \mathbb{E}\left[\mathbb{E}\left[\left(Y^*(1) - Y^*(0)\right) \middle| V, C\right] \cdot \mathbf{1}\{W \in G\}\right] \\
&\qquad \text{by Assumption 1} \\
&\quad = \mathbb{E}\left[\mathbb{E}\left[\left(Y^*(1) - Y^*(0)\right) \middle| V\right] \cdot \mathbf{1}\{W \in G\}\right] \\
&\qquad \text{by Assumption 5} \\
&\quad = \mathbb{E}\left[MTE(V) \cdot \mathbf{1}\{W \in G\}\right].
\end{aligned}$$

Therefore, the policymaker's problem is equivalent to

$$\max_{G \in \mathcal{G}} \mathbb{E}\left[MTE(V) \cdot \mathbf{1}\{W \in G\}\right],$$

implying that focusing on the MTE of duration outcomes is relevant when the policymaker wishes to minimize the cost of recidivism over time.

E.2 Illustrating the Relevance of Duration Outcomes

When analyzing the impact of judicial decisions on recidivism, many authors (Agan et al., 2023; Bhuller et al., 2019; Giles, 2021; Huttunen et al., 2020; Klaassen, 2021; Possebom, 2022) focus on a short time horizon, using a small set of outcome variables that indicate whether the defendant recidivated within a pre-specified number of years. In this paper, we advocate for moving beyond this short time horizon and focusing on quantile or average treatment effects of duration outcomes.

In this appendix, we illustrate why focusing on duration outcomes may provide more information than the standard approach in the empirical literature in crime economics. To do so, we abstract from the MTE heterogeneity (variable V) and focus exclusively on the heterogeneity arising from the distribution of the potential outcomes $(Y^*(0), Y^*(1))$.

We illustrate the relevance of quantile and average treatment effects of duration outcomes by analyzing a simple example with discrete random variables. In this example, focusing on short-term outcomes or long-term quantile treatment effects lead to different conclusions about our policy of interest.

We denote potential time-to-recidivism by $Y^*(0)$ and $Y^*(1)$ and measure it in years. Table E.1 shows the joint probability mass function of $(Y^*(0), Y^*(1))$ and their marginal distributions.

Table E.1: Joint Probability of $(Y^*(0), Y^*(1))$ and their Marginal Distributions

		$Y^*(0) =$					
	$\mathbb{P}[Y^*(0) = \cdot, Y^*(1) = \cdot]$	1	2	3	10	20	$\mathbb{P}[Y^*(1) = \cdot]$
$Y^*(1) =$	1	.10	0	.10	0	0	.20
	2	0	.10	.10	0	0	.20
	3	0	0	0	0	0	0
	10	0	0	0	.10	0	.10
	20	.05	.05	0	.40	0	.50
	$\mathbb{P}[Y^*(0) = \cdot]$	.15	.15	.20	.50	0	1

Note: The last column reports the marginal distribution of $Y^*(1)$. The last row reports the marginal distribution of $Y^*(0)$. The cells in the center of the table report the joint distribution of $(Y^*(0), Y^*(1))$.

Note that, in this example, our judicial decision benefits most defendants. For instance, this treatment strictly increases time-to-recidivism for 50% of the defendants $(Y^*(0) < Y^*(1))$. Moreover, only 20% of the defendants are harmed by this treatment $(Y^*(0) > Y^*(1))$.

However, a short-time horizon analysis would conclude that this treatment is harmful. For example, this treatment increases the probability of recidivism within one year by 5 p.p. and the probability of recidivism within two years by 10 p.p, i.e., $\mathbb{P}[Y^*(1) \leqslant 1] - \mathbb{P}[Y^*(0) \leqslant 1] = 0.05$ and $\mathbb{P}[Y^*(1) \leqslant 2] - \mathbb{P}[Y^*(0) \leqslant 2] = 0.1$.

Differently from the standard empirical analysis, we advocate for focusing on quantile and average treatment effects of duration outcomes. For example, the Quantile Treatment Effect on the Median is equal to seven years because the median of $Y^*(1)$ equal ten years and the median of $Y^*(0)$ equals three years. Moreover, the average treatment effect equals 5.55 years in this example.

Therefore, our proposed analysis would correctly highlight that this treatment benefits at least some agents in our society.

F Identification without Restrictions on Censoring

In this appendix, we focus on which parameters can be point-identified when we do not impose any restriction on the relationship between the censoring variable and the potential outcomes. To compensate for not imposing Assumption 5 nor Assumption 6, we need to allow the $DMTR$ function to depend on the censoring variable.

Specifically, our target parameter is given by:

$$DMTR_d\,(y, v, c) \coloneqq \mathbb{P}\left[Y^*(d) \leqslant y \middle| V = v, C = c\right]$$

for any $d \in \{0, 1\}$, $y < \gamma_C$, $v \in [0, 1]$ and $c \in \mathcal{C}$. Note that our target parameter is interpretable as a conditional distributional marginal treatment response. In particular, the censoring variable C acts similarly to a covariate in the standard MTE analysis (Carneiro, Heckman and Vytlacil, 2011).

In our empirical application, conditioning on the censoring variable is equivalent to conditioning on the defendant cohort or time fixed effects. Considering that most studies about judicial decisions (Agan et al., 2023; Bhuller et al., 2019; Huttunen et al., 2020; Klaassen, 2021) condition on district-by-time fixed effects, they identify the conditional $DMTR$ function for a pre-specified value of y. In this appendix, we discuss how to extend their analysis to consider conditional quantile marginal treatment effects and marginal treatment effects (Remark 1).

To point-identify the conditional $DMTR$ function, we eliminate Assumptions 5 and 6 and impose Assumptions 1-4 only.

Proposition F.1. *If Assumptions 1-4 hold, then*

$$DMTR_d\,(y, p, y + \delta) = (2 \cdot d - 1) \cdot \frac{\partial \mathbb{P}\left[Y \leqslant y, D = d \middle| P\,(Z, C) = p, C = y + \delta\right]}{\partial p}$$

for any $d \in \{0, 1\}$, $y < \gamma_C$, $p \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$.

Remark 1. A direct consequence of Proposition F.1 is the identification of the quantile marginal treatment response function $QMTR_d\,(\tau, p, y + \delta)$ conditional on the censoring variable for any $\tau \in [0, \overline{\tau}_d\,(p, y + \delta))$, where $\overline{\tau}_d\,(p, y + \delta) \coloneqq DMTR_d\,(\gamma_C, p, y + \delta)$. Additionally, if we impose Assumptions 7 and 8, then we straight-forwardly identify the MTE function conditional on the censoring variable.

Remark 2. The comparison between Propositions 4.1 and F.1 illustrate the identifying power of Assumption 5. It allows us to combine multiple values of the censoring variable to identify a single point of the $DMTR$ function through the integral of $\frac{\partial \mathbb{P}\left[Y \leqslant y, D=d \mid P(Z, C)=p, C=y\right.}{\partial p}$ over different values of δ. In our empirical application, it means that we can combine multiple defendant cohorts to identify a single evaluation point in the $DMTR$ function.

Proof. For brevity, we show the proof of Proposition F.1 when $d = 1$.

Fix $y < \gamma_C$, $p \in \mathcal{P}$ and $\delta \in \mathbb{R}_{++}$ such that $y + \delta \in \mathcal{C}$. Note that

$$\begin{aligned}
&\mathbb{P}\left[Y \leqslant y, D=1 \mid P(Z, C)=p, C=y+\delta\right] \\
&\quad=\mathbb{E}\left[\mathbf{1}\{Y \leqslant y\} \mathbf{1}\{P(Z, C) \geqslant V\} \mid P(Z, C)=p, C=y+\delta\right] \\
&\qquad \text{by Equation (1)} \\
&\quad=\mathbb{E}\left[\mathbf{1}\{Y^*(1) \leqslant y\} \mathbf{1}\{p \geqslant V\} \mid P(Z, y+\delta)=p, C=y+\delta\right] \\
&\qquad \text{because } Y_1^* \text{ is not censored when } C>y \\
&\quad=\int_0^1 \mathbb{E}\left[\mathbf{1}\{Y^*(1) \leqslant y\} \mathbf{1}\{p \geqslant v\} \mid P(Z, y+\delta)=p, C=y+\delta, V=v\right] dv \\
&\qquad \text{by the Law of Iterated Expectations and Assumption 3} \\
&\quad=\int_0^1 \mathbf{1}\{p \geqslant v\} \mathbb{E}\left[\mathbf{1}\{Y^*(1) \leqslant y\} \mid P(Z, y+\delta)=p, C=y+\delta, V=v\right] dv \\
&\quad=\int_0^p \mathbb{E}\left[\mathbf{1}\{Y^*(1) \leqslant y\} \mid P(Z, y+\delta)=p, C=y+\delta, V=v\right] dv \\
&\quad=\int_0^p \mathbb{P}\left[Y^*(1) \leqslant y \mid C=y+\delta, V=v\right] dv \\
&\qquad \text{by Assumption 1.}
\end{aligned}$$

Consequently, the Leibniz Integral Rule implies that

$$\begin{aligned}
\frac{\partial \mathbb{P}\left[Y \leqslant y, D=1 \mid P(Z, C)=p, C=y+\delta\right]}{\partial p} &= \mathbb{P}\left[Y^*(1) \leqslant y \mid C=y+\delta, V=p\right] \\
&= DMTR_1(y, p, y+\delta).
\end{aligned}$$

We can prove the same result for $d = 0$ analogously. ■

G Partial Identification under Median Independence

The researcher may believe that imposing that the censoring variable is fully independent of the potential outcomes (Assumption 5) is too strong in many empirical contexts. Alternatively, the researcher may prefer to impose the potential outcome is median independent from the censoring variable conditioning on the value of the latent cost of treatment.

Assumption G.1 (Median Independence). $Med[Y^*(d)|C = c, V = v] = Med[Y^*(d)|V = v]$

Under this assumption, we can identify the sign of the DMTE function for some values of the outcome variable. We discuss how to derive this result below.

From the first part of Lemma 4.1 and the Leibniz Integral Rule for any $C = y + \delta$, we can identify $M_d := Med[Y^*(d)|C = y + \delta, V = v]$ from

$$\frac{\partial \mathbb{P}\left[Y \leqslant M_1, D = 1 | \, P\left(Z, C\right) = p, C = y + \delta\right]}{\partial p} = \frac{1}{2} \tag{G.1}$$

and

$$\frac{\partial \mathbb{P}\left[Y \leqslant M_0, D = 0 | \, P\left(Z, C\right) = p, C = y + \delta\right]}{\partial p} = \frac{1}{2}. \tag{G.2}$$

Consequently, we can know if $M_1 > M_0$ or vice-versa.

Moreover, note then that, for $y \leqslant M_d$,

$$\mathbb{P}\left[Y^*(d) \leqslant y | \, V = v\right] \quad \leqslant \quad 1/2$$

and, for $y \geqslant M_d$,

$$\mathbb{P}\left[Y^*(d) \leqslant y | \, V = v\right] \quad \geqslant \quad 1/2.$$

Then, we have that

$$\mathbb{P}\left[Y^*(1) \leqslant y | \, V = v\right] - \mathbb{P}\left[Y^*(0) \leqslant y | \, V = v\right] \quad \in \quad (-1/2, 1/2) \tag{G.3}$$

if $M_1 > M_0$ and $y \leqslant M_0$,

$$\mathbb{P}\left[Y^*(1) \leqslant y | \, V = v\right] - \mathbb{P}\left[Y^*(0) \leqslant y | \, V = v\right] \quad \in \quad (-1, 0) \tag{G.4}$$

if $M_1 > M_0$ and $M_0 \leqslant y \leqslant M_1$, and

$$\mathbb{P}\left[Y^*(1) \leqslant y \middle| V = v\right] - \mathbb{P}\left[Y^*(0) \leqslant y \middle| V = v\right] \in (-1/2, 1/2) \tag{G.5}$$

if $M_1 > M_0$ and $y \geqslant M_1$.

Similarly, we have that:

$$\mathbb{P}\left[Y^*(1) \leqslant y \middle| V = v\right] - \mathbb{P}\left[Y^*(0) \leqslant y \middle| V = v\right] \in (-1/2, 1/2) \tag{G.6}$$

if $M_0 > M_1$ and $y \leqslant M_1$,

$$\mathbb{P}\left[Y^*(1) \leqslant y \middle| V = v\right] - \mathbb{P}\left[Y^*(0) \leqslant y \middle| V = v\right] \in (0, 1) \tag{G.7}$$

if $M_0 > M_1$ and $M_1 \leqslant y \leqslant M_0$, and

$$\mathbb{P}\left[Y^*(1) \leqslant y \middle| V = v\right] - \mathbb{P}\left[Y^*(0) \leqslant y \middle| V = v\right] \in (-1/2, 1/2) \tag{G.8}$$

if $M_0 > M_1$ and $y \geqslant M_0$,

Thus, the sign of the DMTE function is identified between M_1 and M_0.